APPLIED ELECTRONICS

(*With Solved Examples*)

Vol. I — ELECTRON DYNAMICS AN
ELECTRON TUBES
Vol. II — ELECTRON TUBE CIRCUIT

BY

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Radhakrishan D. Jetwani,

Dedicated to my grandmother
Mrs. Vishambhari Devi

G.K. MITHAL.

PREFACE

The aim of the author in presenting this book is to help the students in grasping the fundamentals of Electronics. A large [illegible] with different aspects [illegible] effort, however, has [illegible]bject in a simple but comprehensive form dealing with the fundamental aspects and assisted by solved examples and exercises wherever possible. It is hoped that the book will prove useful to students preparing for degree and associate membership examinations of various Engineering Institutions. Volume one deals with general theory of Electron Dynamics and Electron Tubes. In addition to the thermionic high-vacuum tubes which are most extensively used in Electronic devices, the book deals with other types of Electron tubes as well, such as photo-electric tubes and mercury arc rectifier tubes.

Volume two deals with general principles of Electron Tube circuits. This includes studies of Rectifiers and power-supplies, Electronic Voltage Stabilizers, Voltage and Power amplifiers of untuned and tuned types, and vacuum-tube oscillators.

The author will be glad to receive comments and suggestions for improving the book in future.

G. K. MITHAL

Jabalpur,
December, 1963

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Group C : Bridge Rectifier Circuits.

Group D : Polyphase Rectifiers ;

Group E : Controlled Rectifiers ;

(i) Thyratron Rectifiers ;

(ii) *Ignitron Rectifiers* ;

(iii) Cold cathode gas triode Rectifiers ;

(iv) Excitron Rectifiers.

VACUUM TUBE RECTIFIER

Full Wave Vacuum Tube Rectifier. Fig. 8.3 gives the [illegible] followed [illegible] using two [illegible] primary winding of a [illegible] secondary side there are usually three windings :

(i) *High voltage centre-tapped winding of predetermined rating*; such as : 250-0-250 volts ; 300-0-300 volts ; 350-0-350 volts etc. The voltage rating 350-0-350 volts means that the a.c. voltage between the centre tap and each end terminal is 350 volts r.m.s.

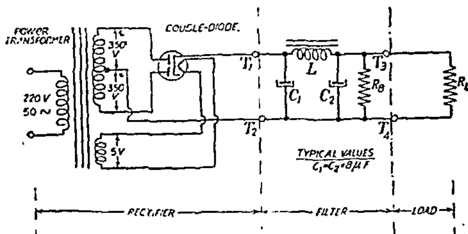


Fig. 8.3 —*Circuit diagram of a full wave vacuum tube rectifier using a power transformer and a double-diode.*

(ii) 5 volts filament winding usually required to supply heating power to the filament of the rectifier tube itself (as in the case of rectifier tube 5Y3)

(*iii*) 6·3 volts filament winding usually required to supply heating power to the filaments of all other tubes.

Usually a colour code is used to distinguish between the terminal wires or leads of different windings.

Each diode section of double-diode tube conducts during half a.c. cycle and the voltage developed across the load resistance in the absence of a filter is shown in Fig. 8.4. During one half cycle of

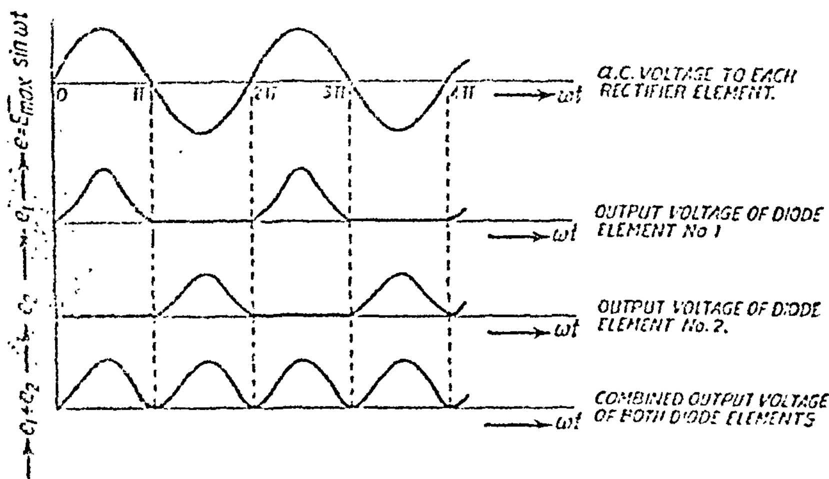


Fig. 8.4.—Waveforms of output voltage of full wave vacuum tube rectifier.

a.c. voltage one diode section conducts and other does not conduct. The output voltage supplied by this conducting diode is shown as e_1 in Fig. 8.4. During the other half period of a.c. voltage waveform, the other diode section conducts and provides the output voltage e_2 shown in Fig. 8.4. The combined output voltage (e_1+e_2) is also shown in the figure and this voltage is a unidirectional voltage of pulsating nature. In order to obtain an almost constant output voltage, a filter circuit is connected between the rectifier and the load resistance R_l as shown in the figure.

The pulsating output voltage of a rectifier consists of a d.c. component and alternating voltage components. The a.c. components of rectified voltage constitute the "ripple voltage". This ripple voltage is not a pure sinusoidal voltage but consists of a fundamental sine wave component and all harmonics thereof in progressively reducing amplitudes. In the case of a full wave rectifier, the fundamental ripple frequency is twice the supply voltage frequency.

Ripple Factor. It is defined as the ratio of the r.m.s. value of the ripple voltage to the algebraic average value of the total rectified voltage.

It is the function of the filter circuit to reduce the ripple factor to as small a value as possible by removing the ripple voltage. The efficiency of a filter in removing the ripple voltage is judged by comparing the ripple factors at the input and output of the filter circuit.

For most of the applications, notably in radio receivers, the filter used is a π-filter shown in Fig. 8.3. There is used a high inductance audio choke in the series arm and there are two electrolytic condensers C_1 and C_2 in the shunt arms, one on either side of the choke. The usual values are indicated in the figure. The choke offers negligible impedance to the flow of d.c. component of output current. The only impedance offered by the choke is due to small resistance component of choke impedance. Thus the d.c. output voltage is developed across the load resistance R_l without appreciable reduction in magnitude. On the other hand, high impedance is offered by the choke to the flow of ripple frequency current. Shunt condensers offer paths of very small impedance to ripple frequency components of output current. Hence the ripple components get bypassed through these condensers. The voltage developed across the load resistance is then constant d.c. voltage with very small ripple voltage superimposed on it.

Half-wave vacuum tube rectifier. Fig. 8.5 gives the circuit diagram of a simple half-wave rectifier. This includes the power transformer and the filter circuit also.

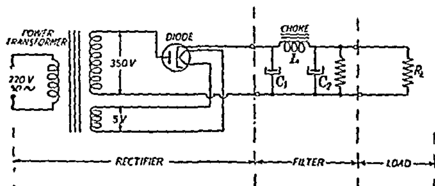
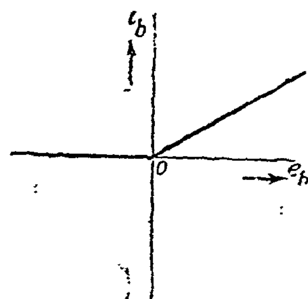


Fig. 8.5.—Circuit diagram of a vacuum tube half-wave rectifier with π-filter

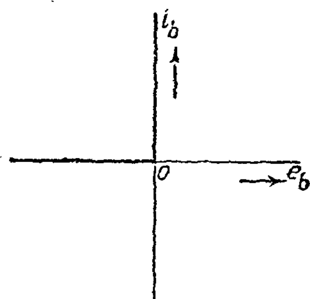
Half wave rectifier conducts only during half-cycle of the input a.c. voltage. The ripple frequency is now the same as the supply voltage frequency. This being the case, a more elaborate filter circuit is required in this case as compared with the full wave rectifier. The waveforms of the output current [illegible] filter are given in Fig. 8.2.

Analysis of a half-wave vacuum tube rectifier with resistance load.

Graphical method of analysis may be used for a rectifier but this method is very cumbersome. Hence for most purposes, an approximate method of analysis is used. Again, the analysis is restricted to the case of a resistance load and no filter circuit. Another assumption made is that the current-voltage characteristic of the vacuum tube is a straight line through the origin. This idealized current-voltage characteristic is shown in Fig. 8.6 (a). The current voltage characteristic of an ideal rectifier is shown in Fig. 8.6 (b). An ideal rectifier does not conduct at all when negative voltage is applied to anode; this is indicated by the horizontal portion of the characteristic to the left of zero. When anode is made positive, the voltage drop in the ideal rectifier tube is zero and the characteristic is along the current axis. An ideal rectifier is indicated by the symbol shown in Fig. 8.6 (c). Idealized vacuum tube rectifier characteristic differs from that of an ideal rectifier in that it has a definite constant slope in the conduction region. If the slope is equal to $1/R_a$, then the vacuum tube rectifier may be represented as an ideal rectifier with a series resistance equal to R_a as shown in Fig. 8.6 (d).



(a) Idealized characteristic of a vacuum tube rectifier.



(b) Characteristic of an ideal rectifier.



(c) Symbol for ideal rectifier.



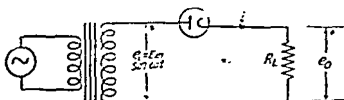
(d) Symbol for an idealized vacuum tube rectifier.

Fig. 8.6.—Characteristics and symbols of ideal rectifier and idealized vacuum tube rectifier.

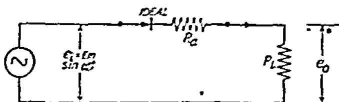
Fig. 8.7 (a) shows the circuit diagram of vacuum tube half-wave rectifier with resistance load. Filter circuit has been eliminated.

Fig. 8.7 (b) shows the idealized circuit. Connections for heater wire etc. have been omitted for clarity. R_l is the load resistance. Sinusoidal voltage is applied at the input and is given by :

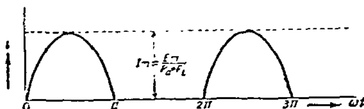
$$e_i = E_m \sin \omega t \quad \text{... (8·1)}$$



(a) Circuit diagram.



(b) Idealized circuit diagram.



(c) Waveform of current.

Fig. 8.7—Circuit diagram, idealized circuit diagram and current waveform of half-wave vacuum tube rectifier with resistance load.

The total impedance in the circuit is equal to $(R_a + R_l)$ and is a pure resistance. Hence the current will be of the same waveform as the applied voltage except that no current will flow during negative half cycle. The maximum value of current is given by :

$$I_m = \frac{E_m}{R_a + R_l} \quad \text{... (8·2)}$$

and the output voltage e_o is given by :

$$e_o = i \cdot R_l \quad \text{... (8·3)}$$

Waveform of e_o is the same as that of current i or applied voltage e_i.

Average value or d.c. value of current i. This is given

$$I_{dc}=\frac{1}{2\pi}\int_o^{2\pi} i\,.\,d\,(\omega t) \qquad \ldots (8\cdot4)$$

$$=\frac{1}{2\pi}\int_o^{\pi} I_m\,.\,\sin\,\omega t\;d(\omega t)+\frac{1}{2\pi}\int_{\pi}^{2\pi} 0.d(\omega t)$$

$$=\frac{I_m}{2\pi}\,(-\cos\,\omega t)_o^{\pi}=\frac{I_m}{\pi} \qquad \ldots (8\cdot5)$$

Substituting the value of I_m :

$$I_{dc}=\frac{E_m}{\pi(R_a+R_l)} \qquad \ldots (8\cdot6)$$

R.M.S. value of current. It is given by :

$$I=\sqrt{\frac{1}{2\pi}\int_o^{2\pi} i^2\,.\,d(\omega t)}=\sqrt{\frac{I_m^2}{2\pi}\int_o^{\pi}\sin^2\omega t\;d(\omega t)} \qquad \ldots (8\cdot7)$$

$$=I_m\sqrt{\frac{1}{4\pi}\int_o^{\pi}(1-\cos\,2\omega t).d(\omega t)}$$

or $$I=\frac{I_m}{2}=\frac{E_m}{2(R_a+R_l)} \qquad \ldots (8\cdot8)$$

Thus the r.m.s. value is larger than the d.c. value. The maximum, average and r.m.s. values of voltage across the load are given by :

Maximum voltage across the load :

$$E_{max}=I_m\;R_l=\frac{E_m}{R_a+R_l}\,.\,R_l=\frac{E_m}{(1+R_a/R_l)} \qquad \ldots (8\cdot9)$$

Average or d.c. voltage across the load :

$$E_{dc}=I_{dc}\,.\,R_l=\frac{E_m}{\pi\left(1+\frac{R_a}{R_l}\right)} \qquad \ldots (8\cdot10)$$

and r.m.s. value of voltage across the load :

$$E=I\,.\,R_l=\frac{E_m}{2\left(1+\frac{R_a}{R_l}\right)} \qquad \ldots (8\cdot11)$$

It is obvious from equation (8·10) that with the change of load resistance R_l, the d.c. value of rectified output voltage changes considerably, *i.e.*, the regulation is poor, unless the load resistance R_l is large compared with the tube resistance R_a.

Frequency components of rectified output. Fourier analysis of the current waveshape of a half-wave rectifier gives the following frequency components :

$$i=I_m\left(\frac{1}{\pi}+\frac{1}{2}\,\sin\;\omega t-\frac{2}{3\pi}\,\cos\,2\,\omega t-\frac{2}{15\pi}\,\cos\,4\,\omega t-\ldots\right) \qquad \ldots (8\cdot12)$$

assuming that the tube conducts during first half of the a c. cycle so that equation (8 1) applies. The first term of the series gives the (direct or the average component and is the same as given by equation (8·6). The second term has its frequency same as the supply frequency and its peak value is $I_m/2$. The effective or r.m.s. value I_1 of this fundamental frequency component is given by :

$$I_1=\frac{\text{Peak value}}{\sqrt{2}}=\frac{I_m/2}{\sqrt{2}}=\frac{I_m}{2\sqrt{2}} \quad \text{... (8·13)}$$

The third term gives the second harmonic term. Its r.m.s. value I_2 is given by :

$$\sqrt{2}\ I_2=\frac{2}{3\pi}I_m \text{ or } I_2=\frac{\sqrt{2}\ I_m}{3\pi} \quad \text{... (8·14)}$$

Similarly the r m s values of the subsequent terms may be calculated. They are found to be of continuously diminishing values.

Rectifier Efficiency. In a rectifier the useful power output is the d.c. power P_{dc} developed across the load and is given by :

$$P_{dc}=I_{dc}^2\times R_l=\frac{I_m^2\times R_l}{\pi^2}=\frac{E_m^2}{(R_a+R_l)^2}\cdot\frac{R_l^2}{\pi^2} \quad \text{... (8·15)}$$

Out of the total power required from the a.c. voltage source, a portion P_p is dissipated at the plate of the rectifier tube and the rest of it P_r is dissipated in the load resistance R_l. Since the rectifier itself is assumed to be ideal, the dissipation is assumed to take place in the resistance R_a. Then we get ·

$$P_p=I^2\times R_a=\frac{E_m^2\cdot R_a}{4(R_a+R_l)^2} \quad \text{... (8·16)}$$

Or $$P_r=I^2\times R_l=\frac{E_m^2\cdot R_l}{4(R_a+R_l)^2} \quad \text{... (8·17)}$$

Hence total input power $P_{in}=P_p+P_r$

$$=\frac{E_m^2}{4(R_a+R_l)^2}\times(R_a+R_l)$$

$$=\frac{E_m^2}{4(R_a+R_l)} \quad \text{... (8·18)}$$

Efficiency of the rectifier is defined as the ratio of the d.c. output power to a.c. input power. Hence the efficiency η is given by :

$$\eta=\frac{P_{dc}}{P_{in}}=\frac{E_m^2\cdot R_l}{\pi^2(R_a+R_l)^2}\Big/\frac{E_m^2}{4(R_a+R_l)}$$

$$=\left(\frac{2}{\pi}\right)^2\cdot\frac{R_l}{R_a+R_l}=\frac{0{\cdot}406}{1+\frac{R_a}{R_l}}$$

$$=\frac{40{\cdot}6}{1+\frac{R_a}{R_l}}\text{ per cent} \quad \text{... (8·19)}$$

The rectifier efficiency increases as the ratio R_a/R_l reduces. Theoretically the maximum value of rectifier efficiency is, therefore, 40·6 per cent corresponding to the value of R_a/R_l equal to zero.

Ripple Factor. The ripple factor is defined as the ratio of the effective value of the a.c. components of voltage or current to the direct or average value of the voltage or current.

Thus the ripple factor gives an idea about the waviness of the rectified voltage. For a wave consisting of a fundamental and a number of harmonic terms, the effective value is given by the square of the effective values of these components. Thus,

$$I = \sqrt{I_{dc}^2+I_1^2+I_2^2+I_3^2 \dots} = \sqrt{I_{dc}^2+I_{ac}^2} \qquad \dots (8\cdot20)$$

where I_1, I_2, etc. are the r.m.s. values of the fundamental, second harmonic, etc. terms,

and I_{ac}^2 is the sum of the squares of r.m.s. values of a.c. components.

Then the ripple factor γ is given by :

$$\gamma = \frac{I_{ac}}{I_{dc}} = \frac{\sqrt{I^2-I_{dc}^2}}{\sqrt{I_{dc}^2}} = \sqrt{\frac{I^2}{I_{dc}}-1} \qquad \dots (8\cdot21)$$

For a half-wave rectifier :

$$I = \frac{I_m}{2} \text{ and } I_{dc} = \frac{I_m}{\pi}$$

Hence form factor

$$F = \frac{I}{I_{dc}} = \frac{\pi}{2} = 1\cdot57$$

and ripple factor

$$\gamma = \sqrt{\left(\frac{I}{I_{dc}}\right)^2 - 1} = \sqrt{(1\cdot57)^2-1} = 1\cdot21 \qquad \dots (8\cdot22)$$

Analysis of full wave vacuum tube rectifier. Fig. 8.8 (*a*) shows the circuit diagram of full wave rectifier without filter and with resistance load. Idealized circuit is shown in Fig. 8.8 (*b*) where E_1 and E_2 are two voltages of equal amplitude and opposite phase.

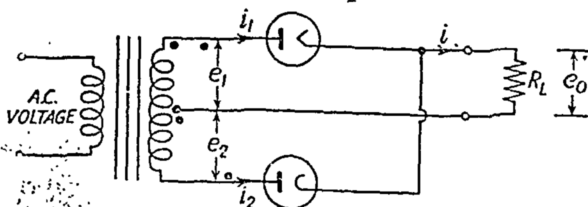
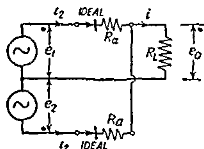


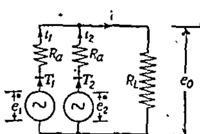
Fig. 8.8.—(*a*) Circuit diagram of full wave vacuum tube rectifier with resistance load but without filter.

The circuit is redrawn in Fig. 8.8 (c). The total current flowing through the load resistance is the sum of the individual currents i_1 and i_2.

Hence, $$i = i_1 + i_2 \qquad \text{... (8·23)}$$



(b) Idealized circuit.



(c) Rearranged form of (b).

Fig. 8.8.—Circuit diagrams of full wave vacuum tube rectifier without filter.

Fig. 8.9 shows the waveforms of input a.c. voltages and currents i_1, i_2 and i. The maximum value of current is given by :

$$I_m = \frac{E_m}{R_a + R_l} \qquad \text{... (8·24)}$$

where E_m is the maximum value of the voltage e_1 or e_2.

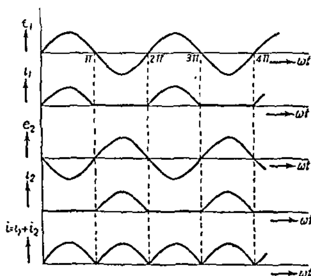


Fig. 8.9.—Waveforms of applied a.c. voltages and output current in a full wave vacuum tube rectifier without filter.

D.C. or average value of current. Since current is of the same form in the two halves,

$$I_{dc} = \frac{1}{\pi}\int_0^\pi i.\,d(\omega t) = \frac{1}{\pi}\int_0^\pi I_m \sin \omega t.\, d(\omega t)$$

$$= \frac{2I_m}{\pi} \quad \ldots (8\cdot25)$$

R.M.S. value of current. This is given by :

$$I = \sqrt{\frac{1}{\pi}\int_0^\pi i^2.d(\omega t)} = \sqrt{\frac{I_m^2}{\pi}\int_0^\pi \sin^2 \omega t\, d(\omega t)}$$

$$= I_m \sqrt{\frac{1}{2\pi}\int_0^\pi (1-\cos 2\,\omega t)\, d(\omega t)}$$

$$= \frac{I_m}{\sqrt{2}} \quad \ldots (8\cdot26)$$

ximum voltage across the load is given by :

$$E_{max} = I_m.R_l = \frac{E_m}{1+\left(\frac{R_a}{R_l}\right)} \quad \ldots (8\cdot27)$$

Average or d.c. voltage across the load is given by :

$$E_{dc} = I_{dc}.R_l = \frac{E_m}{1+\left(\frac{R_a}{R_l}\right)} \times \frac{2}{\pi} \quad \ldots (8\cdot28)$$

R.M.S. value of voltage across the load is given by :

$$E = I \times R_l = \frac{E_m}{1+\left(\frac{R_a}{R_l}\right)} \times \frac{1}{\sqrt{2}} \quad \ldots (8\cdot29)$$

Frequency components of rectifier output. Fourier anlysis of the waveform of current i_1 yields :

$$i_1 = I_m \left[\frac{1}{\pi} + \tfrac{1}{2} \sin \omega t - \frac{2}{3\pi} \cos 2\,\omega t - \frac{2}{15\pi} \cos 4\,\omega t - \ldots \right) \quad \ldots (8\cdot12)$$

i_2 is 180 degrees out of phase with i_1 and hence is given by :

$$i_2 = I_m \left[\frac{1}{\pi} - \frac{1}{2} \sin \omega t - \frac{2}{3\pi} \cos 2\omega t - \frac{2}{15\pi} \cos 4\omega t \ldots \right) \quad \ldots (8\cdot30)$$

The total current i is equal to the sum of i_1 and i_2 so that,

$$i = I_m \left[\frac{2}{\pi} - \frac{4}{3\pi} \cos 2\omega t - \frac{4}{15\pi} \cos 4\,\omega t - \ldots \right) \quad \ldots (8\cdot31)$$

It can be seen from equation (8·31) that the lowest frequency term in the rectified output of a full wave rectifier has an angular frequency of 2ω. This is twice the lowest frequency in the ripple voltage of a half-wave rectifier. Hence it is comparatively easy to filter the ripple voltage components from the rectified output voltage.

Full wave rectifier efficiency. D.C. power output is given by:

$$P_{dc} = I_{dc}^2 . R_l = \left(\frac{2}{\pi}\right)^2 \times \frac{E_m^2 . R_l}{(R_a+R_l)^2} \qquad \ldots (8{\cdot}32)$$

This power is four times the power obtained in a half-wave rectifier provided the peak voltage E_m is the same in each case.

A.C. power input to the rectifier is given by :

$$P_{in} = I^2 (R_a+R_l) = \frac{E_m^2}{2(R_a+R_l)} \qquad \ldots (8{\cdot}33)$$

Hence rectifier efficiency is given by :

$$\eta = \frac{P_{dc}}{P_{in}} = 2\left(\frac{2}{\pi}\right)^2 \times \frac{1}{1+\left(\frac{R_a}{R_l}\right)} = \frac{81{\cdot}2}{1+\left(\frac{R_a}{R_l}\right)} \text{ per cent} \qquad \ldots (8{\cdot}34)$$

Equation (8·34) shows that the rectifier efficiency of a full wave rectifier is twice that of a half-wave rectifier under identical circumstances.

Ripple Factor. Form factor of the rectified output voltage of a full wave rectifier is given by .

$$F = \frac{I}{I_{dc}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11$$

Hence ripple factor is given by :

$$\gamma = \sqrt{(1{\cdot}11)^2 - 1} = 0{\cdot}48 \qquad \ldots (8{\cdot}35)$$

This ripple factor of 0·48 is small compared with 1·21 for a half-wave rectifier.

Comparison of halfwave and full wave rectifiers. The following points are of significance in this connection :

(*i*) In a half-wave rectifier, current flows through the secondary of the power transformer in the same direction always, whereas in a full wave rectifier equal currents flow through the two halves of the centre-tapped secondary of the power transformer in opposite directions. D.C. saturation of the core of the transformer is thus avoided. The saturation of the core increases the magnetising current and hysteresis losses, and produces harmonics in the secondary output.

(*ii*) Half-wave rectifier has the chief advantages of simplicity and low cost compared with a full wave rectifier.

(*iii*) Half-wave rectifier gives high ripple amplitude and low ripple frequency which necessitate the use of comparatively expensive smoothing filter.

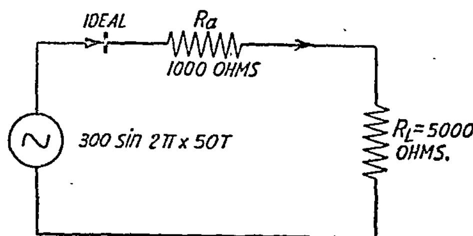
(*iv*) Half-wave rectifier has a low output voltage and low transformer efficiency. Low transformer efficiency results because (*a*) the ripple voltage and currents are high and (*b*) the current flows through the secondary in the same direction and saturates the core.

(v) Efficiency of full wave rectifier is twice that of a half-wave rectifier. Because of its superior performance, a single phase full wave rectifier is most commonly used for developing d.c. voltages not exceeding about 1000 volts and currents not exceeding about one ampere.

SOLVED EXAMPLES

Example 1. *A voltage 300 sin $2\pi \times 50t$ is applied to a half-wave vacuum tube rectifier without filter and a resistance load of 5000 ohms. Rectifier tube may be represented by an ideal rectifier in series with a resistance of 1000 ohms. Calculate (i) peak, average and r.m.s. values of current ; (ii) d.c. power output ; (iii) a.c. power input ; (iv) rectifier efficiency and (v) ripple factor.*

Solution :



Maximum value of current is given by :

$$I_m = \frac{E_m}{R_a + R_l} = \frac{300}{1000 + 5000} = \mathbf{50 \times 10^{-3} \text{ amp.}}$$

D.C. component of current :

$$I_{dc} = I_m/\pi = \frac{50 \times 10^{-3}}{\pi} \text{ amp} = \mathbf{15{\cdot}92 \text{ mA.}}$$

R.M.S. value of current :

$$I = \frac{I_m}{2} = \frac{50}{2} = \mathbf{25 \text{ mA.}}$$

D.C. power output :

$$P_{dc} = I_{dc}^2 \times R_l = (15{\cdot}92)^2 \times 5000 \times 10^{-6} \text{ watt} = \mathbf{1{\cdot}267 \text{ watts.}}$$

A.C. input power :

$$P^2_{in} = I^2(R_a + R_l) = (25 \times 10^{-3})^2 \times (1000 + 5000)$$

$$= \mathbf{3{\cdot}75 \text{ watts.}}$$

Efficiency of rectifier :

$$\eta = \frac{P_{dc}}{} = \frac{1{\cdot}267}{3{\cdot}75} = \mathbf{33{\cdot}79 \text{ per cent.}}$$

Ripple factor :

$$\gamma=\frac{I_{ac}}{I_{dc}}=\frac{\sqrt{I^2-I_{dc}^2}}{\sqrt{I_{dc}^2}}=\sqrt{\left(\frac{I_m/2}{I_m/\pi}\right)^2-1}$$

$$=\sqrt{\left(\frac{\pi}{2}\right)^2-1}=\sqrt{(1.57)^2-1}=1.21.$$

Example 2. *In a full wave vacuum tube rectifier without filter the load resistance is of 4000 ohms. Each diode has idealized characteristic having slope corresponding to a resistance of 800 ohms. Voltage applied to each diode is 240 sin $2\pi\times50t$. Calculate (i) peak average and r.m.s. values of current ; (ii) d.c. power output and total power input ; (iii) rectifier efficiency ; (iv) form factor and (v) ripple factor.*

Solution : (*i*) Maximum value of current :

$$I_m=\frac{E_m}{R_a+R_l}=\frac{240}{800+4000}=50\times10^{-3}\text{ amp}=\mathbf{50\ mA.}$$

Average value of current :

$$I_{dc}=\frac{I_m}{\pi/2}=\frac{50\times2}{3.14}=\mathbf{31.84\ mA.}$$

R.M.S. value of current :

$$I=\frac{I_m}{\sqrt2}=\frac{50}{1.414}=\mathbf{35.36\ mA.}$$

(*ii*) *D.C. power output :*

$$P_{dc}=I_{dc}^2\times R_l=(31.84\times10^{-3})^2\times4000=\mathbf{4.056\ watts.}$$

Total power input :

$$P_{in}=I^2(R_a+R_l)=(35.36\times10^{-3})^2(4800)\text{ watts}$$

$$=\mathbf{6.001\ watts.}$$

(*iii*) Rectifier efficiency .

$$\eta=\frac{P_{dc}}{P_{in}}=\frac{4.056}{6.001}\times100\text{ per cent}=\mathbf{67.59\ per\ cent}$$

(*iv*) Form factor$=\dfrac{I}{I_{dc}}=\dfrac{35.36}{31.84}=\mathbf{1.11}$

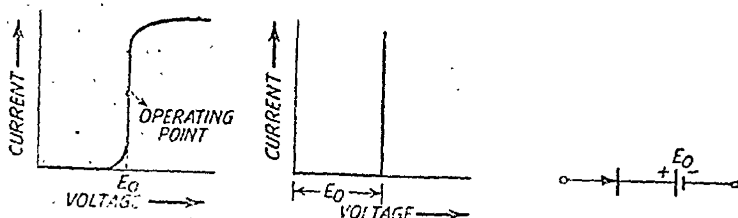
(*v*) Ripple factor :

$$\gamma=\sqrt{\left(\frac{I}{I_{dc}}\right)^2-1}=\sqrt{(1.11)^2-1}=0.48.$$

HOT CATHODE GAS DIODE RECTIFIERS

Fig. 8.10 (*a*) shows the current voltage characteristic of a hot-cathode gas diode. Gas diode conducts only when ionisation potential E_i has been reached. This ionisation voltage E_i is about

12 volts for most of the gas diodes and is the constant voltage drop across the tube when it conducts. Current during conduction is restricted to be within permissible limits by the insertion of resistance in series with the tube. Once the applied voltage falls below E_o, conduction ceases. Fig. 8.10 (*b*) shows the idealized characteristic of a hot cathode gas diode while the symbol used for this diode is shown in Fig. 8.10 (*c*).

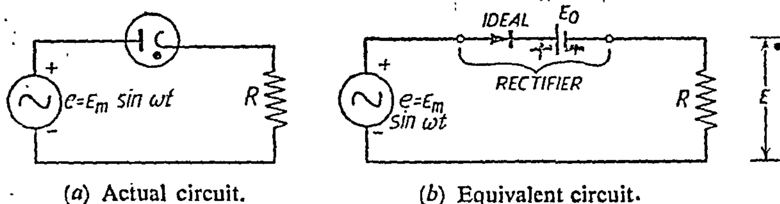


(*a*) Actual characteristic. (*b*) Idealized characteristic. (*c*) Symbol.

Fig. 8.10.—Current-voltage characteristics and Symbol of a hot cathode gas diode.

Half-wave Rectifier using Hot Cathode Gas Diode.

Fig. 8.11 (*a*) shows the circuit diagram of a half-wave rectifier using hot cathode gas diode, and a resistance load. The equivalent circuit is shown in Fig. 8.11 (*b*).



(*a*) Actual circuit. (*b*) Equivalent circuit.

Fig. 8.11.—Half-wave rectifier circuit using hot 7½″ cathode gas diode.

Fig. 8.12 shows the waveforms of the applied alternating voltage and current i through the load resistance. The peak value of load current I_m is given by :

$$I_m = \frac{E_m - E_o}{R} \quad \dots (8{\cdot}35)$$

Let $I_{mo} = E_m/R$. ... (8·36)

Then I_{mo} is the peak current obtained when $E_o = 0$. Obviously I_m is less than I_{mo}. Further the conduction period θ_c is less than 180 degrees. Thus presence of a constant rectifier drop E_o has two effects : (*i*) reduction of peak current and (*ii*) reduction of angle of conduction.

Let $\alpha = E_o/E_m$. ... (8·37)

Then as α increases, θc and I_m reduce.

The current through the rectifier at any time during conduction period is given by :

$$i=\frac{e-E_o}{R}=\frac{E_m \sin \omega t-E_o}{R} \quad \text{... (8·38)}$$

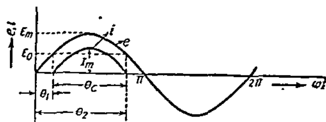


Fig. 8.12.—Waveforms of applied voltage and output current.

Thus during positive half cycle,

$$i=\frac{E_m}{R}\ (\sin \omega t-\alpha) \quad \text{when } e \geqslant E_o \quad \text{... (8·3}$$

and during the negative half cycle,

$$i=0 \quad \text{when } e \leqslant 0 \quad \text{... (8·40)}$$

angle θ_1 is given by the relation,

$$\sin\ \theta_1=E_o/E_m=\alpha$$

or $\theta_1=\sin^{-1} \alpha$ ... (8·41)

By symmetry, $\theta_2=\pi-\theta_1$ ... (8·42)

and angle of conduction $\theta_c=\theta_2-\theta_1$

$$=\pi-2\theta_1=\pi-2 \sin^{-1} \alpha \quad \text{... (8·43)}$$

The d.c. component of current is given by :

$$I_{dc}=\frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_1} i\ d\ (\omega t)$$

$$=\frac{E_m}{2\pi R}\int_{\theta_1}^{\pi-\theta_1}\left[\ (\sin \omega t)-\alpha\ \right] d\omega t \quad \text{...(8·44)}$$

$$=-\frac{E_m}{2\pi R}\Big[\ \cos \omega t+\alpha.\omega t\ \Big]_{\sin^{-1}\alpha}^{\pi-\sin^{-1}\alpha}$$

or $$I_{dc}=\frac{E_m}{\pi R}\left(\sqrt{1-\alpha^2}-\alpha \cos^{-1} \alpha\right) \quad \text{... (8·45)}$$

The d.c. component of voltage across the load resistance R is given by :

$$E_{dc}=I_{dc}\ .\ R=\frac{E_m}{\pi}\left(\sqrt{1-\alpha^2}-\alpha \cos^{-1}\alpha\right) \quad \text{... (8·46)}$$

Study of Eq. (8·46) shows that the ratio E_{dc}/E_m decreases with the increase of α. Fig. 8.13 shows the nature of variation of E_{dc}/E_m with α. Hence α is required to be kept small in all practical rectifiers.

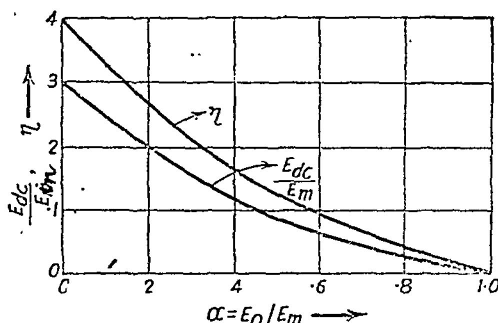


Fig. 8.13.—Variation of E_{dc}/E_m and efficiency with α.

The r.m.s. value of the current is given by :

$$I = \frac{E_m}{R}\sqrt{\frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_1}\left[\left(\sin\omega t\right)-\alpha\right]^2 d(\omega t)} \quad \dots (8·47)$$

$$\text{or } I = \frac{E_m}{R}\sqrt{\frac{1}{2\pi}[(1+2\alpha^2)\cos^{-1}\alpha - 3\alpha\sqrt{1-\alpha^2}]} \quad \dots (8·48)$$

The d.c. power output $P_{dc} = I^2_{dc} \cdot R$ ... (8·49)

Power dissipated in the tube is given by :

$$P_p = \frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_1} E_o\, i\, d(\omega t) = E_o \cdot I_{dc} \quad \dots (8·50)$$

Power input, $P_{in} = I^2.R + P_p = I^2.R + I_{dc}.E_o$

Hence rectifier efficiency is given by :

$$\eta = \frac{P_{dc}}{P_{in}} = \frac{I^2_{dc} \cdot R}{I^2.R + E_o\, I_{dc}} \quad \dots (8·51)$$

Substituting the values of I_{dc} and I, we get,

$$\eta = \frac{(\alpha\cos^{-1}\alpha - \sqrt{1-\alpha^2})^2}{\cos^{-1}\alpha - \alpha\sqrt{1-\alpha^2}} \times 40·6 \text{ per cent} \quad \dots (8·52)$$

Fig. 8.13 shows the variation of efficiency η with α for a half-wave rectifier using hot cathode gas diode. The maximum theoretical efficiency is 40·6 per cent which is the value for vacuum tube half-wave rectifier as well. But in the case of gas tube rectifier the efficiency is almost independent of load resistance and depends only on the ratio of E_o/E_m, whereas in the case of vacuum

tube rectifier the efficiency varies greatly with the load resistance. This dependence of efficiency on E_o/E_m is characteristic of all hot cathode gas diode rectifiers as well as mercury arc rectifiers and hence in these cases in order to obtain high efficiency the operation should be confined to high values of E_m compared with E_o.

Full-wave Rectifier using Hot Cathode Diodes

Fig. 8.14 shows the circuit diagram of a full wave rectifier using thermionic gas diode and a resistance load. Input power transformer is also included in the diagram.

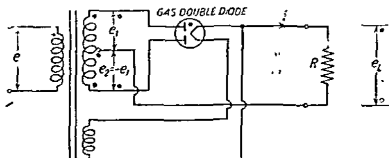


Fig. 8.14.—Circuit diagram of full wave thermionic gas diode rectifier with resistance load.

Fig. 8.15 shows the waveforms of input voltage e and load current i. It is seen that the operation is similar to that in half-wave rectifier except that here there are two current pulses per cycle of input voltage.

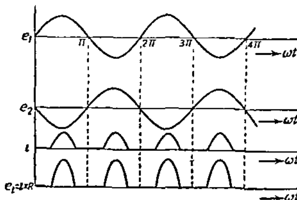


Fig. 8.15—Waveforms of input alternating voltages, load current and output voltage in a full wave thermionic gas diode rectifier with resistance load.

Analysis of this full wave rectifier may be done in the same manner as for the half-wave rectifier. The values of I_m, I_{mo}, and α obtained in this case are the same as in the case of half-wave rectifier. I_{dc} is given by :

$$I_{dc} = \frac{2\ E_m}{\pi\ R}\ [\sqrt{1-\alpha^2} - \alpha \cos^{-1} \alpha] \quad \text{... (8·53)}$$

This value is twice that for the half-wave rectifier circuit.

$$E_{dc} = I_{dc} \,.\, R$$

Hence E_{dc} is also twice as large with the result that E_{dc}/E_m gets doubled. But r.m.s. value of current *i.e.*, I is only $\sqrt{2}$ times as large so that the efficiency is twice as large. Thus maximum theoretical efficiency is 81·2 per cent.

Fig. 8.16 shows the circuit diagram of a full wave thermionic gas diode rectifier with associated load and filter circuit to remove the ripple voltages.

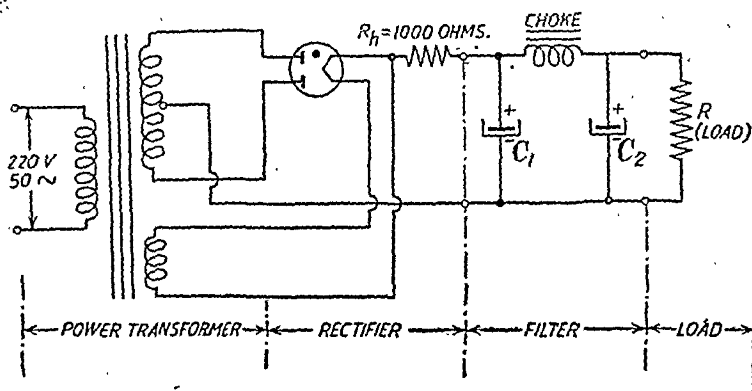


Fig. 8.16.—Practical full wave thermionic gas diode rectifier circuit.

Following precautions are required to be taken in the operation of thermionic gas diode rectifiers :—

(*i*) A high resistance must always be inserted in series with the tube to limit the current.

(*ii*) Cathode must be heated to the normal operating temperature before the plate voltage is applied, otherwise the voltage drop across the tube will exceed the disintegration voltage.

(*iii*) Plate circuit must be protected from even momentary short circuits or overloads because if the current exceeds the maximum rated value for even a few seconds, the cathode of the tube gets damaged.

(*iv*) The temperature of the condensed mercury *i.e.*, the lowest

temperature at any place in the tube envelope must be within prescribed limits.

Hot cathode gas tube rectifier has the following advantages over vacuum tube rectifiers :—

(i) It has lower voltage drop across the tube and hence rectifier efficiency is higher.

(ii) Lower filament power is required.

(iii) Lower first cost, if the anode voltage is greater than 1000 volts and current requirement is high.

The disadvantages of hot cathode gas tube rectifier are :—

(i) Tendency to arcback *i.e.*, conduct in the reverse direction.

(ii) It produces radio frequency transients as the tube ionizes.

(iii) Tube is susceptible to damage even with momentary overloads.

(iv) It is required to heat the cathode first to the operating temperature before the application of anode voltage.

In view of these different properties of vacuum tube rectifiers and hot cathode [illegible] vacuum tube rec[illegible] as for radio recei[illegible] for high power [illegible] are generally used.

Example 3. *In a half-wave hot cathode gas tube rectifier with load resistance of 5000 ohms and no filter, the applied alternating voltage is 500 sin $(2\pi \times 50t)$ and the ionization potential of tube is 10 volts. Assuming idealised rectifier characteristics, calculate (i) peak value of current, (ii) angle of conduction, (iii) d.c. value of current, (iv) d.c. power output, (v) r.m.s. value of current and (vi) rectifier efficiency.*

Solution : $E_m = 500$ volts ; $E_o = 10$ volts ; $R = 5000$ ohms.

(i) Peak plate current $I_m = \dfrac{E_m - E_o}{R} = \dfrac{500-10}{5000}$ amp. $= 98$ mA.

(ii) Angle of conduction $\theta_c = \theta_2 - \theta_1 = \pi - 2\sin^{-1} E_o/E_m$

$$= \pi - 2\sin^{-1} 10/500$$

$$= 177^\circ\ 42'.$$

(iii) $\alpha = E_o/E_m = \dfrac{10}{500} = 0{\cdot}02$

d.c. component of plate current I_{dc} is given by ;

$$I_{dc} = \frac{E_m}{\pi R}\left(\sqrt{1-\alpha^2} - \alpha \cos^{-1} \alpha\right)$$

$$=\frac{500}{\pi\times5000}\left(\sqrt{1-(\cdot02)^2}-\cdot02.\cos^{-1}\cdot02\right)\text{ amp.}$$

$=30{\cdot}86$ **milliamperes.**

(*iv*) D.C. power output $P_{dc}=I^2_{dc}.R=(30{\cdot}86\times10^{-3})^2\times5000$ watts

$=4{\cdot}762$ **watts.**

(*v*) R.M.S. value of current is given by :

$$I=\frac{E_m}{R}\cdot\sqrt{\frac{1}{2\pi}\left[(1+2\alpha^2)\cos^{-1}\alpha-3\alpha\sqrt{1-\alpha^2}\right]}$$

$$=\frac{500}{5000}\sqrt{\frac{1}{2\pi}\left[(1+2\times\cdot0004)\frac{88{\cdot}85\pi}{180}-3\times\cdot02\sqrt{1-(\cdot02)^2}\right]}$$

$=\cdot04883$ amp$=48{\cdot}83$ **milliamperes.**

(*vi*) Rectifier efficiency$=\dfrac{P_{dc}}{P_{in}}=\dfrac{I^2_{dc}\,.\,R}{I^2R+E_oI_{dc}}$

$$=\frac{4{\cdot}762}{(48{\cdot}83\times10^{-3})^2\times5000+(10\times30{\cdot}86\times10^{-3})}$$

$$=\frac{4{\cdot}762}{11{\cdot}92+\cdot3086}=38{\cdot}93\text{ per cent.}$$

Example 4. *A full wave hot cathode gas diode rectifier with resistance load of 6000 ohms uses no filter. The constant voltage drop across each diode during conduction is 12 volts. An alternating voltage of value 400 sin $(2\pi\times50t)$ is applied to the rectifier. Calculate (i) peak value of current, (ii) angle of current flow, (iii) d.c. component of load current, (iv) d.c. power output, (v) r.m.s. value of load current and (vi) rectifier efficiency.*

Solution : Peak load current I_m is given by :

(*i*) $I_m=\dfrac{E_m-E_o}{R}=\dfrac{400-12}{6000}$ amp$=$**64 mA**

(*ii*) $\alpha=E_o/E_m=12/400=\cdot03$

Angle of conduction $\theta_c=\theta_2-\theta_1=\pi-2\,\theta_1=\pi-2\sin^{-1}\alpha$ radians

$=180^\circ-2\times(1^\circ\,43')$

$=$**176° 34′.**

(*iii*) D.C. component of current is given by :

$$I_{dc}=\frac{2E_m}{\pi R}\left[\sqrt{1-\alpha^2}-\alpha\cos^{-1}\alpha\right]$$

$$=\frac{2\times400}{\pi\times6000}\left[\sqrt{1-(\cdot03)^2}-\cdot03\cos^{-1}\cdot03\right]$$

$$=\frac{1}{7{\cdot}5\times\pi}\left[\cdot9997-\cdot0462\right]=\cdot04047\text{ amp}=40{\cdot}47\text{ milliamp.}$$

(*iv*) D.C. power output is given by :

$$P_{dc}=I^2_{dc}.\ R=(40{\cdot}47\times10^{-3})^2\times6000 \text{ watts}=9{\cdot}826 \textbf{ watts.}$$

(*v*) R.M.S. value of load current is given by :

$$I=\frac{\sqrt{2}{\cdot}E_m}{R}\sqrt{\frac{1}{2\pi}\left[(1+2\alpha^2)\cos^{-1}\alpha-3\alpha\sqrt{1-\alpha^2}\right]}$$

$$=\frac{\sqrt{2}\times400}{6000}\sqrt{\frac{1}{2\pi}\left[(1+2\times{\cdot}0009)\cos^{-1}{\cdot}03-3\times{\cdot}03\times\sqrt{1-({\cdot}03)^2}\right]}$$

$={\cdot}04535$ amp$=45{\cdot}35$ **milliamperes.**

(*vi*) Rectifier efficiency is given by :

$$n=\frac{I^2_{dc}.R}{I^2.R+E_o.I_{dc}}$$

$$=\frac{(40{\cdot}47\times10^{-3})^2\times6000}{(45{\cdot}35\times10^{-3})\times6000+(12\times40{\cdot}47\times10^{-3})}$$

$$=\frac{9{\cdot}826}{12{\cdot}34+{\cdot}4856}=\frac{9{\cdot}826}{12{\cdot}82}\times100 \text{ per cent}=76{\cdot}65 \text{ per cent}$$

COLD CATHODE GAS DIODE RECTIFIER

The construction of a cold cathode gas diode suitable for use as a rectifier is given in Chapter VII. The efficiency and power handling capacity of such a rectifier is, however, low and hence it has found little commercial use.

MERCURY ARC RECTIFIER

Construction of mercury rectifier tubes is given in Chapter VII. They are used wherever large power handling is required. For such large power requirements, however, polyphase power input gives better performance. Mercury arc rectifiers are, accordingly, almost always used as polyphase rectifiers. Descriptions of circuit arrangements etc. of such polyphase mercury arc rectifiers is taken up in a later article which deals with polyphase rectifiers using either mercury pool cathodes or hot cathodes.

C. BRIDGE RECTIFIER CIRCUITS

(*Or Dimetric Double-Way Rectifiers*)

A number of rectifier circuit arrangements are possible in

addition to half-wave and full-wave rectifier circuits discussed so far. An important rectifier circuit arrangement is the Bridge Rectifier or Dimetric Double-way Rectifier shown in Fig. 8.17. The bridge rectifier circuit finds two important uses : (*i*) as a power rectifier and (*ii*) as the rectifying system in rectifier-type a.c. meters. When used for power rectification, the rectifying elements employed are thermionic diode either of vacuum type or gas type. When used in rectifying a.c. meter, the rectifying elements usually are the Barrier layer rectifiers either copper oxide type or selenium type.

With reference to the circuit in Fig. 8.17 it is seen that during positive half cycle of the applied a.c. voltage, tubes T_1 and T_3 conduct, while during the other half cycle, tubes T_2 and T_4 conduct. The current, however, flows through the load in the same direction in both halves of the applied a.c. voltage. Hence the waveform of load current is essentially the same as in the case of a full wave rectifier.

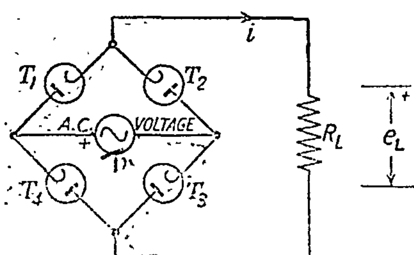


Fig. 8.17.—Single phase Bridge Rectifier Circuit.

The following salient features of a bridge rectifier may be noted :—

(*i*) Current in both the primary and secondary of the plate-supply transformer flows for the entire cycle and hence for a given power output, power transformer of a small size may be used.

(*ii*) No centre tap is required in the transformer secondary.

(*iii*) Since two rectifier elements are present in series in each conduction path, the peak inverse voltage is shared equally by the two elements. Hence Bridge Rectifier circuit is eminently suitable for high-voltage applications.

Bridge rectifier circuit has, however, the following limitations :—

(*i*) Twice the number of rectifier elements are required compared with a full wave rectifier.

(*ii*) Proper insulation must be provided between the transformer windings supplying the heaters of the cathodes of the tubes (when thermionic tubes are used).

Barrier layer rectifiers *i.e.*, selenium rectifiers or copper-oxide rectifiers are more suitable for use in bridge rectifier circuits because they do not require filament heating. These have low peak inverse rating but the bridge circuit reduces the peak inverse voltage per tube to nearly half of that in full wave rectifier.

Fig. 8.18 shows the circuit diagram of a bridge rectifier for use in rectifier type voltmeter. The rectifier elements are selenium or copper-oxide rectifiers and the load a sensitive d.c ammeter. Such a circuit can be used for measurement of a.c. as well d.c. voltages and currents. This d.c. ammeter reads average value of current but the scale may be calibrated to give r.m.s. values of applied sinusoidal voltages. The readings obtained on such an instrument are incorrect if the applied voltage waveform differs from sinusoidal form *i.e.*, when it contains harmonics.

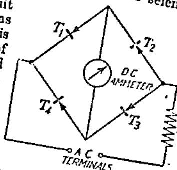


Fig. 8.18.—Bridge Rectifier Type Voltmeter.

D. POLYPHASE RECTIFIERS

When large amount of d.c. power is required, polyphase rectifier circuits are generally used These polyphase circuits are preferred to the single phase rectifier circuits for the following reasons :—

(*i*) According to modern practice, the electrical power is transmitted and distributed as 3-phase power. This suggests the need for a 3-phase rectifier.

(*ii*) Output voltage of a polyphase rectifier is smoother than that of a single phase rectifier. A simpler and cheaper filter circuit may, therefore, be used in conjunction with it.

(*iii*) For the same rectifier output, equipment needed in polyphase rectifier circuit is of lower rating.

For high output powers, the tubes used are those having mercury pool type cathodes but tubes with hot cathodes may be used as well. With mercury pool cathodes, some means of starting the arc, such as an ignitor, is needed.

With a polyphase source of alternating power, the number of possible rectifier circuits is almost unlimited but those commonly used are :—

(*a*) Three-phase half-wave delta-wye rectifier circuit.

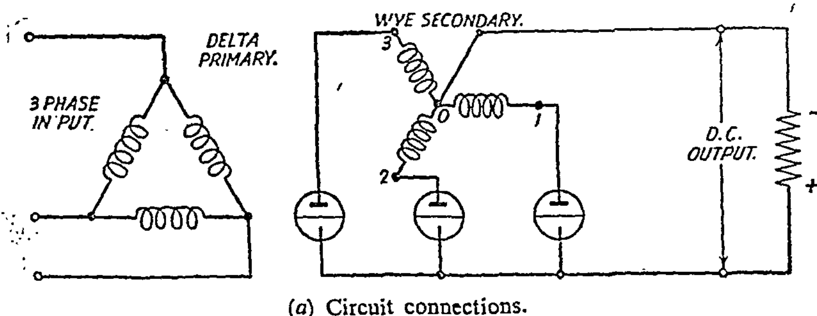
(*b*) Delta six-phase star half-wave rectifier, circuit.

(*c*) Three-phase delta double-wye half-wave rectifier circuit with interphase transformer.

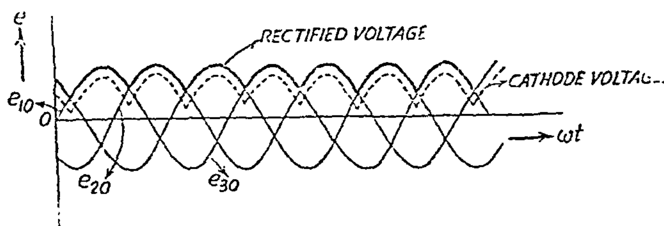
(*d*) Three-phase delta-wye Bridge rectifier circuit.

(a) THREE-PHASE HALF-WAVE DELTA-WYE RECTIFIER

Fig. 8.19 (a) shows a simple three-phase half-wave delta-wye rectifier. Diodes with mercury pool cathodes are shown in the figure but the same circuit applies to hot cathode diodes. Each leg of the secondary wye provides one phase. The transformers in these rectifier circuits serve two functions : (i) providing the voltages on the rectifier anodes and (ii) providing the common connection, called the neutral connection. Voltages are given with respect to this neutral connection. Referring to the circuit of Fig. 8.19 (a) for



(a) Circuit connections.



(b) Waveforms of rectified voltage and cathode voltage.

Fig. 8.19.—Three-phase Delta Star half-wave Rectifier.

sinusoidal input voltages, the voltages applied to the anodes are also sinusoidal, equal in magnitude but shifted from each other in phase by 120°. Each anode voltage is equal to the primary voltage multiplied by the transformer ratio. These input anode voltages e_{10}, e_{20} and e_{30} are shown in Fig. 8.19 (b). It is seen that for certain intervals of time more than one anode is positive with respect to the neutral but only the anode with the highest voltage at the instant conducts. The arc transfers from one anode to the next at the instant their voltages become equal and the order in which this commutation of current takes place is 1, 2, 3, 1, 2 and so on. The difference of voltages $e_{20}-e_{10}$, $e_{30}-e_{20}$, $e_{10}-e_{30}$, when positive in value are called the "commutating voltage". The thick solid curve in Fig. 8.19(b) shows this commutating voltage. If the arc voltage drop is the same for all anodes and does not vary with anode current,

then the potential of the cathode with respect to the neutral *i.e.* output voltage is the rectified anode voltage minus the constant voltage drop. This is shown by the dotted curve in Fig. 8.19 (*b*). ripple frequency of the output voltage is thrice the supply frequency. To avoid d.c. saturation of the transformer, three-ph transformers are used in preference to three single phase transformers.

LTA SIX-PHASE STAR RECTIFIER

It is obvious that better conditions of rectification are obtained by using 3-phase operation. For still better conditions compared with single phase operation. For still better conditions of rectification, number of phases greater than three should be used But most of the power generated these days is three-phase power. Hence the available three-phase power is commonly converted into 6 or 12-phase power by means of transformers. Fig 8 20 (*a*) shows the circuit diagram of Delta six-phase star half-wave rectifier. Here six-phase operation is secured by using 3 centre-tapped secondary windings, each associated with one of the primary windings. The circuit resembles three single-phase full-wave rectifier circuits connected in parallel but having their secondary voltages displaced from one another by 120° electrically. The rectified voltage has a form shown in Fig. 8.20 (*b*) by the thick solid curve and the ripple frequency is six times the supply frequency.

The secondary utilization factor of a rectifier is defined as the ratio of P_{dc}, the direct-current power including the losses in the rectifier to P_r the total voltage-ampere rating of the transformer secondary windings. For a polyphase rectifier this is given by .—

Secondary Utilization factor

$$= \sqrt{\frac{2}{p}} \cdot \frac{\sin \pi/p}{\pi/p}$$

where p is the number of phases.

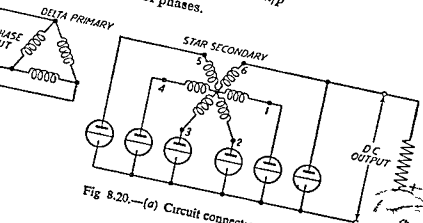
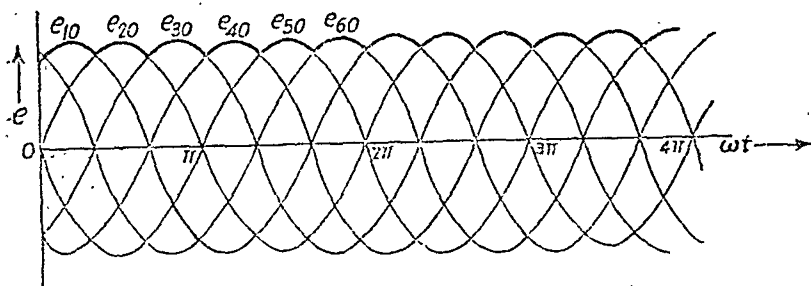


Fig 8.20.—(*a*) Circuit connections.

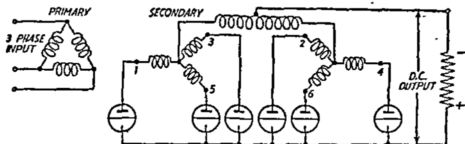


(*b*) Waveforms of anode voltages and rectified voltage.
Fig. 8.20.—Delta six-phase star half-wave rectifier.

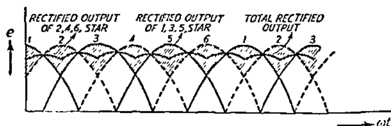
This utilization factor is similar to a power factor except that this applies to a non-sinusoidal transformer current rather than to the phase angle between the sinusoidal current and voltage. This secondary utilization factor is maximum when p is equal to 2·89. The nearest integral value of p is 3. For higher values of p, the secondary utilization factor reduces and this constitutes the major limitation of a large number of phases.

(c) THREE-PHASE DELTA-DOUBLE WYE HALF-WAVE RECTIFIER WITH INTERPHASE TRANSFORMER

Fig. 8.21(*a*) shows the circuit connections of a three-phase delta-double wye half-wave rectifier with interphase transformer (I.P.T.). The two three-phase half-wave circuits are thus in parallel through this interphase transformer. The polarities of the corresponding secondary windings in the two parallel systems are reversed with respect to each other with the result that when the rectified output voltage of one three-phase unit is at its minimum, that of the other is at its maximum as shown in Fig. 8.21 (*b*). In this figure, the solid sine waves are for the 1-3-5 star rectifier whereas the dotted sine waves are for the 2-4-6 star rectifier. The interphase transformer connects the neutral points of two wyes and thus acts as a potential divider across the difference of these two voltages. The instantaneous rectified voltage of centre terminal of the interphase transformer is then the instantaneous average of the rectified voltages of the two individual wyes and is shown by the heavy curve in Fig. 8.21 (*b*). As is obvious from the waveform of rectified voltage, the ripple frequency is 6 times the supply frequency. The circuit is basically 3-phase circuit and hence the secondary utilization factor is high corresponding to p equal to 3. This constitutes a major advantage of this circuit over the 6-phase star connection. The circuit has further advantage that each anode current while flowing is only one-half the load current, whereas without the interphase transformer it is equal to the full load current. The lower anode current reduces the arc voltage drop and hence the loss in the rectifiers.



(a) Circuit Diagram.



(b) Waveforms of rectified output voltages.

Fig. 8.21.—Three-phase Delta-double wye half-wave rectifier with interphase transformer.

(d) THREE-PHASE DELTA-WYE BRIDGE RECTIFIER

Fig. 8.22 (a) shows the circuit diagram of this rectifier. It is [illegible]

from the same 3-phase star connected voltage supply and connected in series. The rectifier elements 4, 5 and 6 produce positive output [illegible] during the positive [illegible] ements 1, 2 and 3 [illegible] ltages and produce

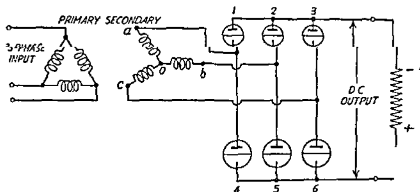
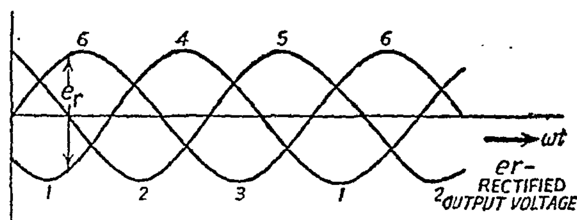


Fig. 8.22.—(a) Circuit Diagram.

negative output voltage with respect to the neutral. The two outputs are added in series. The average value of this total output voltage is then twice that of either taken alone.



(b) Wave forms of anode voltage giving the magnitude of rectified output voltage at any instant.

Fig. 8.22.—3-phase Delta-wye bridge rectifier.

In Fig. 8.22 (b), curves 1, 2, 3 show the waveforms of voltages of anodes 1, 2, 3 while the curves 4, 5 and 6 show the waveforms of anodes 4, 5 and 6 respectively. The rectified output voltage including the tube voltage drops lies between the uppermost and lowermost envelopes. This voltage has the same waveform as that of a six-phase star rectifier. The ripple frequency is six times the supply frequency.

At any instant two rectifier elements conduct, one from each group. Thus each rectifier conducts for 120 electrical degrees and its average current rating is one-third of the average output current. For the phase order *a-b-c*, the rectifier conduct in pairs in the following order : 1-6, 2-6, 2-4, 3-4, 3-5, 1-5, 1-6 and so on.

When one pair of elements, say 1 and 6 are conducting, each of the windings *oa* and *oc* carries the current conducted by these two rectifiers. But this current flows through the windings in opposite directions and hence the average winding current is zero. D.C. saturation of the core is thus avoided. Further each winding carries current for one-third cycle in one direction, for another one-third cycle in the opposite direction and no current in the remaining one-third cycle and hence the secondary utilization factor of the rectifier is quite large.

E. CONTROLLED RECTIFIERS

Often it is required to vary the amount of rectified current in a rectifier. Typical examples of use of controlled rectifier current are : (i) electric welding operations, (ii) lighting-control installation in auditoriums and theatres, (iii) motor speed control, (iv) torque controls etc. Control of the output current of a rectifier may be achieved by one of the following two means : (a) by controlling the voltage to the power transformer such as by using an autotransformer to feed the power transformer and (b) by inserting a controlling resistor in the output circuit. Both methods have their drawbacks. The first method results in expensive equipment while the second method gives poor efficiency. An inexpensive and efficient method of controlling the output current of a rectifier is by using thyratrons, ignitrons, excitrons or cold cathode gas triodes instead of diodes for rectification. Methods of analysis are similar for these tubes and it will suffice to study the control of only one, say the thyratron. In all these cases, average value of rectified current is controlled by controlling the instant in the positive half cycle of the applied voltage at which the conduction starts. Deionisation takes place during the negative half cycles so that conduction may again be controlled in the next half cycle. The point of conduction in the positive half cycle may be delayed or hastened and the average value of the rectified current may thus be controlled. Some of the methods of control by thyratrons are discussed here.

(a) CONTROL BY DIRECT GRID VOLTAGE

To analyse the action of a thyratron in a controlled circuit, it is necessary to use the firing characteristic of the thyratron i.e., the curve showing the critical grid voltage for different values of plate voltage. This curve is reproduced in Fig. 8.23 for a negative control thyratron tube.

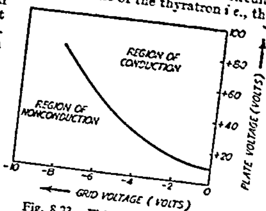
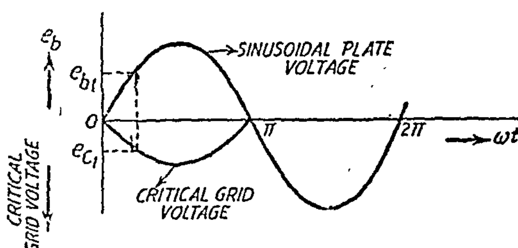


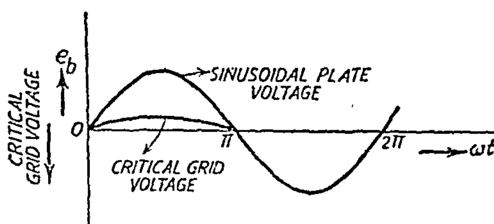
Fig. 8.23.—Firing characteristic of a thyraton.

For different values of plate voltage, this curve gives the minimum grid voltage for conduction to ake place. Now if the late voltage is sinusoidal, is possible by making e of this firing characteristic, to find the critical id voltage for different ints on the a.c. cycle of the plate voltage. It is then possible plot this critical grid voltage as a function of time as shown in . 8.24 for negative and positive control thyratron tubes. These racteristics are called the "Critical grid voltage curves". Thus in

Fig. 8.24 (a) corresponding to any time t_1 in the positive half cycle, the plate voltage is e_{b1}. For this value e_{b1} of plate voltage, the corresponding critical grid voltage e_{c1} is obtained from the firing characteristic of Fig. 8.23. This process is repeated for different values of time t and critical grid voltages are plotted against time t.



(a) Critical grid voltage curve for negative control thyratron.



(b) Critical grid voltage curve for positive control thyratron,

Fig. 8.24.—Critical grid voltage curves for negative and positive control thyratrons for an applied sinusoidal plate voltage.

Study of this critical grid voltage characteristic shows that at any instant of time no conduction will take place unless the grid voltage is made more positive than that given by this characteristic. Then for a constant d.c. grid voltage as shown in Fig. 8.25, as the instantaneous value of applied sinusoidal voltage increases a point is reached when this direct grid voltage exceeds the critical grid voltage and conduction begins. This condition is shown by point P_1 in Fig. 8.26. Once conduction begins, the grid loses all control of the arc and regains control only when the arc is extinguished. In Fig. 8.26, let the conduction begin at angle θ_1 corresponding to point P_1. As soon as conduction starts, plate voltage e_b drops from e_1 to E_o, where E_o is the constant voltage drop of the tube while conducting and lies between 10 to 15 volts. Simultaneously current rises from zero suddenly and subsequently varies according to the variation of the applied voltage. At angle $\pi-\theta_2$, the applied plate voltage just equals E_o and conduction ceases. During the negative half cycle, no conduction takes place and hence the anode voltage e_b

equals the supply voltage so that peak inverse anode voltage is equal to the supply voltage.

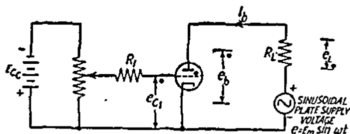


Fig. 8.25.—Thyratron control circuit with a.c. plate voltage and variable d c. grid voltage.

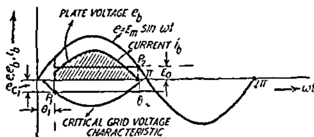


Fig. 8.26.—Waveform of plate voltage, plate current in a thyratron with direct grid voltage and sinusoidal plate voltage.

For a constant voltage drop E_o in the tube, current i_b during conduction is given by the relation :—

$$i_b = \frac{E_m \sin \theta - E_o}{R_l} \qquad \dots (8{\cdot}54)$$

Obviously before angle θ_1, current is zero. Also current falls to zero when applied a.c. plate potential falls to E_o at phase angle $\pi - \theta_2$.

Then the average rectified current is given by :

$$I_{dc} = \frac{1}{2\pi} \int_{\theta_1}^{\pi-\theta_2} i_b \, d\theta$$

$$= \frac{E_m}{2\pi R_l} \int_{\theta_1}^{\pi-\theta_2} \left(\sin \theta - \frac{E_o}{E_m} \right) d\theta.$$

On integration :

$$I_{dc} = \frac{E_m}{2\pi R_l} \left[-\cos \theta - \frac{E_o}{E_m}\, \theta \right]_{\theta_1}^{\pi-\theta_2}$$

$$= \frac{E_m}{2\pi R_l} \left[\cos \theta_1 + \cos \theta_2 - \frac{E_o}{E_m} (\pi - \theta_2 - \theta_1) \right] \qquad \dots (8{\cdot}55)$$

where $$\sin \theta_2 = \frac{E_o}{E_m} \qquad \dots (8\cdot56)$$

If $E_o \ll E_m$, then the ratio $\frac{E_o}{E_m}$ as well as angle θ_2 may be taken as zero. With this condition, Eqn. (8·55) reduces to :

$$I_{dc} \approx \frac{E_m}{2\pi R_l}[1 + \cos \theta_1] \qquad \dots (8\cdot57)$$

I_{dc} is maximum when θ_1 is zero and this maximum value is equal to $\frac{E_m}{\pi R_l}$ so that Eqn. (8·57) may be written as :

$$\frac{I_{dc}}{I_{dc\ max}} = \frac{1+\cos \theta_1}{2} \qquad \dots (8\cdot58)$$

It is apparent from Equations (8·57) and (8·58) that the average rectified current I_{dc} can be varied by varying angle θ_1 *i.e.*, by varying the position in the a.c. cycle at which the d.c. grid potential exceeds the critical grid voltage. If the grid voltage has a large positive value, conduction may take place over almost the complete half cycle, and then I_{dc} has its maximum value equal to $E_m/\pi R_l$. For zero grid voltage, conduction takes place over a large portion of the half cycle and hence I_{dc} has a large value. If the grid voltage equals the maximum negative value of the critical grid voltage, then angle θ_1 is equal to 90 degrees and I_{dc} is then almost half of $I_{dc\ max}$. If grid voltage is made more negative, conduction does not take place at all and I_{dc} is then zero.

Average value of anode voltage is given by :

$$E_{dc} = \frac{1}{2\pi}\int_0^{2\pi} e_b \, d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\theta_1} E_m \sin \theta \, d\theta + \int_{\theta_1}^{\pi-\theta_2} E_o \, d\theta + \int_{\pi-\theta_2}^{2\pi} E_m \sin \theta \, d\theta\right]$$

$$= \frac{E_o}{2\pi}(\pi - \theta_2 - \theta_1) - \frac{E_m}{2\pi}(\cos \theta_1 + \cos \theta_2) \qquad \dots (8\cdot59)$$

If $E_m \gg E_o$, then Eqn. (8·59) reduces to :

$$E_{dc} \approx -\frac{E_m}{2\pi}(1 + \cos \theta_1) \qquad \dots (8\cdot60)$$

The negative sign shows that the cathode is more positive than the anode for most of the cycle.

The r.m.s. value of plate current is given by :

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{0}^{2\pi} i_b^2 . d\theta}$$
$$= \sqrt{\frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_2}\left(\frac{E_m \sin\theta - E_o}{R_l}\right)^2 d\theta} \quad \ldots (8{\cdot}61)$$

This reading will be indicated by an a.c. ammeter placed in the anode lead. Similarly the total power supplied to the plate circuit is given by :

$$P = \frac{1}{2\pi}\int_{0}^{2\pi} e\, i_b\, d\theta$$
$$= \frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_2} E_m \sin\theta \;\; \frac{E_m \sin\theta - E_o}{R_l}\, d\theta \quad \ldots (8{\cdot}62)$$

Two types of control of average value of rectified current are possible by using d c. grid bias (*i*) continuous control and (*ii*) on-off control. These are described below

(*i*) *Continuous control of average rectified current by variable grid bias* : The magnitude of the d.c. or average rectified current of a thyratron may be varied by varying the d.c. grid bias of the tube and applying an a.c. plate supply voltage. The circuit used for this purpose is given in Fig. 8.25. Value of average plate current is given by the Eqn. (8·58) where θ_1 is a function of control grid bias e_c. Fig. 8 27 shows the nature of variation of I_{dc} with e_c. I_{dc} becomes zero when e_c exceeds maximum value of critical grid voltage e_c *max*. A small positive or negative increment of grid bias, produces a corresponding increment of average current A continuous control is thus obtained.

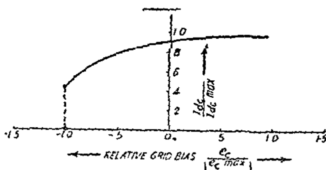


Fig. 8.27.—Variation of average plate current with grid bias

(*ii*) *On-off control of average rectified current by abrupt variation of grid bias* : A number of circuits are possible which prov—

on-off control of average plate current. Such circuits using a thyratron provide an arcless switch or contactor. Fig. 8.28 shows a simple circuit arrangement for on-off switching of grid bias to control the average value of rectified current in a thyratron having a.c. plate supply voltage.

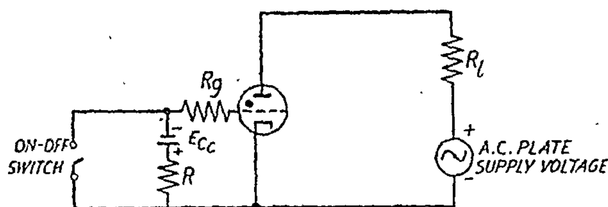


Fig. 8.28.—On-off control in a thyratron with direct grid voltage and alternating plate supply voltage.

When the switch is open, a heavy negative bias E_{cc} greater than critical grid voltage is applied to the grid and the thyratron does not conduct. When the switch is closed, grid is at the same potential as the cathode, and a heavy rectified current flows in the plate circuit. Resistance R is placed in series with E_{cc} battery to prevent a short circuit of the battery when switch is closed.

(b) *Continuous control of average rectified current by phase shift of a.c. grid voltage superimposed on constant grid bias :*

Continuous control of the average rectified current may be attained by using an a.c. grid voltage of the same frequency as the plate supply voltage, in series with a fixed grid bias and by varying the phase of this a.c. grid voltage. Schematic circuit of such an arrangement is shown in Fig. 8.29.

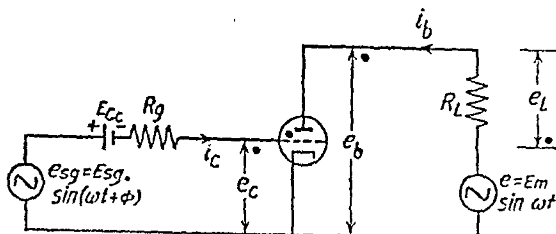
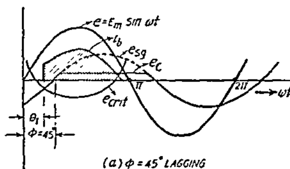


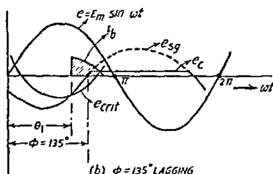
Fig. 8.29.—Schematic circuit of continuous control of average rectified current by phase shift of a.c. grid voltage.

Both the a.c. plate voltage and a.c. grid voltage are taken from the same a.c. voltage source so that they have the same frequency. The phase shifting device in the grid circuit is not shown in the schematic diagram of Fig. 8.29. The point of conduction on the a.c. plate voltage cycle is controlled by varying the phase angle ϕ between the plate and grid potentials. For simplicity grid bias is

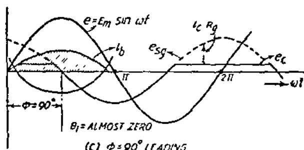
considered to be zero. With fixed zero grid bias, three different [illegible] voltage have been considered. These [illegible] and (c) for phase angle ϕ of grid [illegible] (lagging), —135 degrees (lagging) and +90 degrees (leading) respectively.



(a) $\phi = 45°$ LAGGING



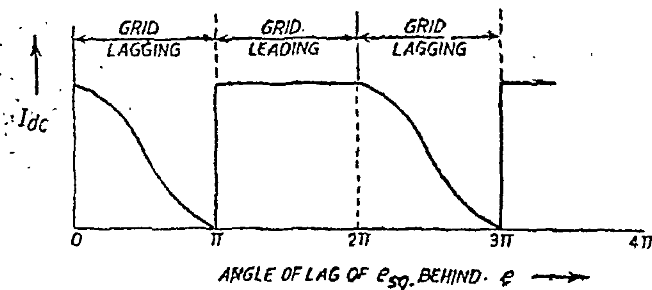
(b) $\phi = 135°$ LAGGING



(c) $\phi = 90°$ LEADING

In Fig. 8-30 (a), the alternating grid potential e_c lags behind the plate voltage e_b by 45 degrees. However, it is to be noted that the grid potential e_c equals the critical grid voltage at an angle θ_1 and conduction begins at this point in the a.c. cycle. Conduction ceases when the applied voltage falls below E_e, the minimum voltage required to maintain conduction.

In most cases, the critical grid potential at the point of conduction is much smaller than the applied grid potential amplitude,



(*d*) Variation of I_{dc} with angle of lag.

Fig. 8.30.—Control of average plate current by phase shift of a.c. grid voltage in a thyratron.

so that angles ϕ and θ_1 are almost equal. It is often justified in assuming, that the critical grid voltage curve coincides with the zero-voltage axis. Again if $E_m >> E_o$, then I_{dc} as given by equation (8-57) shows the dependence of I_{dc} on angle θ_1 (or angle ϕ if the critical grid potential curve coincides with zero voltage axis).

During the positive half cycle of a.c. grid voltage, actual grid to cathode voltage e_c is small and constant, most of voltage drop taking place across the series resistance R_g.

In Fig. 8.30 (*b*), e_c lags behind the plate voltage by 135 degrees. Obviously, the duration of conduction of current is highly reduced and hence average rectified current is also correspondingly reduced.

In Fig. 8·30 (c), e_c leads the plate voltage by 90 degrees (or lags by 270 degrees). Here conduction occurs very nearly at the beginning of each cycle and I_{dc} is then maximum. In fact, this maximum value of I_{dc} is obtainable for all cases of leading grid voltages.

If I_{dc} is calculated for all values of phase angles and is plotted against the angle of lag ϕ, then the curve obtained is as shown in Fig. 8.30 (d). Similar results are obtained for other values of fixed grid bias.

(c) *Continuous control of average rectified current by varying the magnitude of grid bias superimposed on a constant a.c. grid voltage* :— In control method (*a*) angle θ_1 is small and hence small changes in the control characteristics of the tube result in correspondingly larger changes in the average rectified current. The precision of this control method (*a*) is, therefore, inherently poor. Trouble is particularly serious with mercury vapour thyratrons because in these

mercury vapour thyratrons, the temperature and pressure of the gas fluctuate considerably.

The above trouble is eliminated by using an alternating grid voltage in series with the varying grid bias. Fig. 8.31 shows the schematic circuit arrangement.

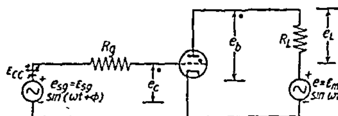


Fig. 8.31.—Schematic circuit arrangement of continuous control of average rectified current by varying the grid bias superimposed on constant a.c grid voltage

Let the alternating grid voltage e_{sg} lag behind the supply voltage by 45 degrees. Then the curve of e_{gt} i.e., sum of e_{sg} and E_{cc} will intersect the critical grid voltage curve at points dependent upon the value of grid bias E_{cc}. Fig. 8.32 (a) shows the various voltage waveforms and nature of plate [illegible] value of angle θ_1 in this case [illegible] responding condition for zero [illegible] d c. grid voltage, angle θ_1 is smaller than 45 degrees and the average value of rectified plate current is reduced still further.

An approximate relation between I_{dc} and E_{cc} may be obtained. Assumption made is that the e_{crit} curve coincides with the zero voltage axis.

Then $$\theta_1 = \frac{\pi}{2} - \sin^{-1}\left(\frac{E_{cc}}{E_m}\right) \qquad \ldots (8{\cdot}63)$$

provided that $|E_{cc}| < E_m$.

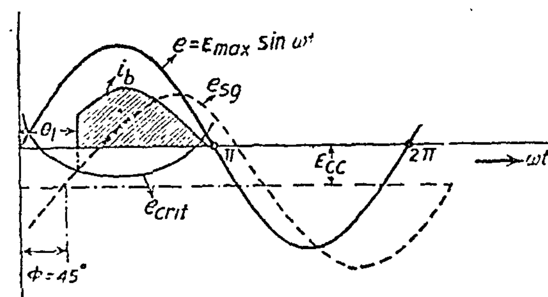
From equation (8·58).

$$\frac{I_{dc}}{I_{dc\,max}} = \tfrac{1}{2}(1+\cos\theta_1)$$

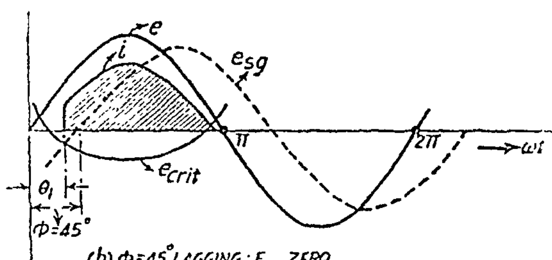
$$= \tfrac{1}{2}\left[1+\cos\left(\frac{\pi}{2} - \sin^{-1}\frac{E_{cc}}{E_m}\right)\right] \qquad \ldots (8{\cdot}64)$$

$$= \tfrac{1}{2}\left[1+\sin\left(\sin^{-1}\frac{E_{cc}}{E_m}\right)\right]$$

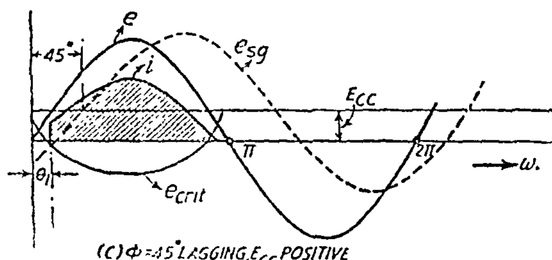
or $$\frac{I_{dc}}{I_{dc\ max}}=\frac{1}{2}\left[1+\frac{E_{cc}}{E_m}\right] \quad \text{... (8·65)}$$



(a) Φ=45° LAGGING, E_{cc} NEGATIVE



(b) Φ=45° LAGGING; E_{cc} ZERO



(c) Φ=45° LAGGING, E_{cc} POSITIVE

Fig. 8.32—Control of average plate current in a thyratron by varying d.c. grid bias superimposed on a constant a.c. grid voltage.

Thus linear variation of I_{dc} with E_{cc} takes place if critical grid voltage is zero everywhere.

A simple circuit arrangement for providing a.c. grid voltage e_{sg} lagging 45 degrees behind the plate supply voltage is shown in Fig. 8.33.

In this circuit if the reactance X_c of the condenser C is equal to the resistance R, then the a.c. grid voltage will lag behind the plate supply voltage by 45 degrees. Further with this arrange-

ment, the a.c. grid voltage amplitude is $\sqrt{2}$ or .707 times the plate supply voltage amplitude.

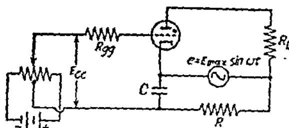


Fig. 8.33.—C...

Methods for producing a.c. grid voltage shifted in phase from plate supply voltage A circuit commonly used for this purpose is given in Fig. 8.34. The phase-shift between grid and plate voltage is controlled by the impedances Z_1 and Z_2.

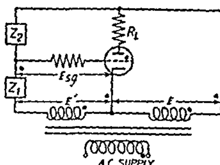


Fig. 8.34.—A simple phase shifting network.

Using a centre tapped transformer, voltages E and E' are equal in magnitude and phase voltage E drives current through the thyratron plate circuit. If out of impedance Z_1 and Z_2, one is chosen as a resistance R while the other is either an inductance or a capacitance, then phase shift of the grid supply voltage E_{sg} may be adjusted by varying the magnitude of either Z_1 or Z_2.

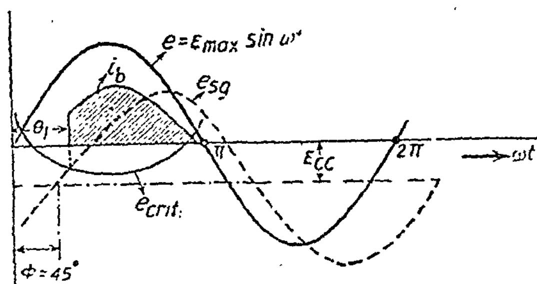
If $\mathbf{I}_s$ is the current through the impedances Z_1 and Z_2, then applying Kirchhoff's law and assuming the grid current through the large resistance R_g to be negligible,

$$\mathbf{I}_s\,(Z_1+Z_2)=E+E' \quad \text{... (8·66)}$$

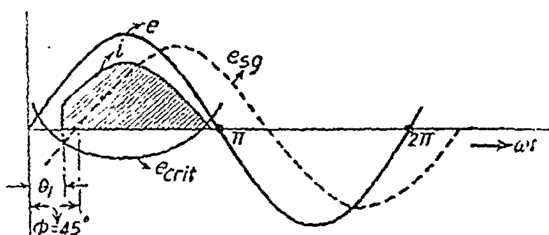
and

$$\mathbf{E}_{sg}=\mathbf{I}_s.\mathbf{Z}_1-\mathbf{E}' \quad \text{... (8·67)}$$

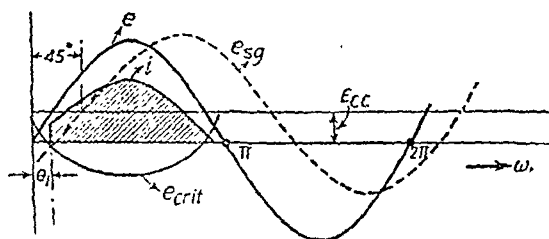
or
$$\frac{I_{dc}}{I_{dc\ max}} = \frac{1}{2}\left[1 + \frac{E_{cc}}{E_m}\right] \qquad \dots (8\cdot 65)$$



(a) Φ = 45° LAGGING; E_{cc} NEGATIVE



(b) Φ = 45° LAGGING; E_{cc} ZERO



(c) Φ = 45° LAGGING; E_{cc} POSITIVE

Fig. 8.32—Control of average plate current in a thyratron by varying d.c. grid bias superimposed on a constant a.c. grid voltage.

Thus linear variation of I_{dc} with E_{cc} takes place if critical grid voltage is zero everywhere.

A simple circuit arrangement for providing a.c. grid voltage e_{sg} lagging 45 degrees behind the plate supply voltage is shown in Fig. 8.33.

In this circuit if the reactance X_c of the condenser C is equal to the resistance R, then the a.c. grid voltage will lag behind the plate supply voltage by 45 degrees. Further with this arrange-

ment, the a.c. grid voltage amplitude is $\sqrt{2}$ or .707 times the plate supply voltage amplitude.

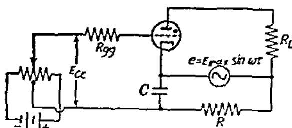


Fig. 8 33 ...

Methods for producing a.c. grid voltage shifted in phase from plate supply voltage. A circuit commonly used for this purpose is given in Fig. 8 34. The phase-shift between grid and plate voltage is controlled by the impedances Z_1 and Z_2.

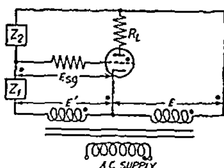


Fig 8.34.—A simple phase shifting network.

Using a centre tapped transformer, voltages E and E' are equal in magnitude and phase voltage E drives current through the thyratron plate circuit. If out of impedance Z_1 and Z_2, one is chosen as a resistance R while the other is either an inductance or a capacitance, then phase shift of the grid supply voltage E_{sg} may be adjusted by varying the magnitude of either Z_1 or Z_2.

If $\mathbf{I}_s$ is the current through the impedances Z_1 and Z_2, then applying Kirchhoff's law and assuming the grid current through the large resistance R_g to be negligible,

$$\mathbf{I}_s\,(Z_1+Z_2)=E+E' \quad \text{... (8·66)}$$

and

$$\mathbf{E}_{sg}=\mathbf{I}_s\,\mathbf{Z}_1-\mathbf{E}' \quad \text{... (8·67)}$$

A vector diagram may be drawn to show phase and magnitude relations of various voltages and currents. This will be considered for the following four combinations of $\mathbf{Z}_1$ and $\mathbf{Z}_2$:—

(*i*) $\mathbf{Z}_1$ resistive and $\mathbf{Z}_2$ inductive.

$$\mathbf{Z}_1 = R + j.0 \qquad \text{and } \mathbf{Z}_2 = 0 + j\omega L.$$

(*ii*) $\mathbf{Z}_1$ resistive and $\mathbf{Z}_2$ capacitive.

$$\mathbf{Z}_1 = R + j.0 \qquad \text{and } \mathbf{Z}_2 = 0 + \frac{1}{j\omega C}$$

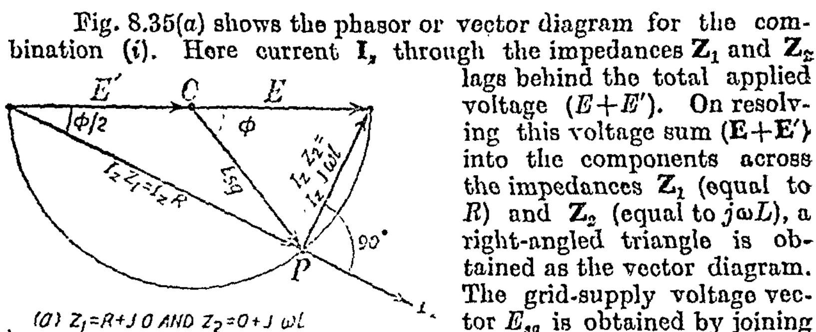
(*iii*) $\mathbf{Z}_1$ inductive and $\mathbf{Z}_2$ resistive.

$$\mathbf{Z}_1 = 0 + j\omega L \qquad \text{and } \mathbf{Z}_2 = R + j.0$$

(*iv*) $\mathbf{Z}_1$ capacitive and $\mathbf{Z}_2$ resistive.

$$\mathbf{Z}_1 = 1 + \frac{1}{j\omega C} \qquad \text{and } \mathbf{Z}_2 = R + j.0$$

Fig. 8.35(*a*) shows the phasor or vector diagram for the combination (*i*). Here current $\mathbf{I}_z$ through the impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ lags behind the total applied voltage $(E+E')$. On resolving this voltage sum $(\mathbf{E}+\mathbf{E}')$ into the components across the impedances $\mathbf{Z}_1$ (equal to R) and $\mathbf{Z}_2$ (equal to $j\omega L$), a right-angled triangle is obtained as the vector diagram. The grid-supply voltage vector E_{sg} is obtained by joining centre C of voltage vector $(\mathbf{E}+\mathbf{E}')$ to vertex of triangle. Obviously this vector represents a value $I_z Z_1 - E'$ and hence satisfies the relation (8·67) and lags behind the vector $\mathbf{E}$ by angle ϕ. It is to be noted that the vertex P of the phasor diagram always lies on the circle on vector $(\mathbf{E}+\mathbf{E}')$ as the diameter. The magnitude of vector $\mathbf{E}_{sg}$ is, therefore, always constant and equal to E. The phase angle ϕ of grid supply voltage $\mathbf{E}_{sg}$ may be controlled by varying either $\mathbf{Z}_1$ or $\mathbf{Z}_2$.



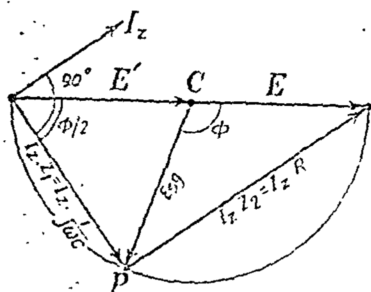


Fig. 8.35.—Vector or phasor diagrams of phase-shifting circuits.

The phase angle of E_{sg} with respect to E is given by the relation :

$$\phi = -2 \tan^{-1} \frac{\omega L}{R} \qquad \dots (8{\cdot}68)$$

Such a phase shifting circuit may be used in control method no. (*b*) for controlling the average value of rectified current in a thyratron. Then as the ratio $\frac{\omega L}{R}$ is increased either by increasing L or by reducing R, then phase angle ϕ of grid supply voltage E_{sg} increases in magnitude *i.e.*, becomes more lagging and hence I_{dc} correspondingly reduces.

Fig. 8.35 (*b*) shows the vector or phasor diagram for combination (*iv*). In this case also it is found that E_{sg} has a constant magnitude equal to E and a phase angle which is lagging and is given by :

$$\phi = -2 \tan^{-1} \omega CR \qquad \dots (8\text{·}69)$$

ϕ may be varied by varying either C or R. If such a phase shifting circuit is used in control method (*b*), then as the ratio $\frac{1}{\omega CR}$ is decreased by increasing either C or R, phase angle ϕ reduces and hence I_{dc} increases

If combination (*ii*) or (*iii*) is chosen, value of phase angle ϕ is positive *i e*, the grid supply voltage leads the plate supply voltage E and then as is seen from Fig. 8.30 (*d*), no control of I_{dc} is possible.

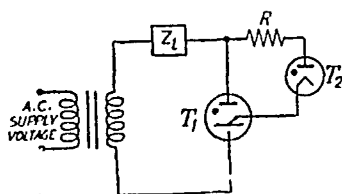
The assumptions made in the above analysis are : (*i*) Load is assumed to be almost resistive and (*ii*) effect of grid current is assumed negligible. In the absence of these conditions, analysis is not valid.

Ignitron Control Circuit. In ignitron tubes it is required to send a pulse of current through the ignitor to initiate conduction [illegible]

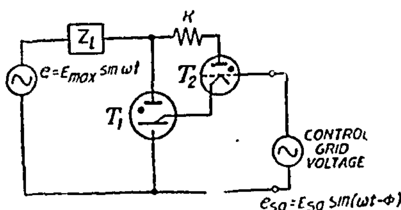
is higher than that required by the grid circuit of a thyratron, and the control method in this case differs from those in thyratron. However, once the main arc is initiated, operation is similar to that in a thyratron and equations (8·54) to (8·62) may be applied to obtain I_{dc} etc. Various methods have been devised for supplying the ignitor current pulses depending upon the use to which ignitron is put. When the ignitron is required to rectify an alternating plate supply voltage, the ignitor pulses must be periodic and must be synchronised with the a.c. anode voltage in such a way that initiation of main arc takes place at the same phase angle in every plate-voltage cycle. For other applications, one or more ignitor current pulses at controlled times may start the arc.

A few common methods for establishing the point of ignition in each plate voltage cycle of an ignitron are illustrated in Figs. 8·36 and 8·37.

(*a*) **Anode firing method or Anode Ignition with Diode.** Fig. 8.36(*a*) gives the circuit arrangement. T_2 is a hot cathode gas diode. During the positive half of applied a.c. voltage, current through T_2 continuously increases until it reaches a critical value at which main arc starts. After ignition of ignitron tube T_1 has taken place, a heavy current flows through the load Z_l, and the voltage applied to T_2 and ignitor decrease to a low constant value. Ignitor current becomes negligibly small. Diode T_2 serves two functions: (*i*) it supplies the ignitor current and (*ii*) it serves to prevent reverse current through the ignitor during the negative half cycle of anode voltage.



(*a*) Anode ignition with hot cathode gas diode.



(*b*) Anod ignition with thyratron

Fig. 8.36.—Simple Ignitor Excitation circuits in which ignitor current flows through the load.

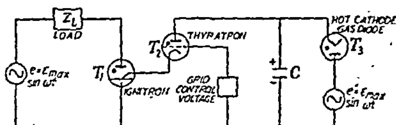
(*b*) **Anode Ignition with Thyratron** :—Fig. 8.36(*b*) shows the circuit arrangement. The control grid voltage e_{sg} applied to the grid of thyratron T_2 is an alternating voltage of the same frequency as anode voltage but delayed in phase. The ignition of thyratron is thus delayed depending upon the delay angle ϕ.

The above two methods have the advantage of simplicity but have the following two disadvantages: (*i*) Ignitor current flows through the load impedance Z_l, restricting its suitability for relatively heavy load currents only and (*ii*) ignition cannot take place at the beginning of the cycle. These defects are avoided in methods (*c*) and (*d*) discussed below.

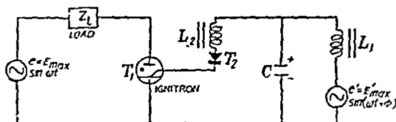
(*c*) **Thyratron and Capacitor Ignition.** Fig. 8.37(*a*) shows the circuit arrangement. With the start of each positive half cycle, capacitor gets gradually charged through the hot cathode gas diode T_3 and discharges through the thyratron tube T_2 as soon as anode voltage of T_3 reaches a definite value depending upon the grid control voltage. A sudden current through ignitor causes ignition in the ignitron. The average value of rectified current may thus be controlled by adjusting the grid control voltage.

(*d*) **Saturating-Reactor Ignition.** In circuit of Fig. 8.37(*b*), during the positive half cycle of the applied voltage condenser C charges through reactor L_1 with the polarity shown. Simultaneously current builds up gradually in the circuit containing reactor L_2,

rectifier T_2 and the ignitor. L_2 is large and hence current in this latter circuit is small. The core of saturable core reactor L_2 is so



(a) Thyratron and capacitor i[illegible]n.



(b) Saturating Reactor Ignition

Fig. 8.37. Ignitor Excitation circuit with no ignitor current through the load

designed that as soon as [illegible] reaches a critical value, the core [illegible] tance decreases. Condenser C [illegible] this circuit containing L_2, T_2 and ignitor and causes ignition. Little saturation takes place in the core of reactor L_1. The purpose of rectifier T_2 is to prevent flow of current in the reverse direction. The phase angle ϕ of voltage applied to reactors may be varied to control the time of ignition in the ignitron plate voltage cycle.

This method eliminates the use of hot-cathode gas tubes which are not economically utilized in ignitor excitation, since ignitor current has large peak to average current ratio. Hot cathode tube with this high peak ignitor current is capable of delivering considerably greater average power than is required for this application.

Example 5. *In thyratron circuit of Fig. 8.25, plate supply voltage has peak value of 600 volts. Constant voltage drop in the tube during conduction is 12 volts. Load resistance is 5,000 ohms. The critical grid voltage curve is then such that for grid bias of* -2 *and* -3 *volts, the angle* θ_1 *of initiation of current is 10 and 20 degrees. Calculate the average value of load current under these bias conditions. As angle* θ_1 *is increased, find the value of* θ_1 *which gives maximum*

Case (D) : $\theta_1 = 70$ degrees,

$$I_{dc} = \frac{500}{2\pi \times 4000}(1+\cos 70°) = \frac{1}{16\pi}(1+{\cdot}342) = 0{\cdot}0267 \text{ amp.}$$

$$= \mathbf{26{\cdot}7\ mA}$$

I_{dc} is maximum, when θ_1 is zero.

Hence $$I_{dc\ max} = \frac{500}{2\pi \times 4000}(1+1) = 0{\cdot}0398 \text{ amp.}$$

$$= \mathbf{39{\cdot}8\ mA.}$$

EXERCISES

1. Draw the circuit diagram of a fullwave vacuum-tube rectifier using a power transformer with centre-tapped secondary for use in a radio receiver. Show a typical filter circuit. Explain the operation of the rectifier and give waveforms of current and output voltage.

2. For a halfwave vacuum tube rectifier using a resistive load with no filter, derive expressions for (*i*) peak, average and r.m.s. values of current, (*ii*) peak, average and r.m.s. values of output voltage, (*iii*) d.c. power output and total power input, (*iv*) rectifier efficiency and (*v*) ripple factor. Assume the diode to have a linear current-voltage characteristic. Calculate the above quantities, given that the r.m.s. value of input a.c. voltage is 300 volts, load resistance is 6,000 ohms and slope of diode characteristic corresponds to 1,000 ohms.

3. A voltage of frequency 50 c.p.s. and effective value 400 volts is applied to each diode of a fullwave vacuum tube rectifier. Diodes have linear $i_b - e_b$ characteristics with slope corresponding to a resistance of 800 ohms. No filter is used. Load is a resistance of 4,000 ohms. Calculate from first principle : (*i*) peak, average and effective values of current, (*ii*) d.c. power output and total power input, (*iii*) rectifier efficiency, (*iv*) form factor and ripple factor.

4. A halfwave rectifier uses hot cathode mercury vapour diode having a constant voltage drop of 12 volts during conduction. The load resistance is 4,000 ohms. Sinusoidal voltage of r.m.s. value 220 volts is applied to the rectifier. Supposing no filter to be used calculate (*i*) peak load current (*ii*) angle of conduction (*iii*) d.c. load current (*iv*) r.m.s. load current and (*v*) rectifier efficiency.

5. A fullwave mercury vapour rectifier uses a load resistance of 3,000 ohms and no filter. The ionisation potential of each diode is 12 volts. An a.c. voltage of peak value 240 volts is applied to the rectifier. Calculate (*i*) angle of conduction, (*ii*) average and r.m.s. values of load current, (*iii*) d.c. power output and (*iv*) rectifier efficiency.

6. In the thyratron circuit of Fig. 8.25, the plate supply voltage has r.m.s. value of 110 volts. Voltage drop across the tube during conduction is 12 volts. Plate load is resistance of 3,000 ohms. The critical grid-voltage characteristic is such that the angles of initiation of plate current for grid bias of −1, −2 and −3 volts are 10, 30 and 50 degrees respectively. Calculate average value of plate current for these bias conditions.

[illegible] itiation of [illegible] to be 40 [illegible] ate current (angle θ_1), if average plate current is required to be increased to 42 and 44 milliamperes. Assume voltage drop in the tube during conduction to be negligible.

CHAPTER IX

POWER SUPPLY SYSTEMS

Introduction. In operation of electron tube devices, power is required for the following purposes :—

(a) Heating the cathodes,

(b) For supply of anode power,

(c) For supplying the grid bias.

The order of these voltages and other qualities like ripple factor, constancy of voltage etc. depend on the use to which the power supply is put. For example, considerable ripple voltage is permissible if the rectifier voltage is used for battery charging whereas for use in a high quality amplifier or oscillator, ripple voltage is required to be very small. For use in ultra high frequency tubes like klystron, highly stabilized d.c. power supply is required.

Cathode heating power. This may be obtained from a d.c. voltage source such as a battery or from an a.c. voltage source. Battery is generally used to supply d.c. cathode heating power in directly heated cathodes *i.e.*, filamentary cathodes. The main disadvantage of the battery supply is that frequent replacement or charging of the battery is required. To avoid this trouble, for most of the applications, a.c. cathode heating is used. This a.c. heating power may conveniently be applied by using a step down transformer fed from mains. When a.c. power is used for heating the cathodes, however, there are two possible troubles that may result, namely (*i*) plate current may develop a component at the second harmonic of the filament supply frequency and (*ii*) the signal being amplified may get modulated by the alternating current used for heating the cathode. These defects result in a low pitched hum in the loudspeaker fed from the output of the system. This low pitched hum is called the "a.c. hum". This a.c. hum is much larger in filament type tubes than in tubes using indirectly heated cathodes. Most of the high power tubes such as in Radio Transmitters etc. use directly heated cathodes and for convenience a.c. heating alone has to be used. In such tubes means should, therefore, be devised to reduce the a.c. hum.

Methods of reducing a.c. hum in filamentary-type tubes. To effect reduction in a.c. hum in filamentary cathode tubes, let us first look into the basic causes for this hum. This a.c. hum may be caused in filamentary tubes due to one of the following reasons :

(*i*) Cyclic variation of filament temperature may take place. Emission from cathode then does not remain constant and develops an a.c. component superimposed on it. The plate current, therefore, also contains this hum. The heat capacity of filaments used in vacuum tubes, however, is generally so large that this variation of filament temperature with 50 c/s filament current is small and causes negligible hum under space charge limited condition.

(*ii*) Action of the magnetic field caused by the a.c. filament current :—This magnetic field deflects the electrons flowing to the plate with the result that the plate current decreases slightly when the filament current flows in either direction. No such reduction of plate current takes place when filament current is zero. This results in a cyclic variation of plate current at a frequency twice the filament supply frequency.

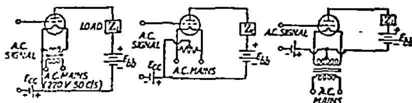
(*iii*) Voltage drop in the filament :—This causes the negative half of the filament to emit more electrons than does the positive half. Further, since the current is space charge limited, the number of electrons actually drawn from any part of filament is proportional t[illegible] Current drawn [illegible] than in the abs[illegible] is extra current from the negative end of the filament more than compensates for the loss of current from the positive end. Thus there is an overall increase of space current when current flows through the filament.

Out of these three causes, first is negligibly small while the second and the third causes result in a.c. hums at the second harmonic of filament power supply frequency and are 180 degrees out of phase with each other. These effects tend to cancel, therefore. There remains, however, considerable amount of residual hum which is undesirable for most of the applications. These may be reduced by the following means :—

(*a*) Special circuit arrangements for feeding the filament power ;

(*b*) Proper choice of filament voltages ;

(*c*) Appropriate filament geometry.



(*a*) Simple arrangement . (*b*) Centre tapped Resistor. (*c*) Centre-tapped Transformer

Fig 9.1.—Filamentary Cathodes heated from a.c. voltage source

Fig. 9.1 (*a*) shows a simple circuit arrangement for supplying the heating power to the filament from an a.c. power source. The a.c. hum in this case is large. This a.c. hum may be minimized by connecting the grid and plate leads to a point that is almost at the same potential as the mid-point of the filament. This may be done by using a centre-tapped resistor or a centre-tapped transformer for feeding the a.c. power to the filament as shown in Fig. 9.1 (*b*) and (*c*).

In high power tubes, often multistrand filaments are used. In such tubes, different strands are usually fed from different phases of a polyphase power source. This technique reduces the hum considerably. Regarding the geometry of the filament, it is desirable to use *V* or *W* shapes in order to reduce the hum.

With the above means used to reduce a.c. hum in filament type tubes, the ratio of a.c. hum power to signal power may be kept low if the signal power is large such as in Radio Transmitters etc. But for filament type tubes carrying low signal power, hum may become quite annoying. For small power handling, either d.c. heating of the filament should be used or indirectly heated cathodes should be used with a.c. heating power. The latter is usually preferred.

A.C. hum in heater type tubes (Tubes having indirectly heated cathodes).

For low power applications indirectly-heated cathodes having high emission efficiency may conveniently be used. A.c. heating is generally used for convenience. In heater type tubes a.c. hum is considerably less than in the filament-type tube primarily because there is no voltage drop in the emitter and the magnetic field of the heater wire is highly reduced by the cathode sleeve surrounding the heater wire. A small amount of residual hum, however, remains because of a small magnetic field produced by the current in the heater and because of leakage resistances from heater to other electrodes. This residual hum is usually negligibly small so that a.c. heating of heater in heater-type tubes may always be used in all commercial applications.

Anode voltage and grid power supplies. Power for supplying the d.c. voltage to anode, screen grid and for bias voltage for control grid are usually obtained by rectification of a.c. power by rectifier-filter assemblies. Rectifier used may be one of the various types discussed in the last chapter depending upon the voltage and current requirements, purpose of use etc. For grid bias purpose, sometimes, batteries are used.

The output voltage of a rectifier is of pulsating nature and consists of a d.c. component and ripple voltages at the fundamental ripple frequency and harmonics thereof with progressively reducing amplitudes. For most of the applications, a constant d.c. voltage almost free from ripple voltages is required and this necessitates

the use of filter circuit following the rectifier. These filters used for removing or at least reducing the ripple voltages, may be of various configurations. In what follows we will first study the harmonic contents of rectifier output and the effect of some of these simple filter circuits on the rectified voltage waveform.

HARMONIC CONTENT OF RECTIFIER OUTPUT

(a) **Halfwave Vacuum Tube Rectifier.** An analytic representation of the output current of any rectifier system is obtained in terms of the Fourier series expansion as given by :

$$i = b_o + \sum_{k=1}^{\infty} b_k \cdot \cos k\,\theta + \sum_{k=1}^{\infty} a_k \cdot \sin k\,\theta \quad \dots (9.1)$$

where $\theta = \omega t$ and the coefficients b_o, b_k and a_k are given by :

$$\left.\begin{aligned} b_o &= \frac{1}{2\pi}\int_o^{2\pi} i \cdot d\theta \\ b_k &= \frac{1}{\pi}\int_o^{2\pi} i \cdot \cos k\theta \cdot d\theta \\ a_k &= \frac{1}{\pi}\int_o^{2\pi} i \cdot \sin k\theta \cdot d\theta \end{aligned}\right\} \quad \dots (9.2)$$

In the case of rectifiers, the term b_o gives the average or d.c. value of the current.

In the case of vacuum tube halfwave rectifier, the output current is given by the relation :

$$\left.\begin{aligned} i &= \frac{E_m}{R_l + R_a} \cdot \sin\theta = I_m \cdot \sin\theta && \text{when } 0 \leqslant \theta \leqslant \pi \\ i &= 0 && \text{when } \pi \leqslant \theta \leqslant 2\pi \end{aligned}\right\} \dots (9{\cdot}3)$$

where R_l is the load resistance and R_a is the constant slope resistance of the idealised characteristic of the diode.

Using the values of i as given by equation (9·3) in eq. (9·2) we get :—

$$b_o = \frac{1}{2\pi}\int_o^{\pi} I_m \cdot \sin\theta \cdot d\theta = \frac{I_m}{\pi} \quad \dots (9{\cdot}4)$$

$$a_k = \frac{1}{\pi}\int_o^{\pi} I_m \cdot \sin\theta \cdot \sin k\theta \cdot d\theta$$

$$= \frac{I_m}{2\pi}\int_o^{\pi} \Big[\cos(k-1)\,\theta - \cos(k+1)\theta\Big] d\theta$$

$$=\frac{I_m}{2\pi}\int_0^\pi (1-\cos 2\theta)d\theta + \frac{I_m}{2\pi}\int_0^\pi [\cos (k-1)\theta-\cos (k+1)\theta]\, d\theta \quad k=2,3,\ldots$$

$$=\frac{I_m}{2\pi}\left(\pi-0\right)+\frac{I_m}{2\pi}\left[\,0\,\right]=\frac{I_m}{2} \qquad \ldots (9\cdot5)$$

$$b_k = \frac{1}{\pi}\int_0^\pi I_m \sin\theta.\ \cos k\theta.d\theta$$

$$= \frac{I_m}{2\pi}\int_0^\pi [\sin(\theta+k\theta)+\sin(\theta-k\theta)]d\theta$$

$$= \frac{I_m}{2\pi}\left[-\frac{\cos(k+1)\theta}{(k+1)}+\frac{\cos(k-1)\theta}{(k-1)}\right]_0^\pi$$

$$= \frac{I_m}{2\pi}\left[\frac{k\{\cos(k-1)\theta-\cos(k+1)\theta\}+\{\cos(k-1)\theta+\cos(k+1)\theta\}}{(k+1)(k-1)}\right]_0^\pi$$

$$= \frac{I_m}{2\pi}\left[\frac{2k\sin k\theta.\sin\theta+2\cos k\theta\cos\theta}{(k+1)(k-1)}\right]_0^\pi$$

$$= \frac{I_m}{\pi}\left[\frac{\cos k\pi\cos\pi-\cos\theta\cos\theta}{(k+1)(k-1)}\right]$$

$$= \frac{-2I_m}{\pi(k+1)(k-1)} \quad \text{for } k=2,4,6,\ldots \qquad \ldots (9\cdot6)$$

For $k=1$, $bk=\frac{1}{\pi}\int_0^\pi I_m.\sin\theta\cos\theta\, d\theta=0.$

Substituting the values of b_o, a_k and b_k in the equation (9·1) we get :

$$i=I_m\left[\frac{1}{\pi}+\frac{1}{2}\sin\omega t-\frac{2}{\pi}\sum_{k=2,4,6,\ldots}^{\infty}\frac{\cos k\omega t}{(k+1)(k-1)}\right] \ldots (9\cdot7$$

The lowest frequency term in the output current has the same frequency as the power supply. The remaining terms are even-harmonic terms of progressively reducing amplitudes.

(*b*) **Fullwave Vacuum tube rectifier.** A fullwave rectifier consists of two halfwave rectifier circuits, one of which conducts during the period when the other is not conducting. Then the current i_2 in one circuit is related to the current i_1 in the other by the relation :

$$i_2(\alpha)=i_1(\alpha+\pi) \qquad \ldots (9\cdot8)$$

The total load current, which is the sum of the currents i_1 and i_2, is given by the expression :—

$$i=I_m\left[\frac{2}{\pi}-\frac{4}{\pi}\sum_{k=2,4,6,\ldots}^{\infty}\frac{\cos k\omega t}{(k+1)(k-1)}\right] \qquad \ldots (9\cdot9)$$

It is seen from eq. (9·9) that the total load current does not contain the fundamental frequency term i.e., the term of angular frequency ω. The lowest frequency term in the total load current has an angular frequency 2ω. The problem of filtering the ripple voltage in the filter circuit is then comparatively easy.

(c) Halfwave and fullwave gasdiode rectifiers. The Fourier series expressions for halfwave and fullwave rectifiers using hot cathode gas diodes may be obtained in a way similar to the above but in these circuits conduction begins at some angle θ_1 and ceases at angle $\pi-\theta_1$, assuming the ionisation and extinction voltages to be equal. The form of Fourier series in these cases is, therefore, comparatively complicated. In practice, however, the angle θ_1 is usually small, particularly when voltage of large amplitude is applied. In that case, equations (9·7) and (9·9) for vacuum tube rectifiers are applicable in this case also.

Halfwave Vacuum tube rectifier with a capacitor filter. Simplest filter arrangement consists of only a capacitor across the load resistance. Fig. 9.2 shows such a filter circuit following a halfwave vacuum tube rectifier. For the purpose of analysis, the halfwave rectifier has been assumed to be ideal. Such a condition is never realized in practice but this does not materially affect the nature of the circuit performance.

The capacitor C gets charged during the conduction period and energy gets stored in the condenser. During the non-conduction period, the condenser discharges through the load resistance delivering energy to it. Thus current flows through the load resistance throughout the a c cycle whereas the rectifier tube current flows only in pulses

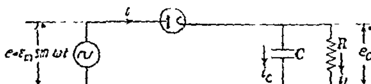


Fig. 9.2.—Halfwave Vacuum tube rectifier with Capacitor filter.

To facilitate understanding the circuit performance, let us first assume the load resistance to be so large as compared with the

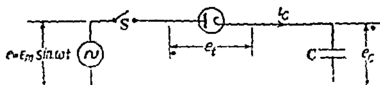


Fig. 9.3.—Halfwave rectifier with filter Capacitor and without load resistance.

reactance of C as to be omitted from the circuit. This simple circuit arrangement is shown in Fig. 9.3.

The applied voltage is sinusoidal and is given by :

$$e = E_m \sin \omega t \quad \text{... (9·10)}$$

Let the switch S be closed at time $t=0$; then the current i_c is given by :

$$i_c = C \frac{de}{dt} = \omega c.\ E_m \cos \omega t \quad \text{... (9·11)}$$

This current i_c leads the applied voltage e by 90 degrees. *Fig.* 9.4 shows the waveforms of applied voltage e and current i_c. At the phase angle $\pi/2$, the condenser is charged to a voltage E_m and the condenser current i_c becomes zero. The rectifier prevents reversal of current and condenser cannot discharge. The voltage across the condenser, therefore, remains constant at the value E_m.

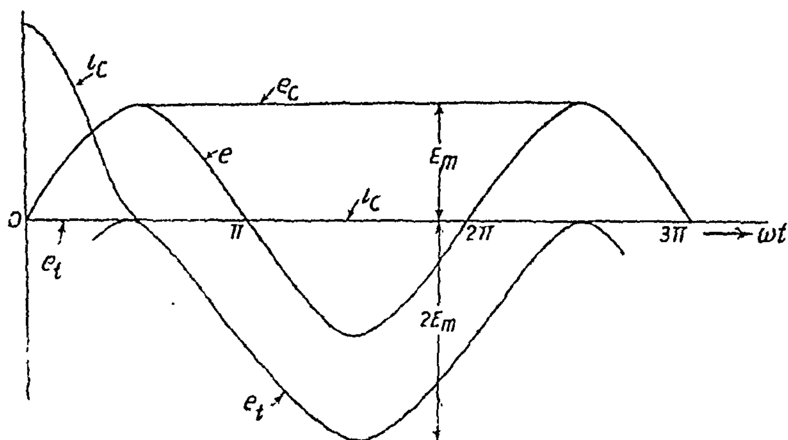


Fig. 9.4.—Waveforms of voltages and currents in a halfwave rectifier with capactior filter and no load resistance.

The voltage e_t across the tube is equal to the difference of voltages e and e_c as given by the equation :

$$e_t = e - e_c \quad \text{... (9·12)}$$

The waveforms of e_t is, therefore, as shown in Fig. 9·4. It reaches a peak negative voltage of $2E_m$.

Circuit of Fig. 9·2 may now be considered. R_l is now in shunt with condenser C and hence during the non-conducting period, the condenser C discharges through the load resistance R_l and loses charge. During conduction, current flows through the rectifier to replace the charge lost *previously*. The waveforms of currents are shown in Fig. 9.5.

Ignition angle θ_1 corresponds to the start of conduction period. Extinction angle θ_2 corresponds to the end of conduction period. Angle θ_c, which is the difference of angles θ_2 and θ_1, gives the duration of conduction.

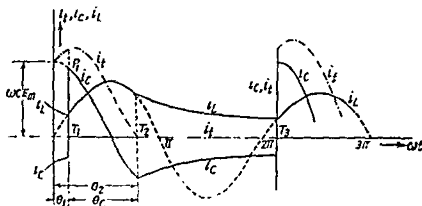


Fig. 9.5.—Waveforms of currents in half wave rectifier of Fig. 9.2.

During conduction period $\quad i_t = i_c + i_L \qquad$ ... (9·13)

During non-conduction period : $i_t = 0$ and hence

$$i_c = -i_L \qquad \text{... (9·14)}$$

Thus during non-conduction period i_c and i_L are equal and opposite.

During Conduction period :

$$i_L = \frac{E_m \sin \omega t}{R_1} \qquad \text{... (9·15)}$$

and $\quad i_c$ is given by $i_c = C \dfrac{de}{dt} = \omega c\, E_m \cos \omega t \qquad$ ... (9·16)

Current i_t through the tube is then given by :—

$$i_t = i_L + i_c = \frac{E_m \sin \omega t}{R} + \omega c\, E_m \cos \omega t \qquad \text{... (9·17)}$$

$$= E_m \sqrt{\left(\frac{1}{R_L}\right)^2 + (\omega c)^2} \sin (\omega t + \tan^{-1} \omega c\, R_L) \qquad \text{... (9·18)}$$

At point T_2 corresponding to angle θ_2 conduction ceases so that e_t becomes zero, i.e. the sum $i_c + i_L$ equals zero.

$$\text{or} \quad \frac{E_m}{R_1} \sin \theta_2 + \omega c\, E_m \cos \theta_2 = 0 \qquad \text{... (9·19)}$$

$$\text{Hence} \quad \tan \theta_2 = -\omega c R_1 \qquad \text{... (9·20)}$$

It is obvious that the value of angle θ_2 must lie somewhere between the limits $\pi/2$ and π (or between $3\pi/2$ and 2π) and its exact value can be determined from ωc and R_l. Beyond θ_2, the condenser c discharges through R_l. The current i_L reduces exponentially with a time constant equal to $C.R_l$ and is given by :—

$$i_L = A \,.\, \epsilon^{-\frac{t}{CR_L}} \qquad \ldots (9{\cdot}21)$$

where A is a constant.

Constant A may be determined from the conditions at the time when $\omega t = \theta_2$, by equating i_L just before θ_2 to just after θ_2. Thus :

$$\frac{E_m}{R_l}\sin\theta_2 = A \,.\, \epsilon^{-\frac{\theta 2}{\omega c R_l}}$$

or
$$A = \frac{E_m}{R_l}\sin\theta_2 \;\epsilon^{\frac{\theta}{\omega c R_l}} \qquad (9{\cdot}22)$$

Substituting the value of A from equation (9·22) into equation (9·21),

$$i_L = \frac{E_m}{R_l}\sin\theta_2 \;\epsilon^{-\frac{(\omega t - \theta_2)}{\omega c R_l}} \qquad \ldots (9{\cdot}23)$$

Non-conduction period ends and the conduction period begins again, when the supply voltage e attains a value equal to $i_l R_l$ as obtained from equation (9·23). This corresponds to point T_3 at phase angle of $2\pi + \theta_1$.

Substituting $\omega t = 2\pi + \theta_1$ and equating $i_l \,.\, R_l$ to e we get :—

$$E_m \sin\theta_2 . \,\epsilon^{-\frac{(2\pi+\theta_1-\theta_2)}{\omega c R_l}} = E_m \sin(2\pi+\theta_1) \qquad \ldots (9{\cdot}24)$$

$$\text{or } \sin\theta_1 = \sin\theta_2 \;\epsilon^{-\frac{2\pi-(\theta_2-\theta_1)}{\omega c R_l}}$$

$$\text{or } \sin\theta_1 = \sin(\tan^{-1} - \omega c R_l) \,.\, \epsilon^{-\frac{2\pi-\theta c}{\omega c R_l}} \qquad \ldots (9{\cdot}25)$$

where $\theta_c = \theta_2 - \theta_1 =$ angle of conduction.

But $\sin(\tan^{-1}\omega c R_l) = \sin(\tan^{-1} - \omega c R_l)$, since $\tan^{-1}\omega c R_l$ is an angle in the first quadrant and is equal to $\pi - \tan^{-1}(-\omega c R_l)$

Equation (9·25) may then be put as :

$$\text{Sin } \theta_1 = \sin(\tan^{-1}\omega c R_l) \;\epsilon^{-\frac{2\pi-\theta_c}{\omega c R_l}} \qquad \ldots (9{\cdot}26)$$

Now $\sin\theta_1$ may be put as $\sin(\theta_2 - \theta_c)$ or $\sin(\pi - \theta_2 + \theta_c)$ and $\pi - \theta_2$ may be put as $\tan^{-1}\omega c R_l$. The equation (9·26) then becomes :—

$$\sin[\tan^{-1}(\omega c R_l) + \theta_c] = \sin[\tan^{-1}\omega c R_l] \;\epsilon^{-\frac{2\pi-\theta_c}{\omega c R_l}} \qquad \ldots (9{\cdot}27)$$

Value of θ_2 may be easily calculated from equation (9·20) but it is difficult to calculate values of θ_1 and θ_e from equations (9·26) and (9·27). Hence in order to find θ_1 and θ_e, often graphical methods are employed. To determine θ_1, the members of equation (9·26), namely $\sin\theta_1$ and $\sin(\tan^{-1}\omega cR_l)\,\epsilon^{-\frac{2\pi-(\theta_2-\theta_1)}{\omega cR_l}}$ are plotted against θ_1 for the given value of ωcR_l. This is shown in Fig. 9.6

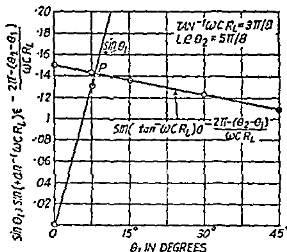


Fig. 9·6.—Variation of $\sin\theta_1$ and $\sin(\tan^{-1}\omega cR_l)$. $\epsilon^{-\frac{2\pi(\theta_2-\theta_1)}{\omega cR_l}}$ with θ_1 in halfwave Vacuum tube rectifier with Capacitor filter

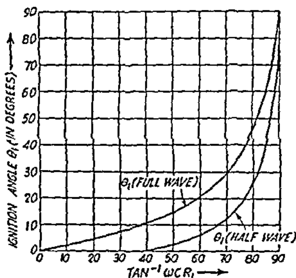


Fig 9 7.—Variation of ignition angle θ_1 with $\tan^{-1}\omega cR_l$

for $\tan^{-1} \omega c R_l$ equal to $3\pi/8$ radians *i.e.*, θ_2 equal to $5\pi/8$ radians. The point of intersection P then gives the value of θ_1 for given value of $\omega c R_l$. This process may be repeated for different values of $\omega c R_l$ and the values of θ_1 so obtained may be plotted against $\tan^{-1} \omega c R_l$ as shown in Fig. 9.7. Thus in any particular case, we need know only the value $\omega c R_l$. Equation 9·20 then enables us to find θ_2 while the value of θ_1 may be read directly from the curve in Fig. 9.7. The value of θ_c may then be found from the relation $\theta_c = \theta_2 - \theta_1$. Values of angles θ_1, θ_2 and θ_c for a half-wave vacuum tube rectifier with condenser filter are plotted against $\tan^{-1} \omega c R_l$ in Fig. 9.8.

The curves of i_c, i_l and i_t in Fig. 9·5 satisfy the values of θ_1, θ_2 and θ_c as found from Fig. 9·8. At point T_1 in Fig. 9·5, conduction begins and then i_l follows the curve e_m/R_l whereas i_c shoots up from a negative value to a high positive value given by point P_1 which is the point of intersection of this vertical line, with curve $\omega c E_m \cos \omega t$. Subsequently curve for i_c follows the cosine law. Current i_t is the sum of currents i_c and i_l. At T_2, corresponding to the extinction angle θ_2, current i_l becomes zero and current i_l reduces exponentially to cut the curve e/R_l at point T_3 corresponding to angle $2\pi + \theta_1$. Current i_c is exactly equal and opposite to current i_l. At T_3, conduction again starts and the conditions at T_1 are repeated.

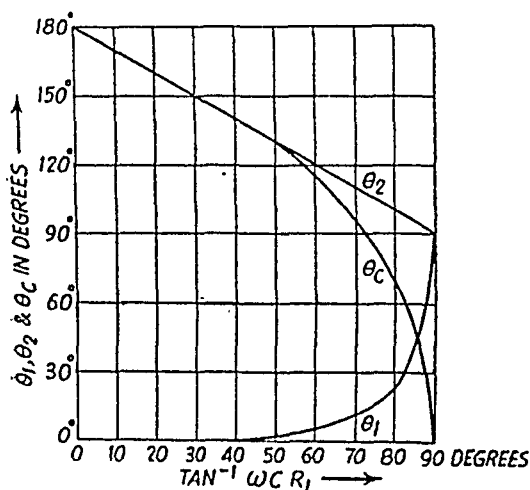


Fig. 9.8.—Graph showing variation of ignition angle θ_1, extinction angle θ_2 and conduction angle θ_c with $\tan^{-1} \omega c R_l$ in a halfwave vacuum tube rectifier.

It is seen from Fig. 9.8, that the ignition angle θ_1 for half-wave rectifier is more or less zero for values of $\tan^{-1} \omega c R_l$ less than 45° *i.e.*, for values of $\omega c R_l$ less than unity. However, as the value of $\tan^{-1} \omega c R_l$ increases beyond 45 degrees, the angle θ_1 increases rapidly and finally becomes 90° for $\tan^{-1} \omega c R_l$ equal to 90°. On the other hand extinction angle is 180° for $\tan^{-1} \omega c R_l$ equal to zero and decreases linearly reaching a value of 90° for $\tan^{-1} \omega c R_l$ equal to 90°. The conduction angle θ_c, being the difference of angles θ_2 and θ_1, reduces with the increase of $\tan^{-1} \omega c R_l$.

Curves of load current i_l *for several values of parameter* ωcR_l are shown in Fig. 9 9. For each of these curves, the values of igni- ... ıgle θ_2 have been determined from ... current i_l show the change in wave- ... capacitance C keeping R_l and E_m constant. It is observed that when ωcR_l is zero, the waveform is the rectified sine wave. With the increase of C, the period of current flow through load increases and the minimum value of i_l rises. Ultimately when ωcR_l becomes infinite, steady direct current without any ripple component flows through the load.

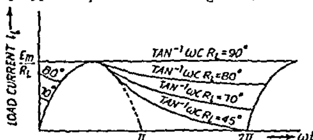


Fig 9 9 —Waveform of load current i_l for different values of the parameter ωcR_l.

Study of Fig. 9·9 reveals that the d.c. component I_{dc} of load current increases with the increase of ωcR_l *i.e.*, with the increase of C if R_l is kept constant. In the limit, when ωcR_l approaches infinity corresponding to $\tan^{-1}\omega cR_l=90°$, then I_{dc} approaches the value $\frac{E_m}{R_l}$.

For halfwave vacuum tube rectifier with capacitor filter, the load current is given by the expression

$$\left.\begin{aligned} i_l &= \frac{E_m}{R_l}\sin\omega t \text{ for } \theta_1<\omega t<\theta_2 \\ \text{and } i_l &= \frac{E_m}{\sqrt{R_l^2+\left(\frac{1}{\omega c}\right)^2}}\,\epsilon^{-\frac{\omega t-\tan^{-1}(-\omega cR_l)}{\omega cR_l}} \\ & \qquad \text{for } \theta_2<\omega t<(2\pi+\theta_1) \end{aligned}\right\} \quad ..(9\cdot28)$$

The d.c. component of load current is then given by :

$$\left.\begin{aligned} I_{dc} = \frac{1}{2\pi}\Bigg[\int_{\theta_1}^{\theta_2}\frac{E_m}{R_l}\sin\omega t.d(\omega t) \\ +\int^{2\pi+\theta_1}\frac{E_m}{\sqrt{R_l^2+\left(\frac{1}{\omega c}\right)^2}}\,\epsilon^{-\frac{\omega t-\tan^{-1}(-\omega cR_l)}{\omega cR_l}}\,d(\omega t) \end{aligned}\right\} \quad ...(9\cdot29)$$

After integration and simplification, equation (9·29) becomes :—

$$I_{dc}=\frac{E_m}{2\pi}\sqrt{\left(\frac{1}{R_l}\right)^2+(\omega c)^2}.\ (1-\cos\theta_c) \quad ...(9·30)$$

The d.c. output voltage E_{dc} is given by :

$$E_{dc}=I_{dc}.R_l=\frac{E_m}{2\pi}\sqrt{1+(\omega cR_l)^2}\ .\ (1-\cos\theta_c) \quad ...(9·31)$$

In Fig. 9·10, the ratio $\frac{E_{dc}}{E_m}$ is plotted against $\tan^{-1}\omega cR_l$. It is observed that the ratio $\frac{E_{dc}}{E_m}$ increases with the increase of capacitance C or load resistance R_l. It is apparent that the regulation of this rectifier with capacitor filter will be very poor.

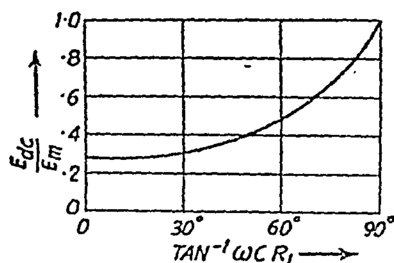


Fig. 9·10—Variation of $\frac{E_{dc}}{E_m}$ with $\tan^{-1}\omega cR_l$.

Since the average value or d.c. value of output voltage increases with the increase of ωcR_l, one is tempted to keep the value of ωcR_l very large. But ωcR_l cannot be increased very much. A study of Fig. 9·5 reveals that as ωcR_l is increased, the rectifier current i_t becomes more peaky and rises to a high value. This large tube current may damage the rectifier tube in spite of the fact that the d.c. component of current may not exceed the rating. This is particularly important for gas tube rectifiers. To reduce the possibility of damage, an inductor or a resistor is often used in series with the rectifier.

Ripple factor in halfwave vacuum tube rectifier with capacitor filter. Calculation of ripple factor using the above methods is difficult and an approximate calculation may be made assuming that the load current decays by only a small percentage during the non-conduction interval. This assumption yields simplified and accurate expressions for E_{dc} and ripple factor. Load current curve

may be taken as formed of straight lines. This waveform is shown in Fig. 9·11.

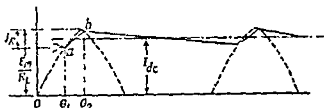


Fig. 9.11.—Approximate load current waveform of halfwave vacuum tube rectifier with capacitor filter.

The approximate load current waveform is seen to have a peak value of almost $\frac{E_m}{R_l}$. During the conduction interval, the load current increases along the straight line ab instead of along the sine curve due to the presence of tube resistance which has been neglected in the analysis. The total change in load current is shown as I_R. The average or d.c. value of this triangular wave is approximately halfway between the maximum and minimum as given by points b and a. Thus we get :

$$I_{dc}=I_m-\frac{I_R}{2} \qquad ...(9·32)$$

where I_m is the maximum value of load current and is equal to E_m/R_l.

During the discharge interval of θ_2 to $(2\pi+\theta_1)$, the capacitor is assumed to lose charge at a constant rate given by current I_{dc}. But voltage across the condenser C is given by $e_c=q/c$ so that the rate of loss of potential across the condenser is given by :

$$\frac{de_c}{dt}=\frac{1}{\omega c}\cdot\frac{dq}{d(\omega t)}=\frac{I_{dc}}{\omega c} \qquad ...(9·33)$$

If $E_R(=I\cdot R_l)$ is the peak-to-peak change in output voltage, then.

$$\frac{de_c}{dt}=\frac{E_R}{2\pi-(\theta_2-\theta_1)}=\frac{E_R}{2\pi-\theta_c} \qquad ...(9·34)$$

From equations (9·33) and (9·34) we get :

$$\frac{E_R}{2\pi-\theta_c}=\frac{I_{dc}}{\omega c}$$

or

$$E_r=\frac{(2\pi-\theta_c)\,I_{dc}}{\omega c} \qquad ...(9·35)$$

The *r.m.s.* value of the ripple component of the triangular wave is independent of the slopes or lengths of the straight lines but depends only on the peak value I_R. Taking the time axis along the I_{dc} curve, it can be shown that for such a triangular wave, the

r.m.s. value of the a.c. component of current is given by :

$$I_{ac}=\frac{I_R}{2\sqrt{3}} \qquad \ldots(9{\cdot}36)$$

The *r.m.s.* value of the a.c. component of output voltage is given by :

$$E_{ac}=\frac{I_R \,.\, R_l}{2\sqrt{3}}=\frac{E_R}{2\sqrt{3}} \qquad \ldots(9{\cdot}37)$$

Hence ripple factor$=\dfrac{E_{ac}}{E_{dc}}=\dfrac{E_R}{2\sqrt{3}.I_{dc}.R_l}$

$$=\frac{(2\pi-\theta_c)\,I_{dc}}{(\omega c)2\sqrt{3}\,I_{dc}.R_l}$$

$$=\frac{(2\pi-\theta_c)}{2\sqrt{3}\,\omega cR_l} \qquad \ldots(9{\cdot}38)$$

If θ_c is small compared with 2π, then the ripple factor is given by the approximate expression :

$$\nu=\frac{2\pi}{2\sqrt{3}\omega cR_l}=\frac{1}{2\sqrt{3}\,fcR_l} \qquad \ldots(9{\cdot}38\ a)$$

The regulation curve is obtained by combining equations 9·32 and 9·35 as below :

$$E_{dc}=I_{dc}.R_l=E_m-\frac{E_R}{2}$$

or

$$E_{dc}=E_m-\frac{(2\pi-\theta_c)\,I_{dc}}{2\omega c} \qquad \ldots(9{\cdot}39)$$

This equation shows a linear fall in potential with d.c. output current. This equation also shows that the regulation is poor unless the capacitance C is large.

Fullwave Vacuum tube rectifier with Capacitor filter. In the case of fullwave rectifier, the analysis is similar to above except that two pulses of current are received by the capacitor from the source in each cycle. Peak inverse voltage across each rectifying element is $2E_m$ similar to the case of a halfwave rectifier. Since there are now two pulses of current per cycle, the load current reduces by a smaller amount before the next pulse is received. The average or the d.c. value of load current I_{dc} is, therefore, increased. The ripple factor is correspondingly reduced. For fullwave vacuum tube rectifier using capacitor filter, the approximate expression for the ripple factor may be found in the same way as for the halfwave circuit, and is given below :

$$\text{Ripple factor } \nu=\frac{(\pi-\theta_c)}{2\sqrt{3}\,\omega cR_l} \qquad \ldots(9{\cdot}40)$$

If θ_c is small compared with π, then ripple factor is given by the approximate expression :

$$\nu=\frac{1}{4\sqrt{3}\,fcR_l} \qquad \ldots(9{\cdot}40\ a)$$

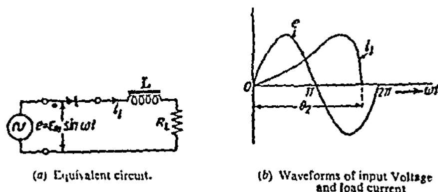
A comparative study of equation (0·38a) and (0·40a) reveals that if the angle of conduction θ_c is small compared with π, then ripple factor for fullwave rectifier is half that for the halfwave rectifier.

The regulation curve for fullwave rectifier may be obtained from the expression for E_{dc} in terms of E_m and I_{dc} as given below :—

$$E_{dc}=E_m-\frac{(\pi-\theta_c)\,I_{dc}}{2\omega C} \quad \dots (0{\cdot}41)$$

Equation 0·41 shows that the regulation is poor in this case as well but is better than that of a halfwave rectifier.

Halfwave rectifier with inductor filter :



(a) Equivalent circuit. (b) Waveforms of input Voltage and load current

Fig. 9 12 Halfwave rectifier with inductor filter.

An inductor in series with the rectifier element and the load, presents another simple filter arrangement. An inductor oppo-es ... ugh it. This means ... when the current is ... red energy when the ... operty is made use ... it that may occur in the circuit is thus smoothed away by this inductor Fig. 9 12 (a) shows the circuit of a halfwave rectifier with resistance load and inductor filter. For simplicity of analysis it is assumed that the rectifier element is ideal i.e., it has no resistance Choke is also assumed to be having no resistance.

The differential equation for the current in the circuit may be written as :—

$$L\frac{di_1}{dt}+R_L i_1=E_m \sin \omega t \quad \dots (0{\cdot}42)$$

Further we assume that the current is zero at time $t=0$. Then equation 0·42 may be solved and the final solution is

$$i_1=I_m\left[\sin(\omega t-\theta_z)-\sin(-\theta_z)\,e^{-\frac{R_L}{L}t}\right]$$

where I_m is the amplitude of the steady-state current and is given by

$$I_m = \frac{E_m}{\sqrt{R_l^2 + (\omega L)^2}} \qquad ...(9{\cdot}44)$$

and θ_z is the impedance angle of the total impedance $(R_l + j\omega L)$ in the circuit and is given by

$$\tan \theta_z = \frac{\omega L}{R_l} \qquad ...(9{\cdot}45)$$

The total load current is the difference of two currents :—

(*i*) the steady state current $I_m \sin(\omega t - \theta_z)$ produced by the voltage $E_m \sin(\omega t)$ in the RL circuit and

(*ii*) current $I_m \sin(-\theta_z)$ which is the value of the steady state current at time t equal to zero.

Thus the development of current is delayed and the current reaches the value zero at an angle θ_c which lies between π and 2π. The current can never become negative because of the presence of rectifier element. Thus current is zero for the angle θ_c to 2π.

The conduction angle θ_c depends upon R and ωL. At θ_c, i_l is equal to zero and hence

$$I_m \sin(\omega t - \theta_z) - I_m \sin(-\theta_z)\, \epsilon^{-\frac{R_l \omega t}{\omega L}} = 0$$

$$\text{or} \quad \sin\left(\theta_c - \tan^{-1}\frac{\omega L}{R_l}\right) + \frac{\omega L}{\sqrt{R_l^2 + (\omega L)^2}}\, \epsilon^{-\frac{R_l \theta_c}{\omega L}} = 0 \qquad (9{\cdot}46)$$

If we put $\theta_c = 2\pi - \theta_c'$ equation (9·46) becomes

$$\sin\left(\tan^{-1}\frac{\omega E}{R_l} + \theta_c'\right) = \sin\left(\tan^{-1}\frac{\omega L}{R_l}\right) \epsilon^{-\frac{R_l(2\pi - \theta_c')}{\omega L}} \qquad ...(9{\cdot}47)$$

This equation is the same as for condenser filter if we put $\frac{\omega L}{R_l}$ equal to $\omega c R_l$. Graphical solution is possible in order to obtain the value of θ_c.

It may be seen that as the value of inductance L is increased, angle θ_c increases. In the limit if the ratio $\frac{\omega L}{R_l}$ is increased to infinity, the angle of conduction θ_c becomes 2π. Fig. 9·13 shows the effect of increase of inductance L on the waveshape of current and on the angle of conduction θ_c. The peak value of the load current is, however, found to decrease with the increase of the angle of conduction.

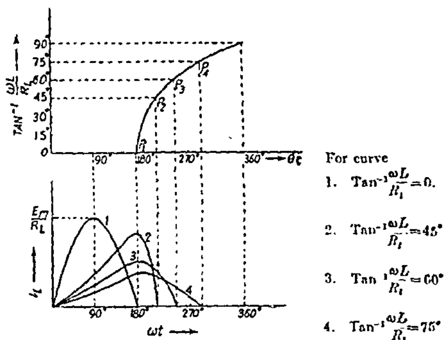


Fig. 9·13. Conduction angle θ_c and load current waveforms for different values of $\tan^{-1}\frac{\omega L}{R_l}$ for a halfwave rectifier with inductance filter

The d.c. component of load current is the area under the current curve divided by the duration of application of voltage namely 2π radians. From Fig. 9·13 it is apparent that the area under curve 1 is maximum and that under the curve 4 is the least. Thus it is seen that the average or d.c. value of load current reduces as the value $\frac{\omega L}{R_l}$ increases whereas the smoothing effect increases with the increase of ωL for a fixed value of load resistance R_L. In the case of capacitor filter, on the other hand, with the increase of $\omega c R_L$, the smoothing effect as well as the d.c component of load current increase.

The expression for the d.c component of load current may be obtained from equation 9·42. Thus we have,

$$i_l = \frac{E_m}{R_l}\sin\omega t - \frac{\omega L}{R_l}\cdot\frac{d\,i_l}{d(\omega t)} \qquad \dots (9\cdot48)$$

Hence $$I_{dc} = \frac{1}{2\pi}\int_0^{\theta_c}\left[\frac{E_m}{R_l}\sin\omega t - \frac{\omega L}{R_l}\cdot\frac{d\,i_l}{d(\omega t)}\right]d(\omega t)$$

or $$I_{dc}=\frac{E_m}{2\pi R_l}(1-\cos\theta_c) \quad \ldots (9\cdot50)$$

As $\tan^{-1}\frac{\omega L}{R_l}$ is increased, θ_c increases and hence I_{dc} reduces.

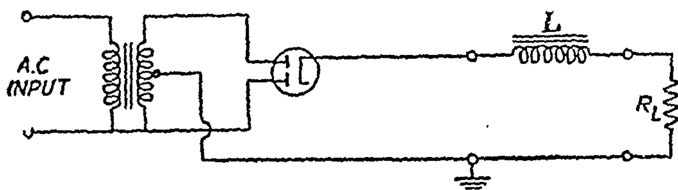
The d.c. component of voltage across the load is given by

$$E_{dc}=I_{dc}\,.\,R_l=\frac{E_m}{2\pi}(1-\cos\theta_c) \quad \ldots (9\cdot51)$$

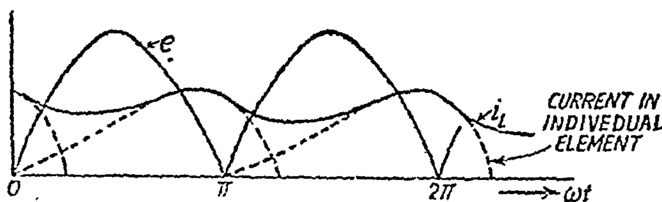
When the conduction ceases, the voltage across the rectifier element changes suddenly to the corresponding negative value of the supply voltage. The maximum peak inverse voltage, therefore, is E_m.

A simple inductance filter is seldom used with a halfwave rectifier mostly because of low d.c. component of load current and high ripple factor.

Fullwave rectifier with inductor filter. Fig. 9·14(*a*) shows the circuit arrangement of fullwave rectifier with inductor filter. Since the conduction period of a single rectifier element extends beyond π radians, the conduction in the second rectifier element begins before the current in first has ceased. Thus the current through the *inductor and* load resistance never drops to zero. Fig. 9·14(*b*) shows the waveshape of the output current. Since current never falls to zero, the analysis is of entirely different form from that in the halfwave rectifier.



(*a*) Circuit diagram.



(*b*) Waveshape of the load current.

Fig. 9·14. Circuit diagram and current waveshape in a fullwave rectifier using inductor filter.

An approximate analysis is given below :—

The voltage at the output of the fullwave rectifier is given by equation 9·9. The lowest frequency term is the second harmonic of the supply frequency. The next term *i.e.*, fourth harmonic term has an amplitude only 20 per cent of the second harmonic amplitude. The higher harmonic terms have still smaller amplitudes. In the approximate analysis we may neglect all the harmonic terms except the second harmonic. An approximate expression for the output voltage is then given by

$$e=\frac{2E_m}{\pi}-\frac{4E_m}{3\pi}\cos 2\omega t \qquad \text{... (9·52)}$$

The impedance to d.c. component of voltage $\frac{2E_m}{\pi}$ is only R_l whereas the impedance to the second harmonic term is $Z_l=R_l+j.2\omega L$. Hence the load current is given by

$$i_l=\frac{2E_m}{\pi R_l}-\frac{4E_m}{3\pi}\cdot\frac{\cos(\omega t-\psi)}{\sqrt{R_l^2+4\omega^2L^2}} \qquad \text{... (9·53)}$$

where $\psi=\tan^{-1}\frac{2\omega L}{R_l}$. (9·54)

Then the ripple factor is given by

$$\gamma=\frac{I_{ac}}{I_{dc}}=\frac{\frac{1}{\sqrt{2}}\left(\frac{4E_m}{3\pi}\right)\times\left(\frac{1}{\sqrt{R_l^2+4\omega^2L^2}}\right)}{\frac{2E_m}{\pi R_l}}$$

$$=\frac{2R_l}{3\sqrt{2}}\times\frac{1}{\sqrt{R_l^2+4\omega^2L^2}} \qquad \text{... (9·55)}$$

or

$$\gamma=\frac{2}{3\sqrt{2}}\times\frac{1}{\sqrt{1+\left(\frac{4\omega^2L^2}{R_l^2}\right)}} \qquad \text{.. (9·56)}$$

If the ratio $\frac{2\omega L}{R_l}$ is large, 1 may be neglected compared with $\frac{4\omega^2L^2}{R_l^2}$ and the equation 9·56 reduces to

$$\gamma=\frac{1}{3\sqrt{2}}\cdot\frac{R_l}{\omega L} \qquad \text{... (9·57)}$$

From equation 9·57 we infer that the filtering operation improves as the load resistance is reduced *i e*, as the load current is increased. The poorest filtering occurs at no load corresponding infinitely large R_l (ratio $\frac{\omega L}{R_l}$ equal to zero) in which case from

equation 9·56 we get

$$\gamma = \frac{2}{3\sqrt{2}} = 0{\cdot}47.$$

The same result is obtained when L is reduced to zero *i.e.*, no filter is used. The ripple factor of the output current of a full-wave rectifier with resistance load and no filter has been found to 0·482. This difference comes in because in the present approximate analysis higher harmonic terms have been neglected.

The d.c. component of output voltage is given by

$$E_{dc} = I_{dc} \cdot R_l = \frac{2E_m}{\pi} = 0{\cdot}637\ E_m \qquad \ldots (9{\cdot}58)$$

A number of assumptions have been made in the above approximate analysis. These assumptions are :—

(*i*) Power transformer leakage reactance is zero.

(*ii*) Power transformer resistance is zero.

(*iii*) Tube resistance and inductor resistance are zero.

and (*iv*) Output potential does not change with load.

The effects of these assumptions are not negligible in general and thus in practice we find that the output potential actually decreases with increase of current.

Inductors are usually used in series with mercury-arc rectifiers using polyphase circuit. The cathode current then never falls to zero and hence the arc never extinguishes.

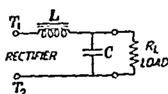
L-C Filters. If only a shunt condenser is used as filter element, then the ripple voltage decreases as the size of the condenser increases assuming a constant load current. But the peak and the effective values of current through the rectifier element tend to increase with the size of the condenser. This is particularly harmful in rectifier using gas-tube and a power source of low impedance. In that case excessively large current may damage the cathode by positive ion bombardment unless the tube current is limited by other means. Another drawback is caused by the increased effective value of rectifier current with increased capacitor size. An increased effective value of current causes increased heating in the windings of the transformer or the power supply circuit, and this decreases the efficiency and the overall direct current rating. On the other hand, a series inductor reduces both the peak and effective values of the rectifier current. This property of series inductor may be utilized to eliminate the undesirable property of a condenser of increasing the peak and the effective values of rectifier current. Thus a series inductor and a shunt condenser may be used very conveniently. Series inductor, however, has the drawback of decreasing the voltage and current output of the rectifier. Hence a full-

wave rectifier with filter using both series inductor and shunt capacitor may be used for giving a very smooth output voltage in small and medium rectified power supply systems *i.e.*, supply systems delivering voltages of 100 to 1000 volts and d.c. currents of upto about 500 milliamperes

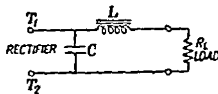
These filters consisting of series inductor and shunt condenser are of two types :

(*i*) *Inductance input filter* :—One using a series inductor immediately following the rectifier tube and followed by a shunt condenser as shown in Fig. 9·15(*a*).

(*ii*) *Condenser input filter* :—One using a shunt condenser immediately across the rectifier circuit and followed by a series inductor as shown in Fig. 9·15(*b*).



(*a*) Inductance Input Filter



(*b*) Condenser Input Filter

Fig. 9·15. Circuits of inductance input and condenser input filters

Inductance input filter An approximate expression for ripple factor in a fullwave rectifier using inductance input filter may be readily obtained by neglecting all other but the first two terms in the Fourier series expression of the output voltage of the rectifier. Thus we may express the output voltage of the full-wave rectifier as below

$$e=\frac{2E_m}{\pi}-\frac{4E_m}{3\pi}\cos 2\omega t \qquad \text{.. (9·58)}$$

It is further assumed that the impedance of the inductor or choke is high compared with the effective parallel impedance of the capacitor and load resistor so that the impedance between terminals T_1 and T_2 is approximately X_l. Then the r.m s value of the current through the circuit is given by

$$I'_{ac}=\frac{4E_m}{3\sqrt{2}\pi}\cdot\frac{1}{X_l}=\frac{\sqrt{2}}{3}E_{dc}\cdot\frac{1}{X_l} \qquad \text{... (9·59)}$$

Again, the reactance of the condenser is assumed to be small compared with the resistance R_l so that all the a.c. current may be assumed to be flowing through the condenser alone and nothing through the load resistance R_l. Then the a.c. potential across the load is the same as the potential across the condenser and is given

by

$$E'_{ac} = \frac{\sqrt{2}}{3} E_{dc} \cdot \frac{X_c}{X_l} \quad \ldots (9\cdot60)$$

Then the ripple factor γ is given by

$$\gamma = \frac{E'_{ac}}{E_{dc}} = \frac{\sqrt{2}}{3} \cdot \frac{X_c}{X_l} = \frac{\sqrt{2}}{3} \cdot \frac{1}{2\omega c} \cdot \frac{1}{2\omega L} \quad \ldots (9\cdot61)$$

For supply frequency of 50 c.p.s., ripple factor γ is given by

$$\gamma = \frac{1\cdot195}{LC} \quad \ldots (9\cdot62)$$

where L is in Henrys and C is in micro-farads.

In the above analysis it has been assumed that the current through the inductor flows throughout the cycle. To obtain this condition the inductance L must be kept above a certain minimum value which may be called the critical inductance. This critica value of inductance depends on load resisalnce R_l and frequency ω. Fig. 9·16 shows the nature of the waveform of current through the inductor L in a fullwave rectifier using inductor input filter for different values of inductance L relative to critical inductance L_c assuming constant I_{dc}.

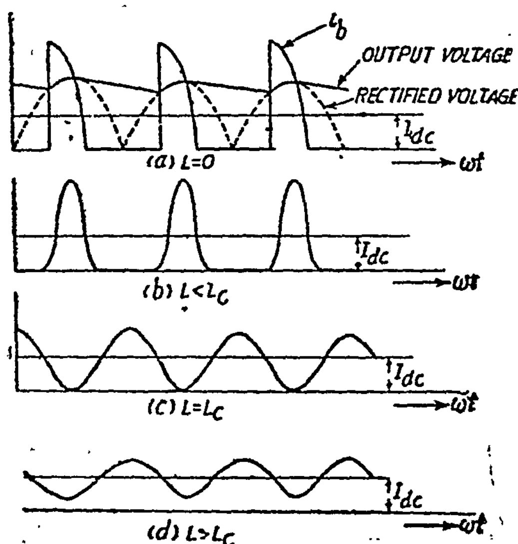


Fig. 9·16. Waveform of current through the inductor in fullwave rectifier using inductor-input filter for different values of inductance L.

In case the current cutout takes place at any time of the cycle, then the analysis would follow the one used for capacitor filter.

It is apparent from Fig. 9·16 that if the current through the inductor is required to flow continuously, then peak a.c. current must be either equal to or less than the d.c. component. The d.c. component of current is E_{dc}/R_l whereas the peak a.c. current is found from equation 9·59 to be $\frac{2}{3} \cdot \frac{E_{dc}}{X_l}$. Hence the condition for continuous flow of current may be put as :

$$\frac{E_{dc}}{R_l} \geqslant \frac{2E_{dc}}{3\ X_l}$$

or

$$X_l \geqslant \frac{2R_l}{3} \qquad \text{... (9·63)}$$

The critical value of inductance is then given by :—

$$L_e = \frac{2\ R_l}{3\ .\ (2\omega)} \qquad \text{... (9·64)}$$

For supply frequency of 50 c/s, we have,

$$L_e = \frac{2\ R_l}{3 \times 2\pi \times 100} = \frac{R_l}{942} \qquad \text{... (9·65)}$$

where R_l is in ohms and L_e is in Henrys.

A number of approximations have been made in this analysis and hence for a safer design it is advisable to use a large value of L_e than is given by Eq. 9·65. In practice a figure of 800 may be used as the denominator in equation 9·65 instead of 942.

The effect of current cutout becomes apparent when we plot the ratio $\frac{E_{dc}}{E_m}$ against inductance keeping R_l constant as shown in Fig. 9·17. When the inductance is zero, the L-C filter reduces to simple capacitor filter and the output potential is almost constant at value E_m. As the inductance is increased the output d.c. potential E_{dc} falls rapidly until L equals the critical value L_e. At this point $E_{dc} = \frac{2\ E_m}{\pi} = 0·637\ E_m$. As L is increased beyond the critical value L_e, there is no change in the value of E_{dc} in the ideal case but because of the presence of resistances of the various elements of the circuit, in practice E_{dc} falls with increase of inductance L.

Equation 9·65 may be used for calculating L_e in all practical cases but under no load condition this will mean infinitely large inductance. This situation is avoided by permanently connecting a resistor called the "bleeder resistor" across the load. The value of this resistance is chosen to satisfy equation 9·65. On application of load resistance, the total shunt resistance would be reduced so that the inductance present in the filter circuit would then exceed the value L_e under this changed condition. By this means E_{dc} does not reduce appreciably on putting the load. Regulation is thus improved by this bleeder resistance. Bleeder resistance serves another purpose of keep-

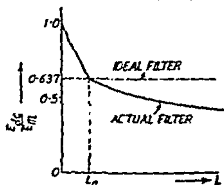


Fig. 9·17. Regulation curve of an inductance input filter as a function of inductance L with constant I_{dc} and R_l.

ing the output voltage from increasing excessively and causing possible puncture of the dielectric of the capacitor if the load resistor gets disconnected accidently. The only drawback of bleeder resistance is the power loss associated with it.

As an alternative to the use of bleeder resistor is the use of swinging choke which has high inductance at low values of d.c. current through it and whose inductance decreases substantially with increased d.c. current. This swinging choke has closed iron core whereas core of a constant inductance choke has a narrow air gap. Fig. 9·18 shows the nature of the variation of inductance with d.c. current in a typical case.

For high load impedance R_l d.c. current is low and consequently this swinging choke has high inductance. Hence the condition that inductance L is equal to or greater than L_c as given by equation 9·65 is satisfied over a wider range of load resistance R_l. However since inductance L is not constant and varies with I_{dc} or R_l, the ripple factor is obviously not independent of load.

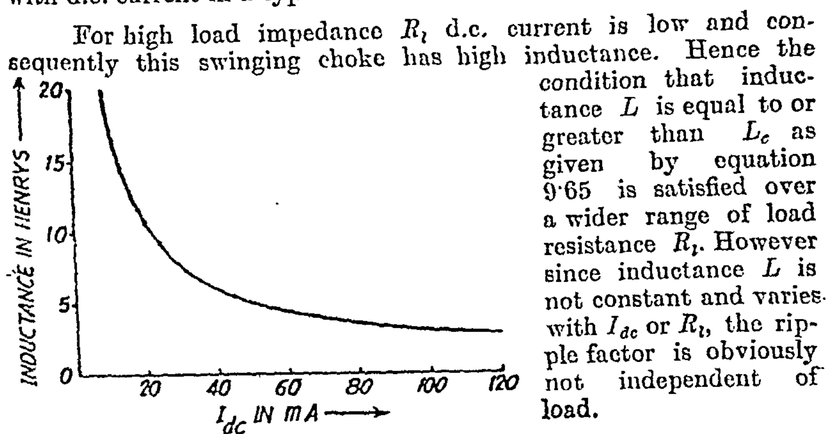


Fig. 9·18. Variation of inductance of a swinging choke with d.c. current through it.

Multi-section inductance input filter :—Two or more sections of inductance input filters are used in cascade when it is required to reduce the ripple factor to very small magnitude *i.e.*, to obtain better smoothness of output d.c. voltage. Fig. 9·19 shows a two-section inductance input filter.

An approximate analysis following the method used for single section inductance input filter is given below:

Assumptions made are: (*i*) Choke impedances are much greater than the capacitor reactances and (*ii*) load resistance R_l is much greater than the reactance of condenser C_2.

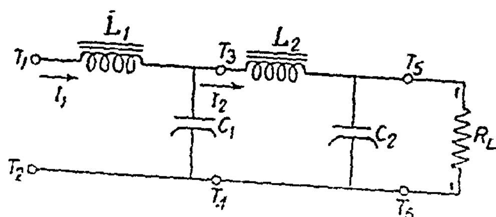


Fig. 9·19. A two-section inductance input filter.

Then the impedance between terminals T_5 and T_6 becomes X_{c2} (re-

actance of condenser C_2). Again the impedance between terminals T_3 and T_4 becomes X_{c1} and that between terminals T_1 and T_2 becomes X_{L1}.

Let I_1 and I_2 be the a.c. currents through chokes L_1 and L_2 respectively. Then I_1 in accordance with equation 9·59 is given by

$$I_1=\frac{\sqrt{2}}{3} \cdot E_{dc} \cdot \frac{1}{X_{L1}} \quad \text{... (9·66)}$$

Hence the a.c. voltage across the condenser C_1 is given approximately by

$$E_{T3\,T4}=I_1 \cdot X_{c1} \quad \text{... (9·67)}$$

The a.c. current I_2 through the choke L_2 is then given approximately by

$$I_2=\frac{E_{T3\,T4}}{X_{L2}}=I_1 \cdot \frac{X_{c1}}{X_{L2}} \quad \text{... (9·68)}$$

The a.c voltage across the load is then given approximately by

$$E_{T5\,T6}=I_2 \cdot X_{c2}=I_1 \cdot \frac{X_{c1}}{X_{L2}} X C_2$$

$$=\frac{\sqrt{2}}{3} \cdot E_{dc} \cdot \frac{X_{C1}}{X_{L1}} \frac{X_{C2}}{X_{L2}} \quad \text{... (9·69)}$$

The ripple factor γ is then given by the expression

$$\gamma=\frac{E_{T5\,T6}}{E_{dc}}=\frac{\sqrt{2}}{3} \frac{X_{C1}}{X_{L1}} \frac{X_{C2}}{X_{L2}} \quad \text{.. (9·70)}$$

In general if n sections of inductance input filter be used, then the ripple factor is given by

$$\gamma=\frac{\sqrt{2}}{3} \cdot \frac{X_{C1}}{X_{L1}} \cdot \frac{X_{C2}}{X_{L2}} \cdots \cdots \frac{X_{Cn}}{X_{Ln}} \quad \text{. (9·71)}$$

If all the sections are similar, so that

$$X_{C1}=X_{C2}= \ldots=X_{Cn}=X_C$$

and $X_{L1}=X_{L2}=X_{L3}= \ \ =X_{Ln}=X_L$,

then $$\gamma=\frac{\sqrt{2}}{3} \cdot \left(\frac{X_C}{X_L}\right)^n=\frac{\sqrt{2}}{3} \cdot \left[\frac{1}{(2\omega C)} \cdot \frac{1}{(2\omega L)}\right]^n$$

$$=\frac{\sqrt{2}}{3} \cdot \frac{1}{(16\pi^2 f^2 LC)^n} \quad \text{... (9·72)}$$

For a 50 c/s power source, the LC product of a n-section, inductance-input filter is given by

$$LC=2\,53 \cdot \left(\frac{0·471}{\gamma}\right)^{1/n} \quad \text{... (9·73)}$$

where L is in Henrys and C is in microfarads.

$X_L \gg X_C$, the output ripple voltage at angular frequency 2ω is given by

$$E(_{2\omega}) = E'(_{2\omega}) \; . \; \frac{X_{C_2}}{X_L} \qquad \dots (9{\cdot}82)$$

Hence ripple factor is given by :

$$\gamma_\pi = \frac{E(_{2\omega})}{E_{dc}} = \frac{E'\,(2\omega).X_{C_2}}{E_{dc}\,.\,X_L}$$

$$= \frac{\sqrt{2}.I_{dc}.X_{C_1}.X_{C_2}}{E_{dc}.X_L} = \frac{\sqrt{2}.X_{C_1}.X_{C_2}}{X_L\,.\,R_L} \qquad \dots (9{\cdot}83)$$

Substituting the value of reactances X_{C_1}, X_{C_2} and X_L in equation 9·83 we get :

$$\pi = \frac{\sqrt{2}}{8\omega^3.C_1.C_2.L.R_L} \qquad \dots (9{\cdot}84)$$

For supply frequency of 50 c/s, Eq. 9·84 reduces to :

$$\gamma_\pi = \frac{5{\cdot}7\times10^3}{C_1\,C_2\,L\,R_L} \qquad \dots (9{\cdot}85)$$

where C_1, and C_2 are in microfarads, L is in Henrys and R_L is in ohms.

For supply frequency of 60 c/s, Eq. 9·84 becomes :

$$\gamma_\pi = \frac{3{\cdot}3\times10^3}{C_1\,C_2\,L\,R_L} \qquad \dots (9{\cdot}85a)$$

Often sufficient filtering is obtained if the inductance L in the π-filter is replaced by a resistor R. Then the ripple factor is given by replacing X_L by R.

$$\text{Hence } \gamma_\pi = \frac{\sqrt{2}.X_{C_1}.X_{C_2}}{R.R_L} \qquad \dots (9{\cdot}87)$$

For supply frequency of 50 c/s,

$$\gamma_\pi = \frac{\sqrt{2}}{(4\pi\times50C_1).(4\pi\times50C_2)R.R_L} = \frac{3{\cdot}6\times10^6}{C_1.C_2.R.R_L} \qquad \dots (9{\cdot}88)$$

where C_1 and C_2 are in microfarads and R and R_L are in ohms.

With the use of this resistor R instead of inductance L, the change of output voltage with load and the value of capacitance required tend to increase. These drawbacks are compensated by the saving in cost, bulk and weight. Sometimes several sections of R-C filter are used to provide good smoothing.

Condenser input filter is used mostly in small single phase power supplies such as for radio receivers. It has the advantages of

(*i*) higher smoothing factor *i.e.*, smaller ripple factor than a multi-section inductance input filter with same total value of capacity and inductance.

(*ii*) higher d.c. output voltage. At light loads the d.c. voltage approximates the peak secondary a.c. voltage E_m.

It has the disadvantages of :

(*i*) higher regulation,

(*ii*) increased peak anode current. This becomes particularly important in mercury vapour tubes,

(*iii*) higher peak inverse voltage. The peak inverse voltage is equal to the crest secondary voltage plus the condenser voltage at the instant under consideration. Since under light loads, the condenser voltage approximates to the crest secondary voltage, the peak inverse voltage approaches a value twice the crest secondary voltage *i.e.*, $2E_m$.

Example 1. *A halfwave vacuum tube rectifier has a load resistance of 10,000 ohms and a shunt filter condenser of 10 microfarads. The applied voltage has amplitude of 200 volts and frequency of 50 cycles per second. The angle of conduction is 24 degrees. Assuming the rectifier to be ideal, calculate (a) extinction angle* θ_2 *(b) ignition angle* θ_1 *(c) d.c. output voltage* E_{dc} *and (d) ripple factor using approximate expression assuming load current curves to be straight lines. Compare this value of ripple factor with the one obtained without filter condenser.*

Solution. (*a*) Extinction angle θ_2 is given by

$$\tan\theta_2 = -\omega CR_L$$

Substituting the values we get

$$\tan\theta_2 = -(2\pi\times 50)\,.\,(10\times 10^{-6})\times 10^4$$
$$= -10\pi$$

Hence $\theta_2 = 91{\cdot}8°$

(*b*) Ignition angle θ_1 is given by :

$$\theta_c = \theta_2 - \theta_1$$

Hence $\theta_1 = \theta_2 - \theta_c = 91{\cdot}8° - 24° = 67{\cdot}8°$

(*c*) d.c. output voltage E_{dc} is given by

$$E_{dc} = \frac{E_m}{2\pi}\sqrt{1+(\omega C\,R_L)^2}\,.\,(1-\cos\theta_c)$$

$$= \frac{200}{2\pi}\sqrt{1+(31{\cdot}42)^2}\,.\,(1-0{\cdot}9135)$$

$$= \frac{200\times 31{\cdot}42\times 0{\cdot}0865}{2\pi} = 86{\cdot}5 \textbf{ volts}$$

(*d*) Ripple factor of halfwave vacuum tube rectifier with condenser filter is given by :

$$\gamma = \frac{2\pi-\theta_c}{2\sqrt{3}\,\omega C\,R_L}$$

θ_c expressed in radians is $= \frac{24\pi}{180} = \frac{2\pi}{15}$

Hence $\gamma = \frac{2\pi - (2\pi/15)}{2\sqrt{3}(10\pi)} = 0{\cdot}054$

The ratio of this ripple factor to that for rectifier without filter condenser is $= 0{\cdot}054/1{\cdot}21 = 0{\cdot}044$.

Example 2. *A halfwave vacuum tube rectifier uses filter inductance of 30 henrys and load resistance is 10 kilo-ohms. A sinusoidal voltage of amplitude 200 volts and frequency 50 c/s is applied to the rectifier. If the corresponding angle of conduction θ_c is 220 degrees, calculate the d.c. component of load current and the d.c. output voltage.*

Solution. D.C. load current in halfwave vacuum tube ectifier with inductance filter is given by

$$I_{dc} = \frac{E_m}{2\pi R_L}(1 - \cos\theta_c)$$

Substituting the values of E_m, R_L and θ_c :

$$I_{dc} = \frac{200}{2\pi \times 10^4}(1 - \cos 220°) = \frac{1}{100\pi}(1 + 0{\cdot}766)$$

$$= 0{\cdot}005623 \text{ ampere} = \mathbf{5{\cdot}623 \text{ miliamp.}}$$

The d.c. output voltage is then given by

$$E_{dc} = I_{dc}.R_L = 5{\cdot}623 \times 10^{-3} \times 10^4 \text{ volts}$$

$$= \mathbf{56{\cdot}23 \text{ volts.}}$$

Example 3. *A fullwave vacuum tube rectifier uses filter inductance of 25 henrys and load resistance of 10 kilo-ohms. A sinusoidal voltage of amplitude 400 volts and frequency 50 c/s is applied to the input. Assuming the rectified output to be devoid of harmonics higher than the second, find (a) the d.c. load current (b) ripple factor and (c) d.c. output voltage.*

Solution. The rectified voltage in a fullwave rectifier is given by :

$$e = \frac{2E_m}{\pi} - \frac{2E_m}{3\pi}\cos 2\omega t.$$

(*a*) the d.c. component of load current is then given by :

$$I_{dc} = \frac{2E_m}{\pi R_L} = \frac{2 \times 400}{\pi \times 10} = \frac{8}{100\pi} \text{ amperes}$$

$$= \mathbf{25{\cdot}47 \text{ milli-amperes}}$$

(*b*) the a.c. component of load current is given by :

$$I_{ac} = \frac{\frac{1}{\sqrt{2}}\left(\frac{4}{3}\frac{E_m}{\pi}\right)}{\sqrt{R_L^2 + 4\omega^2 L^2}} = \frac{\frac{4 \times 400}{3\sqrt{2}\pi}}{\sqrt{(10^4)^2 + (2 \times 2\pi \times 50 \times 30)^2}}$$

$$= \frac{800 \times 1.414}{3\pi \times 10^4 \times 1.163} \text{ amperes} = 10.31 \text{ milliamperes}$$

Hence ripple factor $= \frac{I_{ac}}{I_{dc}} = \frac{10.31}{25.47} = 0.405$

(c) d.c. output voltage E_{dc} is given by :

$$E_{dc} = I_{dc}.R_L = 25.47 \times 10^4 \times 10^{-3} \text{ volts} = 254.7 \text{ volts.}$$

Example 4. *A fullwave vacuum tube rectifier uses an inductance input filter with inductance L equal to 30 Henries and condenser C equal to 10 micro-farad. Calculate the ripple factor assuming that (i) the current flows throughout the cycle, (ii) all harmonic terms except the second harmonic may be neglected, (iii) reactance X_L of inductor is much greater than the impedance of the parallel combination of C and R_L. (iv) reactance of C is much smaller than R_L. The applied voltage frequency is 50 c/s. Calculate also the critical value of the inductance given that the load resistance is 8 kilo-ohms.*

Solution : The rectified voltage is given by :

$$e = \frac{2\,E_m}{\pi} - \frac{4\,E_m}{3\,\pi} \cdot \cos 2\omega t$$

With the above assumptions, r.m.s. value of the current through the circuit is given by

$$I'_{ac} = \frac{4\,E_m}{3\pi\sqrt{2}} \cdot \frac{1}{X_L} = \frac{\sqrt{2}}{3} \cdot E_{dc} \cdot \frac{1}{X_L}$$

The a.c. potential across the load is then :

$$E'_{ac} = I'_{ac} \cdot X_c = \frac{\sqrt{2}}{3} \cdot E_{dc} \cdot \frac{X_c}{X_L}$$

Hence ripple factor

$$\gamma = \frac{E'_{ac}}{E_{dc}} = \frac{\sqrt{2}}{3} \cdot \frac{X_c}{X_L} = \frac{\sqrt{2}}{3} \times \frac{1}{2\omega L} \times \frac{1}{2\omega C}$$

$$= \frac{\sqrt{2}}{3} \cdot \frac{1}{(4\pi \times 50)^2 . (30 \times 10 \times 10^{-6})} = 0.004.$$

For critical value of inductance, the peak a.c. current equals the d.c. component of current so that

$$\frac{E_{dc}}{R_L} = \frac{4\,E_m}{3\pi \, . \, X_L} = \frac{2\,E_{dc}}{3\,X_L}$$

or

$$X_L = \frac{2\,R_L}{3}$$

Hence critical inductance

$$L_c = \frac{2\,R_L}{3 \times 2\omega} = \frac{2 \times 8 \times 10^3}{3 \times 2 \times (2\pi \times 50)}$$

$$= 8.5 \text{ Henries.}$$

Example 5. *A fullwave vacuum tube rectifier is followed by a 3-section inductance-input filter. Each section of the filter has an inductor of 10 henries and capacitor of 8 micro-farad. Calculate the ripple factor of the output voltage if the applied voltage has a frequency of 50 cycles per second.*

Solution : The ripple factor for multi-section inductance-input filter rectifier is given by the expression :

$$\gamma=\frac{\sqrt{2}}{3}\cdot\left(\frac{X_c}{X_L}\right)^n$$

where X_c and X_L are the reactances of condenser and inductor elements at the second harmonic.

Substituting the values :

$$\gamma=\frac{\sqrt{2}}{3}\cdot\left[\frac{1}{(2\omega C)\,.\,(2\omega L)}\right]^3$$

$$=\frac{\sqrt{2}}{3}\cdot\frac{1}{(16\pi^2\times50^2\times10\times8\times10^{-6})^3}=1\cdot5\times10^{-5}.$$

Example 6. *A π-filter having condensers of value 8 micro-farad each and a choke inductance 15 henries follows a fullwave vacuum tube rectifier. The load is a resistance of 12 kilo-ohms. The input voltage has a frequency of 50 c/s. Calculate the ripple factor of the output voltage assuming all harmonics but the second to be absent from the rectified voltage. Repeat the calculations if the choke is replaced by a resistance of 5 kilo-ohms.*

Solution : Expression for the ripple factor in this case is :

$$\gamma=\frac{\sqrt{2}\,.\,X_{c1}\,.\,X_{c2}}{X_L\,.\,R_L}$$

where X_{c1} and X_{c2} are the condenser reactances and X_L is the choke reactance at the second harmonic.

Substituting the values :

$$\gamma=\frac{\sqrt{2}}{8\times(2\pi\times50)\,.\,(8\times10^{-6})\,.\,(8\times10^{-6})\times15\times12\times10^3}$$

$$=\frac{5\cdot7}{11520}=4\cdot95\times10^{-4}.$$

When the choke has been replaced by the resistor, ripple factor is given by :

$$\gamma=\frac{\sqrt{2}\,.\,X_{c1}\,.\,X_{c2}}{R\,.\,R_L}$$

Substituting the values we get :

$$\gamma=\frac{2}{(4\pi\times50)^2\,.\,C_1\,.\,C_2\,.\,R\,.\,R_L}$$

$$=\frac{3\cdot6\times10^5}{8\times8\times(5\times10^3)\times(12\times10^3)}$$

$$=9\cdot38\times10^{-4}.$$

VOLTAGE MULTIPLYING RECTIFIER CIRCUITS

By voltage multiplying rectifier circuit is meant a rectifi circuit capable of delivering a d.c. voltage two or more times th peak value (amplitude) of the applied alternating voltage. Suc circuits find use in power supplies for X-Ray tubes, in ac-dc radi receivers and other devices where use of power transformer is not permitted.

Common voltage mul[illegible] doublers a[illegible]

Volt[illegible] D[illegible] [illegible] voltage doublers are of two types :— (a) Fullwave voltage doubler or symmetrical voltage doubler and (b) Halfwave voltage doubler or cascade voltage doubler. —(i) Voltage

Fullwave voltage Doubler. Fig. 9 21 shows the circuit of a fullwave voltage doubler. During the positive half cycle of the applied a c voltage, only diode T_1 conducts and charges the condenser C_1 to the peak value E_m of the applied a.c. voltage in the polarity shown. During the negative half cycle of applied a c. voltage, only diode T_2 conducts and charges condenser C_2 to the peak voltage E_m in the polarity shown. The output voltage e_o is the sum of the voltage across the condensers C_1 and C_2. If the load resistance is large, the output voltage e_o is almost twice the peak value of the a c. voltage. The capacitor C_1 and C_2 continually discharge in series through the load resistance R_L. For large values of load resistance, variation in voltage across the condensers is small. These condensers serve the dual purpose of developing the output voltage and smoothing out the pulsations from the output voltage.

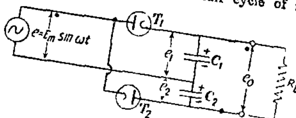


Fig. 9 21. Fullwave voltage doubler.

Fig. 9 22 shows diagrammatically the method of deducing the output voltage from the waveform of input a c. voltage. Dotted line in the diagram shows the applied a c voltage while the solid lines give the voltages e_1 and e_2 developed across the condensers C_1 and C_2. e_1 is positive and e_2 is negative with respect to the potential of point P,

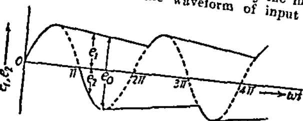


Fig 9 22 Output voltage in a fullwave voltage doubler.

joining the two condensers. The total output voltage e_o across the load resistance R_L equals the difference (e_1-e_2) and is given in the diagram by the vertical spacing between the curves for e_1 and e_2 as shown in Fig. 9·22.

Both halves of the applied a.c. voltage drive current through the output to provide the d.c. output voltage and hence the circuit is generally referred to as "Fullwave voltage doubler". The circuit may also be arranged in the symmetrical form shown in Fig. 9·23 and hence it is sometimes called "Symmetrical voltage doublers". The fullwave voltage doubler has the disadvantage that the load R_L and source have no common point which may be earthed.

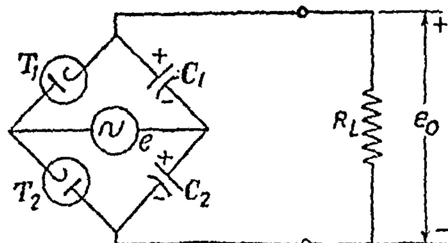


Fig. 9·23. Symmetrical arrangement of fullwave voltage doubler.

Fullwave voltage doubler has the following advantages :—

(*i*) ripple frequency is twice the supply frequency and hence the process of smoothing the output voltage is easier,

(*ii*) maximum voltage across each condenser is only E_m, the peak a.c. voltage and

(*iii*) voltage regulation is better than in halfwave voltage doubler.

Halfwave voltage doubler or cascade voltage doubler :—

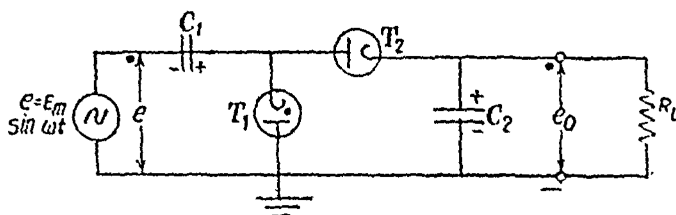


Fig. 9·24. Halfwave voltage doubler.

Fig. 9·24 shows the circuit of a halfwave voltage doubler. During the negative half cycle of the applied a.c. voltage e_1, diode T_1 alone conducts charging the condenser C_1 to the peak value E_m in the polarity shown. During the subsequent positive half cycle, this voltage E_m across the condenser C_1 gets added to the input voltage and this total voltage drives current through diode T_2 charging the condenser C_2 to a voltage equal almost to $2E_m$ in the polarity shown. Condenser C_1 discharges during this process but this charge deficiency is made up during the subsequent negative half cycle. With each cycle of operation, charge transferred from C_1 to

C_2 during the positive half cycle reduces and finally becomes zero when condenser C_2 gets charged to $2E_m$.

This halfwave voltage doubler has the advantage of a common source and load terminal which may be earthed. It has, however, the following disadvantages : (*i*) ripple frequency is the same as the power-supply frequency since only one charging pulse is fed to the output condenser C_2 per cycle, (*ii*) maximum voltage across the condenser C_2 is $2E_m$ and hence condenser of higher voltage rating is required to be used and (*iii*) voltage regulation is poor.

Voltage quadrupler. Fig. 9·25 shows the circuit of a simple voltage quadrupler. Essentially it consists of two cascade voltage

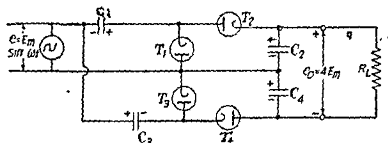


Fig 9 25 Voltage quadrupler circuit

doublers. The first cascade voltage doubler consists of diodes T_1 and T_2 and condensers C_1 and C_2 while the other cascade voltage doubler consists of diodes T_3 and T_4 and condensers C_3 and C_4 Voltage equal almost to $2\ E_m$ is developed across each of the condensers C_2 and C_4 so that the total output voltage e_o is equal to $4\ E_m$.

Voltage Tripler. In the circuit of Fig 9 25 if tube T_3 is removed and the condenser C_3 is short circuited, then the circuit reduces to that of a voltage tripler.

Because of the difference in potential between the cathodes of
sep
fier
coils.

In all the voltage multiplying rectifiers actual d c. voltage developed across the load depends upon the voltage drop in the rectifier tubes. As the load current increases by moderate amount, high voltage drop takes place in the tubes and hence the output voltage drops rapidly. The voltage regulation of these voltage multiplying rectifiers is accordingly poor

Plate supply voltage from low voltage d.c. source. In mobile communication systems radio transmitters and recei-

vers may be housed in an aeroplane, or a moving automobile. In such cases, high d.c. voltage for plates of different electron tubes is required to be derived from low voltage d.c. source, usually a storage battery. Two devices most commonly used for such a purpose are : (*i*) Dynamotor and (*ii*) vibrator (synchronous and non-synchronous types).

Dynamotor. Dynamotor, also called rotary transformer, is a small d.c. machine with two sets of brushes, two armature windings and a common field and it acts like a motor-generator. One armature winding is operated from a low voltage d.c. power source, usually a storage battery, to produce motor operation. The other armature winding serves as a d.c. generator to produce the desired high d c. voltage.

Non-synchronous Vibrator. Fig. 9·26 gives the circuit of a non-synchronous vibrator. Here a storage battery B is used in conjunction with a vibrator contact to change direct current from the battery into an alternating voltage of square waveform. As soon as switch S is closed, current flows through the high resistance coil L. This current energizes a magnet which in its turn attracts the reed towards terminal T_1. When the reed makes contact

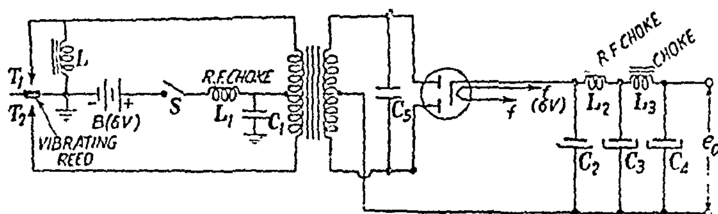


Fig. 9·26. Nonsynchronous vibrator circuit.

with the terminal T_1, coil L gets short circuited and hence the reed is thrown back. The reed then vibrates at its frequency of mechanical resonance and makes contact with terminals T_1 and T_2 alternately. When the reed makes contact at terminal T_1, current flows through the upper half of the primary winding of the transformer. On the other hand when the reed makes contact at terminal T_2, current of reverse polarity flows through the lower half of the primary winding of the transformer. Thus an alternating current of square waveform is applied to the primary winding. A stepped up a.c. voltage of square waveform appears at the secondary terminals. This secondary voltage is applied to a fullwave rectifier. The output of the rectifier is filtered for both the r.f. components and ripple frequency components. The output so obtained is a high d.c. voltage which may be used as plate supply voltage for electron tubes in receivers and transmitters.

R.F. choke L_1 and condenser C_1 constitute the R.F. filter used

for removing r.f. components from the current through the transformer primary.

Synchronous Vibrator. This differs from the non-synchronous vibrator in that instead of using a fullwave rectifier on the secondary side, rectifier effect is achieved by using a second set of contact mounted on the same vibrating reed as shown in Fig. 9·27.

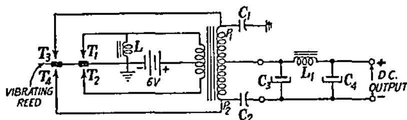


Fig. 9 27. Synchronous vibrator circuit

When the vibrating reed makes contact at terminals T_1 and T_3, current flows through the upper half of primary winding and point P_1 gets earthed through terminal T_3 Hence voltage across the upper half of secondary winding is applied to the π-filter (consisting of L_1, C_3 and C_4). When the reed makes contact at terminals T_2 and T_4, current flows through the lower half of the primary winding and point P_2 gets earthed through terminal T_4. Hence voltage across the lower half of secondary winding is applied to the π-filter in the same direction.

Filter circuit uses a capacitor at the input to avoid sparking at the instant when contacts break the circuit.

EXERCISES

1. For a halfwave vacuum tube rectifier with capacitance filter, obtain an [illegible] load current curve [illegible] non-conduction inte[illegible] [illegible]0 c/s, filter condenser is of 8 micro-farad capacity and load resistance is 8 kilo-ohms, calculate the ripple factor if the angle of conduction is [illegible]6 radian.

2. Derive an expression for the d c load current in a halfwave vacuum tube rectifier with inductance filter and resistance load. Calculate the values of d.c load current and d.c. output voltage, given that the peak value of applied sinusoidal voltage is 220 volts, load resistance is 8 kilo-ohms and the angle of conduction is 240 degrees.

Explain why a simple inductance filter is never used in practice in conjunction with a halfwave rectifier.

vers may be housed in an aeroplane, or a moving automobile. In such cases, high d.c. voltage for plates of different electron tubes is required to be derived from low voltage d.c. source, usually a storage battery. Two devices most commonly used for such a purpose are: (*i*) Dynamotor and (*ii*) vibrator (synchronous and non-synchronous types).

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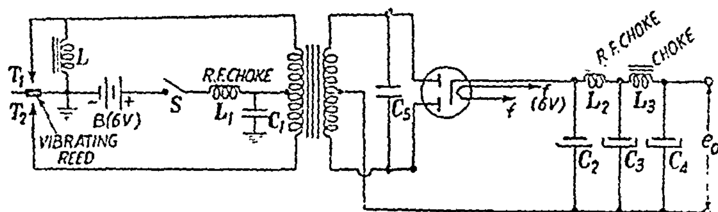


Fig. 9·26. Nonsynchronous vibrator circuit.

with the terminal T_1, coil L gets short circuited and hence the reed is thrown back. The reed then vibrates at its frequency of mechanical resonance and makes contact with terminals T_1 and T_2 alternately. When the reed makes contact at terminal T_1, current flows through the upper half of the primary winding of the transformer. On the other hand when the reed makes contact at terminal T_2, current of reverse polarity flows through the lower half of the primary winding of the transformer. Thus an alternating current of square waveform is applied to the primary winding. A stepped up a.c. voltage of square waveform appears at the secondary terminals. This secondary voltage is applied to a fullwave rectifier. The output of the rectifier is filtered for both the r.f. components and ripple frequency components. The output so obtained is a high d.c. voltage which may be used as plate supply voltage for electron tubes in receivers and transmitters.

R.F. choke L_1 and condenser C_1 constitute the R.F. filter used

CHAPTER X

VOLTAGE STABILIZERS AND AUTOMATIC VOLTAGE REGULATORS

Definition. In accordance with British Standard Specifications, a voltage regulator is defined as "a device for varying at will, the voltage of a circuit or for automatically maintaining it at, or near, a prescribed value".

A voltage stabilizer may be defined as "a circuit, usually including one or more electron vacuum tubes or gas discharge tubes which maintain an almost constant voltage across its output terminals in spite of variations in the load and in the supply voltage."

Thus voltage regulator is a more elaborate device and serves the purpose of regulating or varying the voltage of a circuit and at the same time keeping this voltage at the desired value constant in spite of variations in the supply or in the load. Voltage stabilizers, on the other hand, simply maintain the output terminal voltage of a circuit almost constant It is, therefore, a comparatively simpler device. Usually the term voltage stabiliser is reserved for an electronic apparatus of small power output, having good stability.

Voltage may be controlled or regulated either at the source (at the generator) or near the load. Accordingly automatic voltage regulators may be classified as :

(*i*) Generator-Automatic Voltage Regulator—used for controlling the output voltage of a generator.

(*ii*) Supply-Automatic Voltage Regulator—used for controlling the supply voltage to the load.

Supply-Automatic Voltage Regulators are more common. These Supply-Automatic Voltage Regulators or voltage stabilizers may be put into the following three categories :

Type A . It maintains the output voltage constant in spite of changes in the input.

Type B : It maintains the output voltage constant in spite of variations in load.

Type C : It corrects for changes in the input as well as in the load.

Short-period accuracy of a stabilizer. Short-period accuracy of a stabilizer is the accuracy over a period of minutes, due to changes of input and/or load. Short-period accuracy may be

3. Derive an approximate expression for ripple factor in a fullwave vacuum tube rectifier using inductance filter and resistance load.

To one such rectifier a sinusoidal voltage of amplitude 300 volts and frequency 50 c/s is applied. The load resistance is 10 kilo-ohms. Calculate the ripple factor if filter inductance is of 40 henries and also when it is increased to 60 henries. Calculate also the d.c. output voltage.

4. Derive an approximate expression for the ripple factor in a fullwave vacuum tube rectifier using inductance input filter. State clearly all assumptions made. In such a power supply circuit if the values of inductance L and condenser C are 15 henries and 10 micro-farad respectively, calculate the ripple factor.

5. Derive an expression for the critical inductance in a full-wave rectifier using one section inductance-input filter. If the value of load resistance is 8 kilo-ohms, calculate the critical inductance for a supply frequency of 50 cycles per second.

6. A fullwave vacuum tube rectifier is followed by a two-section inductance-input filter. Find a suitable product LC of each section if the ripple factor at the output is required to be 0·001. Supply frequency is 50 cycles per second.

7. A fullwave rectifier using π-filter has condensers of value 8 micro-farad and choke of 8 henries. The load resistance is 5 kilo-ohms. Calculate the ripple factor of the output voltage at the input voltage frequency of 50 c/s assuming all harmonics but the second to be absent from the rectifier output. Calculate also the ripple factor if the choke is replaced by a wire-wound resistance of 3 kilo-ohms.

8. Explain the working of halfwave and fullwave voltage doublers and discuss their relative merits and demerits.

9. Draw the circuit diagram and discuss the working of a non-synchronous multivibrator. Explain why it is preferred to a synchronous vibrator.

In a true automatic voltage regulator the two basic units, namely, the measuring unit and the regulating unit, can be separately identified. In many circuits, however, the two units cannot be distinguished. These are commonly called voltage stabilizers. This constitutes another significant difference between automatic voltage regulators and voltage stabilizers. These voltage stabilizers find extensive use in electronic devices, in spite of the fact that the accuracy of stabilization obtainable is usually poor.

Voltage Stabilizer. In a voltage stabilizer constant output voltage is obtained by the use [illegible] such a way that the increase [illegible] ing non-linear device just compensates for the increase in the supply voltage and *vice-versa*. Such voltage stabilizers, in general, have a large regulation factor and are, therefore, more suitable for a constant load.

The non-linear devices used in stabilizers may be put into two groups as follows :—

Group 1 non-linear devices. This includes non-linear devices in which the impedance decreases with applied voltage so that the current increases more rapidly than the applied voltage as shown in Fig 10·1. A few examples of devices of this group are : (*a*) cold cathode gas tubes, (*b*) Thermistors, (*c*) crystal diodes, (*d*) carbon filament lamps, (*e*) copper oxide and selenium rectifiers and, (*f*) saturated reactor and transformers.

Group 2 non-linear devices. This includes such non-linear devices in which the impedance increases with applied voltage *i.e.*, the current decreases less rapidly than the applied voltage as shown by the dotted line curve in Fig. 10 1. Typical examples of devices of this group are : (*a*) barreters, (*b*) tungsten filament lamps.

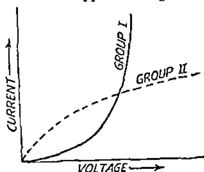


Fig 10 1. Current-voltage characteristic of non-linear impedances

Basic voltage stabiliser circuit using non-linear potential divider. Input voltage to be stabilized is fed to two elements A and B in series and output is taken from across the element B. Then the fraction of the input voltage fed to the output is $Z_B/(Z_A+Z_B)$, where Z_A and Z_B are the impedance of elements A and B respectively. By making either or both Z_A and Z non-linear, this ratio may be made to change in such a way as to compensate for the changes in the input voltage. Such an arrangement will be ideally suited for a constant load but considerable stabilization may still be obtained in spite of load variations. With reference to circuit of Fig. 10·2, it may be seen that three combinations of elements A and B are possible that provide stabilization.

defined for Type A, B and C voltage regulator and stabilizers as below :

Short-period accuracy of Type A stabilizer. In this case the accuracy of stabilization is expressed by the "regulation factor". The regulation factor is defined as the ratio of percentage change in output voltage to percentage change in input voltage.

$$\text{Thus regulationf actor} = \frac{d\,v_o/v_o}{dv\,/_i v_i} \qquad \ldots (10\cdot1)$$

where v_o and v_i are the output and input voltages respectively of the voltage stabilizer or regulator.

Reciprocal of regulation factor is the "Stabilization ratio".

$$\text{Hence stabilization ratio} = \frac{d\,v_i/v_i}{d\,v_o/v_o} \qquad \ldots (10\cdot2)$$

Since v_i is approximately equal to v_o, stabilization ratio S may be approximately written as $S = d\,v_i/d\,v_o$ ... (10·2a)

Short-period accuracy of Type B stabilizer. In this case the accuracy of stabilization is expressed as the ratio dv_o/di_o *i.e.*, the rate of change of output voltage with output or load current. This ratio is nothing but the internal impedance of the regulator or stabilizer and may be indicated by Z_i.

Short-period accuracy of Type C stabilizer. In this case the accuracy of stabilization may be indicated by both the stabilization ratio and internal impedance of the stabilizer.

Long period accuracy of a stabilizer. The long-period ccuracy of a voltage stabilizer or regulator is the accuracy over a period of hours or days and is indicated by the change of output voltage due to :

(i) a specified change of ambient temperature, (ii) a specified change of frequency and (iii) aging, instability of components and vibration over a specified period of time.

The principle of Automatic Voltage Regulators. An automatic voltage regulator consists of two units : (a) the measuring unit and (b) the regulating unit.

The measuring unit performs two functions : (i) detects and measures the change in the input or output voltage of the automatic voltage regulator and (ii) produces a signal to operate the regulating unit.

The regulating unit receives signal from the measuring unit and acts to correct the output voltage of the regulator to a constant or predetermined value with least possible variation.

Sometimes a third unit called "anti-hunting unit" is also used to provide smooth regulation without hunting or continual fluctuations.

In a true automatic voltage regulator the two basic units, namely, the measuring unit and the regulating unit, can be separately identified. In many circuits, however, the two units cannot be distinguished. These are commonly called voltage stabilizers. This constitutes another significant difference between automatic voltage regulators and voltage stabilizers. These voltage stabilizers find extensive use in electronic devices, in spite of the fact that the accuracy of stabilization obtainable is usually poor.

Voltage Stabilizer. In a voltage stabilizer constant output voltage is obtained by the use of a non-linear compensating device in such a way that the increase in voltage drop across this compensating non-linear device just compensates for the increase in the supply voltage and *vice-versa*. Such voltage stabilizers, in general, have a large regulation factor and are, therefore, more suitable for a constant load.

The non-linear devices used in stabilizers may be put into two groups as follows :—

Group 1 non-linear devices. This includes non-linear devices in which the impedance decreases with applied voltage so that the current increases more rapidly than the applied voltage as shown in Fig. 10 1. A few examples of devices of this group are : (*a*) cold cathode gas tubes, (*b*) Thermistors, (*c*) crystal diodes, (*d*) carbon filament lamps, (*e*) copper oxide and selenium rectifiers and, (*f*) saturated reactor and transformers.

Group 2 non-linear devices. This includes such non-linear devices in which the impedance increases with applied voltage *i.e.*, the current decreases less rapidly than the applied voltage as shown by the dotted line curve in Fig. 10 1. Typical examples of devices of this group are : (*a*) barreters, (*b*) tungsten filament lamps.

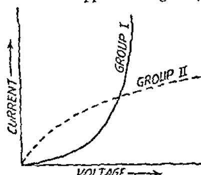


Fig 10 1 Current-voltage characteristic of non-linear impedances

Basic voltage stabiliser circuit using non-linear potential divider. Input voltage to be stabilized is fed to two elements A and B in series and output is taken from across the element B. Then the fraction of the input voltage fed to the output is $Z_B/(Z_A+Z_B)$, where Z_A an[illegible] ment [illegible] [illegible]her or both Z_A and Z [illegible] in such a way as to compensate for the changes in the input voltage Such an arrangement will be ideally suited for a constant load but considerable stabilization may still be obtained *in spite of load variations.* With reference to circuit of Fig. 10 2, it may be seen that three combinations of elements A and B are possible that provide stabilization.

(i) *Element A a linear element and element B a nonlinear element of Group 1.* With the increase of input voltage e_i, impedance Z_B reduces and the fraction of input voltage available at the output reduces tending to compensate for the increase of input voltage.

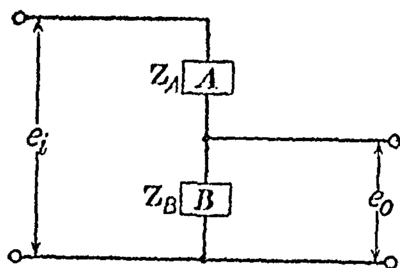


Fig. 10.2. Basic potential divider circuit of a voltage stabilizer.

(ii) *Element A a non-linear device of Group 2 and element B a linear device.* With the increase of input voltage, the impedance Z_A reduces and the fraction of input voltage available at the output reduces tending to compensate for the increase of input voltage.

(iii) *Element A a non-linear device of Group 2 and element B a non-linear device of Group 1.* With an increase of input voltage, Z_A increases and Z reduces. Both these effects reduce the fraction of input voltage available at the output tending to compensate for the increase of input voltage. In each of the above three cases, with decrease in voltage in the input voltage, fraction of input voltage available at the output increases and provides the necessary compensation.

Simple d.c. voltage stabilizer circuits. We shall consider here some of the simple voltage stabilizer circuits used for stabilizing the output voltage of a rectifier. These include—(*a*) Glow discharge tube voltage stabilizer, (*b*) Vacuum tube stabilizers of one of the following types :

(*i*) degenerative voltage stabilizer, (*ii*) μ-bridge voltage stabilizer, (*iii*) g_m-bridge voltage stabilizer, (*iv*) two-stage degenerative voltage stabilizer (also called Electronic Voltage Stabilizer).

Glow discharge tube voltage stabilizer :—

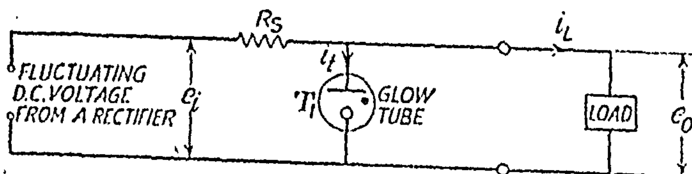


Fig. 10·3. Voltage stabilizer using glow discharge tube.

A glow discharge tube has almost a constant voltage drop across it for widely varying current through it. When connected in the manner shown in Fig. 10·3, the voltage drop across the load remains more or less constant, in spite of variations in the applied input voltage. This occurs because any variation in current caused

variation in the input voltage is bypassed through this glow charge tube with the result that the current through the load mains constant. The glow discharge tube actually behaves as a on-linear impedance of Group 1 corresponding to the impedance B of Fig. 10 1.

Further, for a constant input supply voltage, if the load varies, he output voltage tends to vary. This glow discharge tube, however, bypasses current variation through it and maintains the current through the resistance R essentially constant. The output voltage across the load then remains essentially constant. Thus the glow discharge tube maintains the output voltage almost constant in spite of changes in either the supply voltage or changes in the load.

A tube designed for such a purpose is called a "Voltage Regulator" (VR) tube and should have a large cathode area in order to give a large range of normal current.

For satisfactory operation of this voltage stabilizer, the following conditions must be satisfied :—

(i) Voltage applied to the voltage regulator tube to start the discharge must be at least equal to starting voltage which is 10 to 30 per cent greater than the constant operating voltage

(ii) At no time should the current i_t through the glow tube exceed the maximum permissible value i_{max} otherwise permanent change in the tube characteristics may result due to overheating

The current i_t through the tube is given by

$$i_t = \frac{e_i - e_t}{R_S} - i_L \qquad (10{\cdot}3)$$

where e_i is the applied voltage,

e_t is the voltage across the glow tube when conducting,

R_s is the series resistance,

and i_L is the current through the load

Let the load current i_L vary between known limits and may then be expressed as

$$i_L = I_L \pm \Delta i_L \qquad (10{\cdot}4)$$

where I_L is the mean load current and Δi_L is the variation of load current on either side

Similarly let the fractional change in e_i be k ; then

$$e_i = E_i(1 \pm k) \qquad .. \ (10{\cdot}5)$$

Hence equation (10 3) may be written as

$$i_t = \frac{E_i(1 \pm k) - e_t}{R_s} - (I_L \pm \Delta i_L) \qquad .. \ (10$$

Current i_t through the voltage regulator tube is maximu when $e_i = E_i(1+k)$ and $i_L = (I_L - \Delta i_L)$.

Hence $$\frac{E_i(1+k)-e_t}{R_s}-(I_L-\Delta i_L)\leqslant i_{max} \quad ...(10\cdot7)$$

The maximum allowable current through voltage regulator tube is of the order of 40 milliamperes.

(*iii*) i_t should never fall below a minimum prescribed value i_{min} otherwise stabilization deteriorates or becomes unstable. This value i_{min} is of the order of 5 milliamperes for most of the voltage regulator tubes.

Current through the V.R. tube is minimum when, $e_i=E_i(1-k)$ and $i_L=(I_L+\Delta i_L)$.

Hence $$\frac{E_i(1-k)-e_t}{R_s}-(I_L+\Delta i_L)\geqslant i_{min} \quad ...(10\cdot8)$$

Thus when the supply voltage is constant and the voltage stabilizer is required to maintain the output constant in spite of variation in load current, then the current through the series resistance R_s must remain constant *i.e.*, the variation in current i_L must be equal and opposite to the current variation in V.R. tube. Hence maximum permissible variation of i_L is equal to $(i_{max}-i_{min})$.

The cost of rectifier reduces with the value of the rectified voltage. Hence for a given voltage regulator tube and load current, the minimum permissible value of e_i should be selected. The value of R_s also reduces as e_i is reduced.

The resistance R_s is shown associated with the voltage regulator circuit but this may as well include the internal resistance of the rectifier. Hence it is possible to use a rectifier with high internal resistance and consequent poor voltage regulation when a voltage regulator is used in conjunction with it. The high internal impedance of the rectifier which normally results in poor voltage regulation, is now completely included in the regulator series resistor R_s.

Again, since the voltage stabilizer removes all random variations of supply voltage, it must also reduce periodic ripple voltage in the rectifier output. Hence a comparatively cheaper and simple filter circuit may be used whenever a voltage stabilizer is used.

To obtain a stabilized output voltage higher than can be provided by one V.R. tube, several such tubes are operated in series. Thus the use of a VR-105 in series with a VR-150 will provide a constant 255-volts source.

Stabilization Ratio of a glow tube stabilizer. Fig. 10·4 shows typical current voltage characteristic of glow tube (cold cathode gas diode). Curve in the region *AB* is a straight line. Ideally the curve *AB* should be perfectly vertical in which case the stabilization ratio would be infinite. But region *AB* of the characteristic may be more correctly represented by a straight line only slightly inclin-

from the vertical and may be represented by the equation

$$e_t = e_A + \alpha(i_t - i_A) \quad \ldots (10.9)$$

where $\alpha = \dfrac{e_B - e_A}{i_B - i_A}$ ... (10.10)

where e_A and e_B are the voltages at points A and B on the characteristic,

and i_A and i_B are the currents at points A and B.

Equation (10.9) may be put in the following alternative form :

$$e_t = \alpha i_t + \beta \quad \ldots (10.11)$$

where $\beta = e_A - \alpha i_A$ ... (10.12)

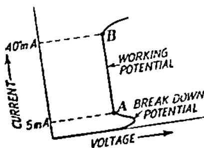


Fig 10·4. A typical glow tube current-voltage characteristic.

From the circuit given in Fig. 10·3,

$$e_i = (i_t + i_L)\,R_s + e_t \quad \ldots (10.13)$$

and $$e_t = e_o = R_L \cdot i_L = \alpha i_t + \beta \quad \ldots (10.14)$$

From equation (10·14),

$$i_t = \frac{e_o - \beta}{\alpha} \quad \ldots (10.15)$$

and $$i_L = \frac{e_o}{R_L} \quad \ldots (10.16)$$

Substituting the values of i_t and i_L from Eqs. 10·15 and 10·16 into Eq. 10·13,

$$e_i = \left[\frac{e_o - \beta}{\alpha} + \frac{e_o}{R_L}\right] . R_s + e_o \quad \ldots 17$$

Rearrangement of Eq. 10·17 yields :

$$e_o = \frac{\alpha R_L e_i + \beta R_L R_s}{R_L R_s + \alpha R_s + \alpha R_L} \quad \text{... (10·18)}$$

Equation 10·18 expresses the stabilized voltage e_o as a function of the input voltage e_i and circuit constants.

Differentiating Eq. 10·18 with respect to e_i, we get :

$$\text{Regulation factor} = \frac{de_o}{de_i} = \frac{\alpha . R_L}{R_L (R_s + \alpha) + \alpha . R_s} \quad \text{... (10·19)}$$

Stabilization ratio S is reciprocal of regulation factor, and hence is given by :

$$\text{Stabilization ratio } S = \frac{de_i}{de_o} = \frac{R_L (R_s + \alpha) + \alpha R_s}{\alpha R_L} \quad \text{... (10·20)}$$

Substitution of value of e_o from Eq. 10·18 gives :

$$S = \frac{\alpha e_i + \beta R_s}{\alpha e_o} \quad \text{... (10·21)}$$

For good regulation *i.e.*, for large value of stabilization ratio, both e_o and α should be small and β, e_i and R_s should be large.

Degenerative vacuum-tube voltage stabilizer. Fig. 10·5 shows the circuit of a simple degenerative vacuum tube stabilizer. Increase of output voltage e_o may be caused either by an increase of the supply voltage e_i or by an increase in the load resistance. In either case, this increase in the output voltage e_o results in making the grid still negative, thus causing a reduction in the plate current. This reduced plate current flowing through the load resistance R_L results in smaller output stabilized voltage e_o. This tends to neutralise the initial increase of voltage e_o. In fact, the increased output voltage e_o gets fedback to the triode grid in such a way as to cancel the increase and hence the name degenerative vacuum tube stabilizer is given to it.

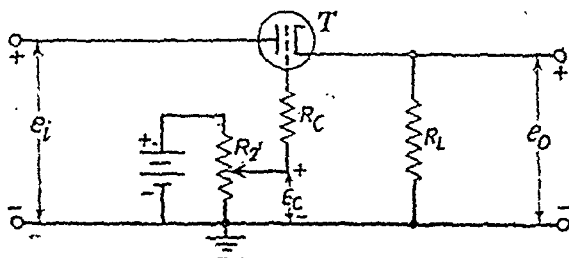


Fig. 10·5. Degenerative vacuum tube stabilizer.

In so far as the stabilization of input voltage is concerned, circuit may be viewed as a potential divider of Fig. 10·2, in which Z_B is the fixed resistance R_L and Z_A is the d c. resistance, $=e_b/i_b$, of the tube T. This resistance increases with an increase the supply voltage and hence constitutes a non-linear impedance Group 2. Greater voltage drop then takes place across the tube . For a properly designed circuit, the increased potential drop across T is approximately equal to the increase of potential e_i and the stabilized output voltage e_o then remains substantially constant.

The stabilized output voltage e_0 may be varied by varying the value of positive voltage E_c on the grid of value T. Let the voltage across the load be at the desired value. Then, the cathode is positive with respect to ground by a potential e_o. The grid may be made positive with respect to ground by a potential E_c by adjustment of potentiometer R_2. However, E_c always remains less than e_0 during operation Value of E_c is adjusted until the bias on tube T is such as to make the tube T pass requisite load current through R_L producing the desired output voltage e_0. The circuit, therefore, constitutes a simple voltage regulator rather than simply a voltage stabilizer.

If the input voltage e_i is removed, a high grid current may flow and may damage the tube. To limit this current, a resistance R_c is placed in series with the grid.

Battery and potentiometer R_2 combination is not satisfactory since battery voltage may reduce with time. In practice the battery is replaced by a glow tube (V.R. tube) as shown in Fig. 10 6.

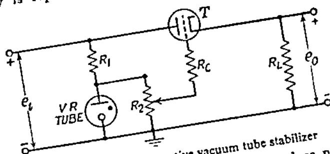


Fig. 10 6 Modified degenerative vacuum tube stabilizer

The output voltage of a glow tube stabilizer does not remain absolutely constant, since an increase in cathode potential is necessary in order that the stabilizer may work. With careful utilization of tube T characteristics, however, variation of output voltage may be made small.

Vacuum tube for use in this stabilizer should preferably be such that beam resistance varies rapidly with small changes in bias. Necessity of such tubes is, however, obviated by using a d c. amplifier in the circuit to amplify the minute changes in output potential before these changes are applied to tube T degeneratively.

Rearrangement of Eq. 10·17 yields :

$$e_o = \frac{\alpha R_L e_i + \beta R_L R_s}{R_L R_s + \alpha R_s + \alpha R_L} \quad \dots (10{\cdot}18)$$

Equation 10·18 expresses the stabilized voltage e_o as a function of the input voltage e_i and circuit constants.

Differentiating Eq. 10·18 with respect to e_i, we get :

$$\text{Regulation factor} = \frac{de_o}{de_i} = \frac{\alpha . R_L}{R_L \ (R_s + \alpha) + \alpha . R_s} \quad \dots (10{\cdot}19)$$

Stabilization ratio S is reciprocal of regulation factor, and hence is given by :

$$\text{Stabilization ratio } S = \frac{de_i}{de_o} = \frac{R_L (R_s + \alpha) + \alpha R_s}{\alpha R_L} \quad \dots (10{\cdot}20)$$

Substitution of value of e_o from Eq. 10·18 gives :

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For good regulation *i.e.*, for large value of stabilization ratio, both e_o and α should be small and β, e_i and R_s should be large.

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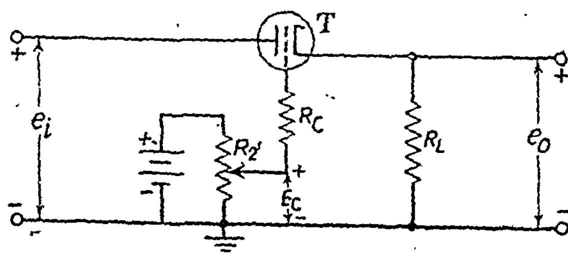


Fig. 10·5. Degenerative vacuum tube stabilizer.

In so far as the stabilization of input voltage is concerned, the circuit may be viewed as a potential divider of *Fig.* 10·2, in which Z_B is the fixed resistance R_L and Z_A is the d c. resistance, $r_b = e_b/i_b$, of the tube T. This resistance increases with an increase of the supply voltage and hence constitutes a

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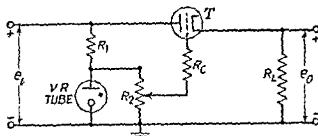


Fig. 10·6 Modified degenerative vacuum tube stabilizer

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μ-Bridge Stabilizer :

Fig. 10·7 shows a simple μ-bridge stabilizer. Ratio R_2/R_1 is made equal to amplification factor μ of the vacuum tube T_1. Then

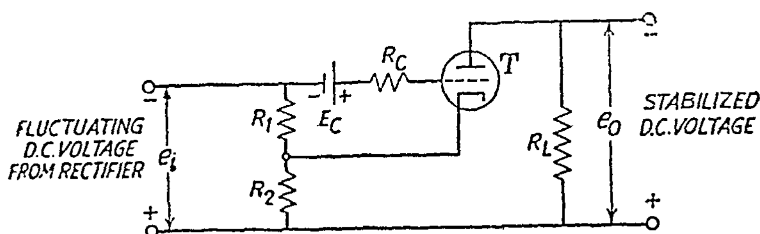


Fig. 10·7 μ-Bridge Stabilizer

for a small change in the input voltage e_i, the change in plate voltage is μ-times the change in the grid voltage but in the opposite direction. Hence the plate current remains constant. This constant current flowing through the load resistance R_L produces constant voltage drop across it. Condition for good stabilization in this circuit is that μ should remain constant over the range of operation of the tube.

Resistance R_c serves the purpose of limiting the grid current in case the applied input voltage becomes zero. The output voltage e_o falls short of input voltage e_i by an amount equal to the sum of voltage drop in tube T and that in resistance R_1. To reduce this drop in resistance R_1, high μ tube should be used since $R_1 = R_2/\mu$. But high μ tubes in general, give low plate current. Hence this stabilizer is suitable for low load current applications only.

g_m-bridge voltage Stabilizer :

Fig. 10·8 shows a simple g_m-bridge voltage stabilizer. Glow tube T_2 maintains cathode of vacuum tube T_1 at a fixed positive

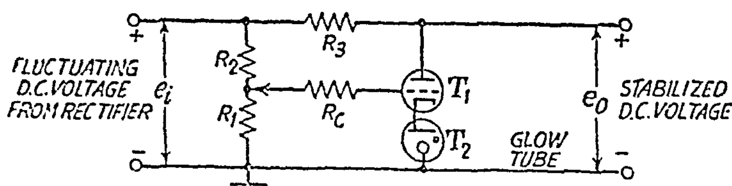


Fig. 10·8. g_m-Bridge voltage Stabilizer.

potential above ground. If the input voltage e_i rises, the potential of grid of tube T_1 rises and results in higher plate current. This increased plate current results in increased voltage drop across resistor R_3 and reduces the output voltage e_o. By suitable selection

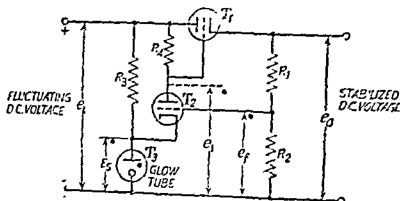
of circuit constants, this increase in voltage drop across R_1 may be made equal to the increase in input voltage. An almost constant output voltage may thus be obtained. For good stabilization, the relationship between the circuit constants is found on analysis to be

$$R_2 = \frac{(R_1 + R_2)}{R_1 \cdot g_m} \qquad \text{—(10·22)}$$

where g_m is the mutual conductance of tube T_2.

It is necessary that g_m remain constant throughout the range of operation of tube T_2. Output voltage e_o falls short of input voltage e_i by an amount equal to voltage drop in R_1 and this may be reduced by reducing the value of R_1. R_1 may be reduced by using high value of transconductance g_m in accordance with equation (10·20).

Two-stage degenerative voltage stabilizer (also called electronic voltage stabilizer). Fig. 10·9 shows basic elements



ig. 10·9. Basic Electronic Voltage Stabilizer Circuit.

onic voltage stabilizer. This is nothing but a modified egenerative voltage stabilizer of Fig. 10·6. This, howes an amplifier to amplify the feedback voltage before it o the grid of the series control tube T_1. T_2 is the ampli. The operation of this circuit is as follows: The fluctuatoltage from rectifier and filter combination is fed to the ninals of the stabilizer and same is available at the output s reduced only by a voltage equal to the voltage drop in ontrol tube T_1. A fixed fraction K_1 of the output voltage e_o ied by a potential divider constituted by resistors R_1 and R_2. tage e_f is applied to the control grid of the amplifier tube be cathode of this amplifier tube T_2 is applied a standard voltage E_s which in the circuit is the constant voltage drop across a voltage regulator tube T_3. For instance, if this voltage

tube is VR-150, then E_s is equal to 150 volts. The grid to cathode voltage of amplifier tube T_2 is then equal to the difference of voltage e_f and E_s. This difference voltage is amplified in the high-gain d.c. amplifier consisting of tube T_2 and load resistance R_4 in the plate circuit. Amplified output voltage is developed across R_4 and appears between plate and grid of series control tube T_1. In fact, the grid voltage of T_1 then falls short of plate voltage by a voltage equal to the voltage drop across R_4. The rest of the stabilization action is then similar to that in degenerative stabilizer of Fig. 10-5. Thus if the input voltage increases, the control grid of amplifier tube T_2 rises in potential and voltage across R_4 increases. The grid of series control tube T_1 then falls in potential with the result that the voltage drop across tube T_1 increases compensating for the rise in input voltage e_i. The output voltage e_o then remains almost constant.

By the use of high-gain d.c. amplifier variation of grid voltage of series control tube T_1 is made large and hence it avoids the necessity of using a series control tube in which d.c. resistance of the tube ($r_{dc}=e_b/i_b$) varies rapidly with changes in grid bias. This tube should, however, be capable of passing the requisite high load current even at the highest voltage drop across it corresponding to the highest input voltage. A power triode is commonly used as tube T_1, although a pentode or a beam tetrode connected as a triode may be used with equal success. Tubes commonly used as series control tube are 6*A*5, 6*A*3, 6*AS*7*G*, 6*L*6, 6*Y*6*G* etc. Such tubes can conveniently carry current as large as 70 *mA* without overheating the plate.

Separate transformer windings, insulated to stand high voltages, are used for cathodes of tubes T_1 and T_2 because of large potential difference between cathodes. Further resistance R_3 should be chosen high enough to keep the current through the glow tube within the permissible limits.

Sometimes R_3 is connected at the output end instead of input end. This gives slightly better stabilization and lower glow tube current but increases the tendency of relaxation oscillations in T_3 at low output voltages.

Modified Electronic Voltage Regulator. Fig. 10·10 shows a typical circuit arrangement that is a modified version of the circuit of Fig. 10·9 and may be used in practice giving satisfactory results over a considerable range of input voltage and load current. It shows several circuit refinements over the basic circuit of Fig. 10·9. Firstly for good regulation d.c. amplifier uses a pentode 6*SJ*7 instead of a triode because its grid impedance is much higher than that of a high-gain triode. A high grid impedance is desirable specially if the full gain capacity is to be utilized. Provision for variation of stabilized output voltage is made by inclusion of a variable resistance R_6 in-between R_1 and R_2. Another modification consists in the use of condenser C_1. Presence of C_1 increases the fraction of the output

voltage applied between grid and cathode of T_1. The reactance of this condenser C_1 at ripple frequency may be made small compared

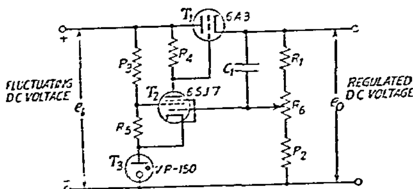


Fig. 10·10. Electronic Voltage Regulator.

with the resistance in shunt with it. Then this condenser helps the stabilizer in reducing the ripple. This condenser should not, however, be made very large otherwise undesirable transient oscillations may result in the output voltage.

In order to obtain a practically ripple-free output and an almost perfect regulation, greater d.c. amplification is required to be used. Two stages of d.c. amplification are then provided for this purpose.

Analysis of Electronic Voltage Stabilizer. Operation of the circuit of Fig. 10·9 may be analysed in terms of the slope of the tube characteristics at points of operation. This is given below for two cases : (*A*) varying input voltage and (*B*) varying load.

(*A*) Varying Input Voltage :

(*i*) R_3 connected to the input side of the stabilizer.

Assumptions made are :

(*a*) output load remains constant.

(*b*) input voltage to the stabilizer varies either due to fluctua- [illegible] ltage in [illegible]

It is required to find the stabilization ratio.

Let g_{m1} and r_{p1} be the mutual conductance and dynamic plate resistance of tube T_1. Then small variation i_{p1} in plate current of T_1 is related to small increments in grid voltage and plate voltage by the relation

$$i_{p1}=g_{m1}\cdot e_{g1}+\frac{1}{r_{p1}}\cdot e_{p1} \qquad \text{... (10·23)}$$

But $$e_{g1}=de_1-de_0$$

tube is *VR*-150, then E_s is equal to 150 volts. The grid to cathode voltage of amplifier tube T_2 is then equal to the difference of voltage e_f and E_s. This difference voltage is amplified in the high-gain d.c. amplifier consisting of tube T_2 and load resistance R_4 in the plate circuit. Amplified output voltage is developed across R_4 and appears between plate and grid of series control tube T_1. In fact, the grid voltage of T_1 then falls short of plate voltage by a voltage equal to the voltage drop across R_4. The rest of the stabilization action is then similar to that in degenerative stabilizer of Fig. 10-5. Thus if the input voltage increases, the control grid of amplifier tube T_2 rises in potential and voltage across R_4 increases. The grid of series control tube T_1 then falls in potential with the result that the voltage drop across tube T_1 increases compensating for the rise in input voltage e_i. The output voltage e_o then remains almost constant.

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Separate transformer windings, insulated to stand high voltages, are used for cathodes of tubes T_1 and T_2 because of large potential difference between cathodes. Further resistance R_3 should be chosen high enough to keep the current through the glow tube within the permissible limits.

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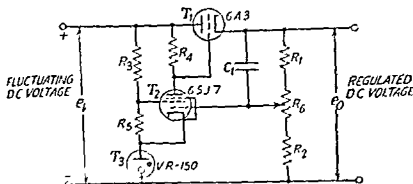


Fig. 10 10 Electronic Voltage Regulator

... resistance in shunt with it. Then this condenser helps the ... d not, how- ... oscillations

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(A) Varying Input Voltage :

(i) R_1 connected to the input side of the stabilizer.

Assumptions made are :

(a) output load remains constant.

(b) input voltage to the stabilizer varies either due to fluctuating a.c. supply voltage to the rectifier or due to ripple voltage in the output of the rectifier resulting from inadequate filtering.

It is required to find the stabilization ratio.

Let g_{m1} and r_{p1} be the mutual conductance and dynamic plate resistance of tube T_1. Then small variation i_{p1} in plate current T_1 is related to small increments in grid voltage and plate volta... the relation

and $e_{p1}=de_i-de_o$ ... (10·25)

Also $i_{p1}=\dfrac{de_o}{R_L}$ ... (10·26)

where R_L is the load resistance which is assumed to be constant.

Combining equations (10·20) to (10·23), we get

$$\frac{de_o}{R_L}=g_{m1}(de_1-de_o)+\frac{1}{r_{p1}}(de_i-de_o) \qquad \text{... (10·27)}$$

Rearranging terms in Eq. (10·24), we get

$$de_o\left(\frac{r_{p1}}{R}+\mu_1+1\right)=\mu_1 \, . \, de_1+de_i$$

or $$de_o=\frac{\mu_1 \, . \, de_i}{\mu_1+1+\dfrac{r_{p1}}{R_L}} \qquad \text{... (10·28)}$$

In order to find the ratio $\dfrac{de_i}{de_o}$, the term de_1 in Eq. (10·25) should be replaced by its equivalent expression in terms of de_i, de_o and circuit constants. To get the expression for de_1, it may be observed that de_1 results because of two voltage variations de_i and de_f. Increment de_i causes a change in the voltage at the plate of T_2 amounting to $+\,de_i \, . \, \dfrac{r_{p2}}{R_3+r_{p2}}$ where r_{p2} is the dynamic plate resistance of tube T_2. Increment de_f is the change in the voltage fed to the grid of tube T_2 from the junction of resistors R_1 and R_2. This incremental voltage after amplification in tube T_2 results in an increment of voltage at the plate of tube T_2 amounting to $de_o . \dfrac{R_2}{R_1+R_2} \, . \, A_2$ where A_2 is the voltage gain of amplifier using tube T_2.

Hence $$de_1=\frac{r_{p2}}{R_3+r_{p2}} \, . \, de_i+\frac{R_2}{R_1+R_2} \, . \, A_2 \, . \, de_o \qquad \text{... (10·29)}$$

or $$de_1=\alpha_2 \, . \, de_i+\beta \; A_2 \, . \, de_o \qquad \text{... (10·30)}$$

where $\alpha_2=\dfrac{r_{p2}}{R_3+r_{p2}}$, $\beta=\dfrac{R_2}{R_1+R_2}$ and $A_2=\dfrac{-\mu_2 R_3}{r_{p2}+R_3}$

Substituting the value of de_1 from Eq. (10·30) into Eq. (10·28) we get

$$de_o=\frac{\mu_1(\alpha_2 \, . \, de_i+\beta . A_2 . de_o)+de_i}{\mu_1+1+\dfrac{r_{p1}}{R_L}} \qquad \text{... (10·31)}$$

On rearranging Eq. (10·28), we get

$$de_o=\frac{de_i(\mu_1\alpha_2+1)}{\mu_1(1-\beta A_2)+1+\dfrac{r_{p1}}{R_L}} \qquad \text{... (10·32)}$$

Hence the stabilisation ratio S is given by :

$$S=\frac{de_i}{de_o}=\frac{\mu_1(1-\beta A_2)+1+\frac{r_{p1}}{R_L}}{\mu_1\alpha_2+1} \quad \ldots (10\cdot33)$$

Under normal working conditions, $\beta A_2>>1$,

and $\mu_1(1-\beta A_2)>>1+\frac{r_{p1}}{R_L}$ and hence we may write

$$S=\frac{-\beta\mu_1 A_2}{\mu_1\alpha_2+1} \quad \ldots (10\cdot34)$$

For single-stage d.c. amplifier, stabilization ratio S is of the order of 500. If almost ripple free output is desired, two-stage d.c. amplifier should be used. This provides a stabilization ratio of the order of 20,000.

***(ii)* R_3 connected to the output side of the stabilizer.**

Under this condition,

$$de_1=\beta.A_2\, de_o+\alpha_2\, de_o \quad \ldots(10\cdot35)$$

But $\alpha_2<<\beta A_2$ and hence Eq. (10·32) may be written as

$$de_1=\beta.A_2.de_o \quad \ldots (10\cdot36)$$

All other relations remain the same as for case (*i*) above. Hence stabilization ratio S is given by

$$S=\frac{de_i}{de_o}=\mu(1-\beta A_2)+1+\frac{r_{p1}}{R_L} \quad \ldots(10\cdot37)$$

Corresponding approximate expression for S is

$$S\doteq-\beta\, A_2.\mu_1 \quad \ldots(10\cdot38)$$

The value of S as given by Eq. (10 33) is greater than that obtained from Eq. (10·30) for case (*i*) with R_3 connected to the input side of the stabilizer. In practice, however, the d c. amplifier is more linear and gives more gain under condition (*i*) Hence there exists no appreciable difference in results obtained by the two methods of connection of R_3.

(*B*) Varying Load · (*i*) R_3 connected to the input side of the stabilizer.

Here it is assumed that the input voltage remains constant but change in output potential may take place due to change in the load current. Stated otherwise, the stabilizer has an internal resistance resulting in the drop of output voltage with increase of load current. It is desired that this internal resistance of the stabilizer be as small as possible. Expression for this internal resistance of the stabilizer is derived as under :

Here again use is made of equations (10 22) and (10 23) *i.e.*,

$$dI_s=i_{p1}=g_{m1}.e_{g1}+\frac{e_{p1}}{r_{p1}} \quad \ldots(10\cdot39)$$

$$e_{g1}=de_1-de_o \qquad \ldots(10\cdot40)$$

But in this case due to stabilization, and due to constant input voltage e_i, variation in plate voltage *i.e.*, e_{p1} is very small.

Hence in Eq. (10·20) $\frac{e_{p1}}{r_{p1}}$ may be neglected compared with $g_m \cdot e_{g1}$ and it may then be written as

$$dI_p=g_{m1}.e_{g1}=g_{m1}(de_1-de_o) \qquad \ldots(10\cdot41)$$

From equation (10·22) by putting de_i equal to zero, we get

$$de_1=\beta.A_2.de_o \qquad \ldots(10\cdot42)$$

Combining equations (10·41) and (10·42) we get

$$dI_p=g_{m1}(\beta A_2-1)de_o \qquad \ldots(10\cdot43)$$

Then the effective internal resistance of the regulated power supply is given by

$$R_o=-\frac{de_o}{dI_p}=\frac{1}{g_{m1}(1-\beta A_2)} \qquad \ldots(10\cdot44)$$

Usually $|\beta A_2| >> 1$ so that Eq. (10·38) may be written as

$$R_o \doteq \frac{1}{-g_{m1}\beta A_2} \qquad \ldots(10\cdot45)$$

(ii) R_3 connected to the output side of the stabilizer

Here again de_i is given by

$$de_1=\alpha_2.de_o+\beta.A_2.de_o$$

or

$$de_1=(\alpha_2+\beta A_2).de_o \qquad \ldots(10\cdot46)$$

All other expressions remain the same as in the case of B (*i*). Hence,

$$dI_p=g_{m1}[\beta A_2+\alpha_2-1]de_o \qquad \ldots(10\cdot47)$$

Effective internal resistance of the stabilizer is then given by

$$R_o=-\frac{de_o}{dI_p}=\frac{1}{g_{m1}[1-(\alpha_2+\beta A_2)]} \qquad \ldots(10\cdot48)$$

However in practice βA_2 is much larger than either α_2 or unity so that in this case also the expression for effective internal resistance comes out to be

$$R_o \doteq \frac{1}{-g_{m1}\beta A_2} \qquad \ldots(10\cdot49)$$

The value of this effective internal resistance may be as small as 0·5 ohm.

In the above analysis it has been assumed that the input voltage to the stabilizer remains constant in spite of the variation of the load current. Stated otherwise, this means that the internal impedance of the voltage source driving the regulator is negligible. In practice the voltage sources always have some internal impedance. Let this internal resistance of the voltage source driving the stabilize

be R_r. Then the expression for the effective internal impedance of the stabilizer gets modified as below.

$$R_o = \frac{1}{-g_{m1}\beta A_2} + \frac{R_s}{S} \qquad ...(10\cdot50)$$

But stabilization ratio for case $A(ii)$ is given by

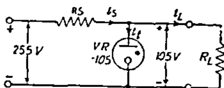
$$S = \beta A_2 \mu_1 \qquad ...(10\cdot51)$$

Hence $$R_o = \frac{1}{-g_{m1}\beta A_2}\left[1 + \frac{R_s}{r_{p1}}\right] \qquad ...(10\cdot52)$$

Typical value of R_s may be taken as 500 ohms and that of stabilization ratio S may be taken as 500, so that the additional component added to the effective internal resistance of the stabilizer by the presence of R_s is 500/500=1·0 ohm.

Example 1. *A VR.-105 tube is used in a glow tube stabilizer. The normal operating current range for this tube is taken as 5 to 40 mA. The input voltage remains constant at 255 volts but the load current can vary from 35 to 55 mA. Load resistance is connected directly across the glow tube, and the glow tube in series with a resistance R_s is connected to the input d c. voltage source. Assuming stabilizer tube characteristics to be ideal, find the maximum and minimum possible values of series resistance R_s if current through the glow tube is to remain within limits.*

Solution. For ideal glow tube characteristics, output voltage across the tube must remain constant at 105 volts. Hence current



i_s through R_s must remain constant at a value given by

$$i_s = \frac{255-105}{R_s}$$

Further $i_s = i_t + i_L$

Current i_t will then be maximum when i_L is minimum and vice versa.

Hence $$\frac{255-105}{R_s} - i_{L\,min} < i_{t\,max}$$

and $$\frac{255-105}{R_s} - i_{L\,max} > i_{t\,min}$$

or $$\frac{255-105}{R_s} < i_{t\,max} + i_{L\,min} < (40+35)mA$$

and $$\frac{255-105}{R_s} > i_{t\,min} + i_{L\,max} > (5+55)mA$$

Hence maximum value of $R_s = \dfrac{150\ V.}{60mA} = 2500$ ohms

and minimum value of $R_s = \dfrac{150\ V}{75mA} = 2000$ ohms

Example 2. *A 150-V neon tube stabilizer is connected in series with a resistance R_s to a 240 volts d.c. supply. The load R_L is connected directly across the stabilizer tube. If load resistance R_L is 15000 ohms and current i_t through the neon tube is 20 mA, find the resistance of R_s and power dissipated in it.*

If now the load resistance is reduced to 10000 ohms, find the new value of current through the neon tube, assuming that it operates ideally.

Solution. Circuit arrangement is the same as given in circuit of Example 1.

Voltage drop across load resistance R_L is 150 volts. Hence current i_L through load resistance $= \frac{150}{15000}$ amp. = 10 milli-amperes.

Current i_t through neon tube = 20 mA

Hence current through R_s = 10+20=30 mA

Further voltage drop across R_s = 240−150=90 volts.

Hence $R_s = \frac{90 \text{ volts}}{30 \times 10^{-3} \text{ amp.}} = 3000$ ohms,

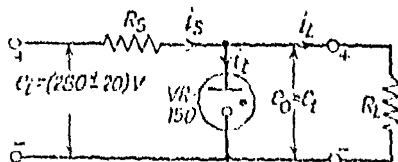
and power dissipated across R_s = 90 volts × 30 mA = 2·7 watts.
When load resistance is reduced to 10000 ohms, load current becomes

$$\frac{150 \text{ volts}}{10000 \text{ ohms}} = 15 \, mA$$

and current through R_s still remains at 30 mA. Hence current i_t through neon tube becomes (30−15)=15 mA.

Example 3. *A VR-150 regulator tube is connected in series with a resistor R_s of 2000 ohms to a 280 volts d.c. supply. The load resistance is connected directly across the VR tube. The input d.c. voltage varies by ± 20 volts. Find the maximum and minimum values of load resistance that may be used keeping current through VR tube within its operating limits of 5 to 40 mA.*

Solution. Let the maximum and minimum values of load



current be $i_{L\,max}$ and $i_{L\,min}$.

$$\text{Then} \quad \frac{(280+20)-150}{2000} - i_{L\,min} \leqslant 40 \, mA$$

or $i_{L\,min} \geqslant \dfrac{(280+20)-150}{2000} - 40\times10^{-3}$ amperes

$$\geqslant (75-40)\ mA$$
$$\geqslant 35\ mA$$

Again $\dfrac{(280-20)-150}{2000} - i_{L\,max} \geqslant 5mA$

or $$i_{L\,max} \leqslant \frac{(280-20)-150}{2000} - 5\times10^{-3} \text{ amperes}$$

$$\leqslant (55-5)\ mA.$$
$$\leqslant 50\ mA.$$

Hence $R_{L\,max} = \dfrac{150}{i_{L\,min}} = \dfrac{150 \text{ volts}}{35\times10^{-3} \text{ amp.}} = 4285$ **ohms.**

and $R_{L\,min} = \dfrac{150}{i_{L\,max}} = \dfrac{150 \text{ volts}}{50\times10^{-3} \text{ amp.}} = 3000$ **ohms.**

Example 4 *A glow tube stabilizer has a neon tube and a resistance R_s of 2000 ohms in series connected across 250 volts voltage supply. The load resistance is connected directly across the neon tube. The volt-ampere characteristic of the neon tube in the region of operation is given by*

$$e_t = 20.\ i_t + 150.$$

Calculate the stabilization ratio of the stabilizer when the load resistance R_L is 4000 ohms. If the input voltage increases by 10 volts, find the output voltage of the stabilizer.

Solution : Stabilization ratio is given by the relation

$$S = \frac{R_L(R_s+\alpha) + \alpha R_s}{\alpha R_L}$$
$$= \frac{4000\,(2000+20) + 20\times2000}{20\times4000} = \mathbf{101{\cdot}5}$$

If next the input voltage is increased by 10 volts, de_i becomes 10 volts. The $de_o = de_i/S = 10$ volts/101·5

$$= \mathbf{0{\cdot}1\ volt} \text{ approx.}$$

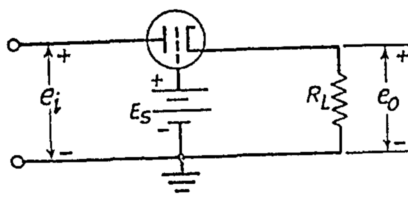
Example 5. *For simple degenerative stabilizer of Fig. 10·5, derive expressions for stabilization ratio and effective internal impedance of the stabilizer.*

One such circuit uses a tube having amplification factor. and r_p of 10000 ohms. Load resistance is 4000 ohms. Calculate lization ratio. Also calculate the effective internal stabilizer.

Solution. *Case (A)* : **Varying input voltage**

$$i_p = g_m.\ e_g + \frac{1}{r_p}.\ e_p$$

$$e_g = -de_o \qquad e_p = de_i - de_o \qquad i_p = de_o/R_L$$



Hence $de_o/R_L = g_m\,(-de_o) + \dfrac{1}{r_p}\,(de_i - de_o)$

Rearranging terms : $de_o\,(r_p + \mu\, R_L + R_L) = R_L\, de_i$

Hence stabilization ratio is given by

$$S = \frac{de_i}{de_o} = \frac{r_p + R_L\,(\mu+1)}{R_L}$$

Case B : **Varying Load**

Here input voltage e_i is assumed to be constant.

$$dI_p = i_p = g_m.e_g + \frac{e_p}{r_p}$$

$$e_g = -de_o$$

e_p is small. Hence $dI_p = g_m.\ e_g = -g_m.de_o$.

Hence effective internal resistance of the stabilizer

$$= R_o = -\frac{de_o}{dI_p} = \frac{1}{g_m}$$

For the given values of μ, r_p and R_L we have

$$S = \frac{de_i}{de_o} = \frac{r_p + R_L\,(\mu+1)}{R_L}$$

$$= \frac{10000 + 4000\,(20+1)}{4000}$$

$$= (2{\cdot}5) + 21 = 23{\cdot}5$$

Effective internal resistance of this circuit is given by

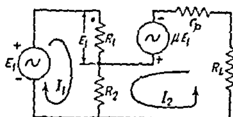
$$R_o = \frac{1}{g_m} = \frac{r_p}{\mu} = \frac{10000}{20}$$

$$= \mathbf{500\ ohms.}$$

Example 6. *For the μ-bridge stabilizer of Fig. 10·7, consider the variation de_i in the input voltage as a sinusoidal voltage and hence find the corresponding alternating output voltage. Thus drawing the a.c. equi-*

valent circuit for the stabilizer, find the stabilization ratio which for the method adopted may be written as $E_i/I_2.R_L$.

Solution. In the equivalent circuit shown above the tube has been replaced by its equivalent a.c. generator of voltage E_1 and internal impedance r_p where E_1 is the voltage applied between grid and cathode and hence is the voltage drop across resistor R_1 and r_p is the dynamic plate resistance of the tube.



Let I_1 and I_2 be the a c. currents through the input and output loops as shown in the equivalent circuit.

Then $$E_i = I_1 R_1 + (I_1+I_2).R_2 \quad ... (1)$$

$$\mu E_1 = (I_1+I_2).R_2 + I_2(r_p+R_L) \quad .. (2)$$

and $$E_1 = I_1 R_1 \quad ... (3)$$

Equation (1) may be written as

$$I_1(R_1+R_2) + I_2 R_2 = E_i \quad ... (4)$$

Equations (2) and (3) may be combined to give

$$I_1(R_2-\mu R_1) + I_2(r_p+R_L+R_2) = 0 \quad ... (5)$$

Eliminating I_1 from Equations (4) and (5) we get

$$I_2.\frac{(r_p+R_L+R_2)(R_1+R_2)}{(\mu R_1-R_2)} + R_2 I_2 = E_i \quad ... (6)$$

Hence stabilization ratio S is given by

$$S = \frac{E_i}{I_2.R_L} = \frac{R_2}{R_L} + \frac{(R_1+R_2)(r_p+R_L+R_2)}{(\mu R_1-R_2)}$$

Example 7. *In the electronic voltage stabilizer of Fig. 10·9, the d.c. voltage source has internal resistance of 100 ohms, series control tube T_1 has amplification factor of 2·5, dynamic plate resistance r_p of 300 ohms. The load resistance is 1000 ohms, the gain of d.c. amplifier is −50, minus sign indicating phase reversal. Dynamic plate resistance of d.c. amplifier tube is 500 kilo-ohms. Resistances R_1, R_2 and R_3 are respectively 200, 200 and 500 kilo-ohms. Calculate the stabilization ratio. If the d.c. supply voltage changes by 10 volts, find the corresponding change in output d.c. voltage. Also calculate the internal resistance of the stabilizer.*

Solution. Stabilization ratio for the given circuit is expressed by the relation

$$S=\frac{\mu_1(1-\beta A_2)+\frac{r_{p1}}{R_L}}{\mu_1\alpha_2+1}$$

$$=\frac{2\cdot5(1+0\cdot5\times50)+\frac{300}{1000}}{(2\cdot5\times0\cdot5)+1}=\frac{65\cdot3}{2\cdot25}=29\cdot02.$$

For the input voltage change of 10 volts, the output voltage change will be $=\frac{10}{29\cdot02}=$ **0·3445 volt.**

Effective internal impedance of the stabilizer is given by :

$$R_o=\frac{R_s}{S}+\frac{1}{g_{m1}(1-\beta A_2)}=\frac{100}{29}+\frac{1}{\frac{2\cdot5}{300}(1+0\cdot5\times50)}$$

$$=3\cdot448+2\cdot308$$

$$=5\cdot756$$

EXERCISES

1. With the help of a circuit diagram explain the working of a simple glow tube stabilizer.

A glow tube stabilizer uses a series resistance R_s and a VR-150 tube. The input voltage remains constant at 270 volts but the load current varies between the limits 40 mA to 55 mA. Normal operating range of the glow tube is 5 to 40 mA. The output is required to be maintained constant at 150 volts. Calculate a suitable value of resistance R_s so that the current through the glow tube remains within prescribed limits.

2. A 105-volt neon stabilizer tube is connected in series with a resistor R_s of 3000 ohms to a 255 volts d.c. supply. The load resistance R_L is connected directly across the stabilizer. Normal operating current range of stabilizer tube is 5 to 40 mA. Find the maximum and minimum values of load resistance R_L that may be used keeping the current through the neon-tube within prescribed limits. Assume ideal operation of stabilizer.

3. A glow tube stabilizer uses VR-105 regulator tube. This tube in series with a resistor R_s is connected to a d.c. voltage of 250 volts d.c. The load resistance is put directly across the VR tube. The d.c. source voltage varies by ±15 volts and load current is required to be varied between 30 and 50 mA. The normal operating range of the VR tube is 5 to 40 mA. Find a suitable value of series resistor R_s such that the VR tube operates within the prescribed limits. Assume ideal operation of stabilizer.

4. A neon tube stabilizer uses a series resistance R_s and the two are connected across the input d.c. voltage source of voltage e_i. Load resistance R_L is connected directly across the neon tube. The volt-ampere characteristic of the neon tube in the operating region is assumed to be of the form : $e_t = \alpha i_t + \beta$. Show that the output voltage e_o and the input voltage e_i are related by the expression :

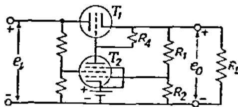
$$e_o = \frac{\alpha R_L . e_i + \beta . R_L . R_s}{\alpha (R_s + R_L) R_L . R_s}$$

Hence prove that the stabilization ratio $S = \dfrac{de_i}{de_o}$ for this circuit is given by the expression :

$$S = \frac{\alpha\, e_i + \beta\, R_s}{\alpha e_o}.$$

5. Draw the circuit diagram of a simple degenerative vacuum tube stabilizer and explain its stabilization action. Derive expression for the stabilization ratio and effective internal resistance of the stabilizer.

6. For the electronic voltage stabilizer shown below, derive expression for the stabilization ratio. If amplification factor μ_1 of the series control tube T_1 is 2·5, its dynamic plate resistance r_{p1} is 250 ohms, load resistance is 500 ohms, gain of d.c. amplifier is, -60 and $R_1 = R_2 = 0{\cdot}2$ meg-ohms, calculate stabilization ratio of the stabilizer. If input voltage changes by 20 volts, find the change in output voltage.



7. For the electronic voltage stabilizer of exercise 6, derive expression for effective internal resistance of the stabilizer. If the internal resistance of the input d.c. voltage source is assumed to be zero, calculate the effective internal resistance of the stabilizer for the component values given in exercise 6.

CHAPTER XI

VACUUM TUBE AMPLIFIER PRINCIPLES

Introduction. An amplifier is a device which amplifies or increases in magnitude any current or voltage applied at its input.

An amplifier is termed as a vacuum tube amplifier if it makes use of one of the grid controlled vacuum tubes, typically a triode, tetrode or pentode, as the basic amplifying device. On the other han if it utilises one or more transistors as the basic amplifying devce, it will be called a transistor amplifier. In this and the following chapters we will deal with only vacuum tube amplifier.

It has been seen that a grid controlled vacuum tube is able to amplify a small signal voltage applied between its control grid and cathode, because control grid exerts far greater influence on the flow of electrons from cathode to anode than does the anode itself. This results because of the closeness of control grid to cathode. Any small increment of input signal results in an increment in the plate current which flowing through the load impedance in the plate circuit produces an increment of voltage across the load impedance. This incremental or a.c. voltage developed across the load impedance is, in general, far greater than the incremental or a.c. signal voltage iapplied on the grid side, resulting in voltage amplification.

Fig. 11·1 shows the basic circuit of a vacuum tube amplifier using a triode. Triode is shown here, it being the simplest grid controlled vacuum tube. Tetrode and pentode are, of course, *also* used commonly.

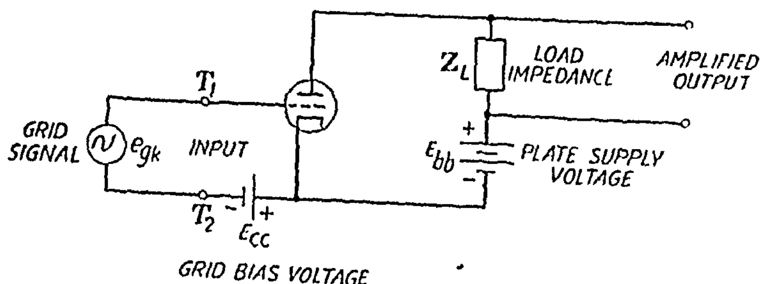


Fig. 11·1. Basic vacuum tube amplifier using triode.

In the circuit shown, grid of the triode is always kept negative with respect to the cathode by means of the grid bias voltage E_{cc} (also called *C* voltage). On this fixed grid bias voltage E_{cc} is super-

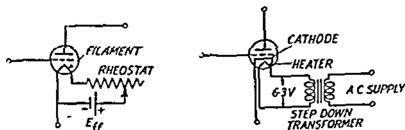
imposed the voltage e_{gk} which is required to be amplified. This voltage is called the signal voltage and in general it is a periodic voltage. The simplest case of a periodic voltage is a pure sinusoidal voltage. This signal voltage is connected at terminal T_1 and T_2, in series with the bias voltage E_{cc}. On the plate side, a positive d.c. voltage E_{bb} is applied between plate and cathode through a load impedance Z_L. This voltage E_{bb} is called the "plate supply voltage" or "*B*-voltage". The load impedance in its simplest form may be a pure resistance, but it may alternative be a complex impedance.

In the absence of a signal voltage *i e.*, when e_{gk} is zero, grid voltage is constant at value E_{cc} and a constant plate current, therefore, flows in the plate circuit. On application of a signal voltage, plate current varies in accordance with the variations in signal voltage. This incremental or a.c. component of plate current flowing through the load impedance Z_L develops the amplified output voltage across Z_L, as shown in the circuit.

Vacuum tube amplifier as a linear device. In the above simple description of the working of a vacuum triode as an amplifier, it is assumed that a c. component of plate current is proportional to input signal voltage with the result that amplified output voltage is proportional to the input signal. Thus if the signal amplitude is doubled, the amplitude of the output voltage also gets doubled. It may then be said that the amplifier is a linear device. Such a condition is possible provided that the characteristics of the tube are linear, parallel and equidistant for equal increments of the parameter. Obviously such a condition is not satisfied by any tube. There exists considerable non-linearity in the tube characteristics. [illegible] ltage are small [illegible] uantities, then [illegible], characteristic curves may be treated as almost linear. With this restriction, the vacuum tube amplifier may be treated as a linear device.

Symbols and Conventions :

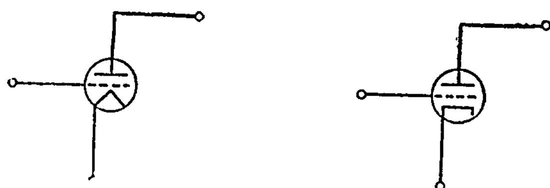
(*a*) **Symbol for Tubes.** Fig. 11·2 (*a*) shows two methods of



(*a*) Two common methods of heating cathodes

Fig. 11·2. Symbols for triodes.

heating the cathodes of filamentary cathodes and indirectly heated cathodes. For the sake of brevity the cathode heating circuits are eliminated and symbols as shown in Fig. 11·2 (*b*) are used.



(*b*) Symbols for triodes using directly and indirectly heated cathodes.
Fig. 11·2. Symbols for triodes.

(*b*) **Symbols for voltages and currents.** (*i*) Instantaneous values of varying voltage and current are usually represented by lower case symbols *e* and *i* respectively followed by the subscripts "*g*" and "*p*" for grid and plate respectively.

Thus e_g, e_p—represent instantaneous values of varying or a.c. components of grid voltage and plate voltage respectively.

i_g, i_p—represent instantaneous values of the varying components of grid and plate current respectively.

It may be noted here that grid and plate voltages are taken with respect to the cathode. Hence sometimes a second subscript "*k*" is also used. Thus instead of e_g and e_p, sometimes symbols e_{gk} and e_{pk} are used.

(*ii*) Instantaneous total values of current and voltage are represented by lower case letters *e* for voltage and *i* for current followed by subscripts "*b*" and "*c*" for plate and grid respectively.

Thus,

e_b, e_c—represent instantaneous total values of plate and grid voltages respectively.

i_b, i_c—represent instantaneous total values of plate and grid currents respectively.

These total values consist of two parts (*a*) no signal component or d.c. component and (*b*) instantaneous value of the time-varying component.

(*iii*) No signal voltage and current are indicated by capital letters *E* and *I* respectively followed by subscript "*b*" for plate and "*c*" for grid.

Thus E_b, E_c—represent no signal plate and grid voltages respectively.

I_b, I_c—represent no signal plate and grid currents respectively.

Further let e_d, e_l and E_d represent respectively the instantaneous total value, instantaneous value of varying component and zero signal value of voltage developed across load impedance.

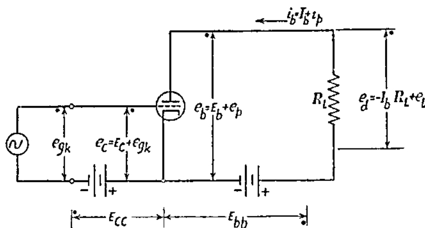


Fig. 11 3. Assigned positive reference directions for currents and voltages in a basic vacuum triode linear amplifier

Fig. 11·3 shows the symbols for different electrode voltages and currents. To each voltage and current a positive reference direction is assigned. This is done to establish the algebraic signs in the analysis of such an amplifier. With reference to the assigned positive direction of e_c, for example, whenever grid is at a higher voltage than the cathode, e_c is given a positive sign and vice-versa. The selection of positive reference directions of voltages is entirely arbitrary. The currents are however assigned positive reference directions on the basis of the fact that conventional current in a vacuum tube always flow from plate to cathode or from a grid to cathode. Usually the grid bias in vacuum triode is kept quite negative and grid signal is of such small amplitude that the grid current does not flow. Grid current is avoided to prevent power drainage from the source of voltage to be amplified.

No signal operation of a vacuum tube amplifier. Circuit of a simple vacuum triode amplifier using resistance load is shown in Fig. 11·3. To analyse the behaviour of such an amplifier it is desirable to study it under two conditions : (*i*) No-signal condition : In this case e_{gk} is zero and only voltage on the grid side or the input side of the tube is the grid bias voltage E_{cc}. Corresponding plate current and plate voltage will also have only steady or d c. com-

This article deals with the performance of the amplifier under zero-signal condition or quiescent condition. Fig. 11·4 shows the circuit arrangement and the assigned positive directions of different voltages and currents under such a zero-signal condition. Dots indicate the assigned positive directions of the voltages whereas arrow heads indicate the assigned positive directions of the currents. Thus in circuit of Fig. 11·4, E_{cc} is negative and since no signal is applied, $e_c = E_c = E_{cc}$.

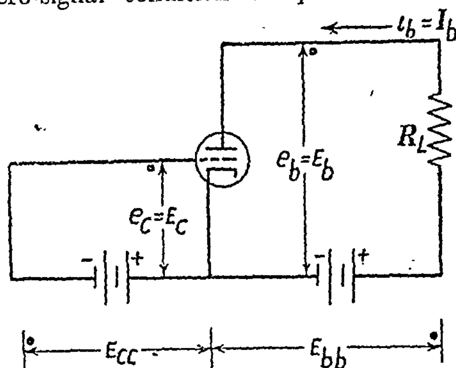


Fig. 11·4. Circuit diagram showing zero-signal operation of a vacuum triode amplifier.

This circuit consists of the amplifier tube and the associated circuit. In the analysis of this amplifier, therefore, characteristics of the tube as well as the associated circuit must be considered.

To analyse the operation under no signal condition, we apply Kirchhoff's voltage law to the plate circuit *i.e.*, circuit consisting of plate, cathode, plate supply battery and load resistance R. We get :

$$e_b + i_b \cdot R_L = E_{bb} \quad \ldots (11·1)$$

or

$$i_b = \frac{E_{bb} - e_b}{R_L} \quad \ldots (11·2)$$

Again the plate current i_b at any instant is a function of instantaneous values of total plate voltage e_b and total grid voltage e_c. This may be put mathematically as :

$$i_b = f(e_c, e_b) \quad \ldots (11·3)$$

or

$$i_b = f(E_{cc}, e_b) \quad \ldots (11·4)$$

Since grid voltage is constant at value E_{cc} in this case. Equating the expression (11·2) and (11·4), we get :

$$f(E_{cc}, e_b) = \frac{E_{bb} - e_b}{R_L} \quad \ldots (11·5)$$

It is required to find the values of i_b, and e_b that satisfy the equations (11·2) and (11·4) simultaneously *i.e.*, satisfy the equation (11·5). A graphical solution may be obtained by drawing curves for equations (11·2) and (11·4) and finding the point of intersection. This is shown in Fig. 11·5 on the next page.

The curve corresponding to equation (11·4) is nothing but the static plate characteristic curve of the tube for $e_c = E_{cc}$ as

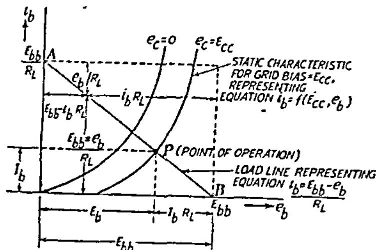


Fig 11·5. No signal operating condition of a triode amplifier with resistive load

shown in Fig 11·5. The curve corresponding to Eq (11·2) is a straight line which cuts the current and voltage axes at points A and B as shown in Fig. 11 5 The coordinates of these points A and B may be obtained by putting $e_b = 0$ and $i_b = 0$ in equation (11·2). Thus the coordinates of these points A and B are respectively $\left(0, \frac{E_{bb}}{R_L}\right)$ and $(E_{bb}, 0)$. This line joining the points A and B is called the "load line" or "resistance line" and the reciprocal of the slope of this line equals the load resistance R_L .

The two curves intersect at the point P which is called the "point of operation". This point P satisfies both the equations (11·2) and (11·4) simultaneously. The ordinate of this point P then gives the no-signal plate current I_b for grid bias of E_{cc}, plate supply voltage E_{bb} and load resistance R_L . The abscissa of point P gives the no-signal plate voltage E_b. These no-signal plate current and plate voltage are indicated by capital letters I and E respectively followed by subscript b pertaining to plate It may be noted that no signal plate voltage E_b is less than the plate supply E_{bb} by an amount equal to the d.c. voltage drop $I_b . R_L$ across the load resistor R_L . On the grid side, E_{cc} being negative no grid current flows and further since there is no resistor in the grid circuit, grid voltage E_c under no-signal condition is the same as the grid bias voltage E_{cc}.

If the load resistance R_L is changed to some new value R_L' the load line will still pass through the point E

or down so as to intersect the current axis at point E_{bb}/R_L'. On the other hand if the grid voltage E_{cc} changes, point P shifts along the load line up or down depending upon whether the value of E_{cc} is made less or more negative. The point of operation may be found by noting the point of intersection of load line and the static characteristic curve for this new value of bias. For instance in Fig. 11·5. point P_1 is the point of operation for zero grid bias. Finally keeping R_L and E_{cc} unaltered, let the plate supply voltage be changed. Then the load line gets shifted horizontally by the same amount. The operating point P then shifts along the plate characteristic curve for the given grid bias. Point P moves up the plate characteristic for an increase in the value of plate supply voltage E_{bb} and vice versa.

Working of vacuum-tube amplifier with small a.c. grid signal. Referring now to the circuit of Fig. 11·3, a small amplitude a.c. signal is applied between grid and cathode in series with the grid bias. Voltages E_{bb} and E_{cc}, and load resistance R_L are so chosen that the operating point lies over the linear region of the characteristics. Linear amplification is then obtained provided that signal voltage has so small an amplitude that operation is confined essentially to linear region of characteristic curves. Amplified output voltage then appears across the load resistor R_L. We here take up a graphical study of the relation between the grid-signal voltage and the amplified output voltage.

Let signal voltage be sinusoidal. The total grid to cathode voltage e_c consists of two parts (*i*) constant grid bias voltage E_{cc} $(=E_c)$ and (*ii*) a.c. signal voltage of instantaneous value e_{gk}, as given by the relation

$$e_c=E_c+e_{gk} \qquad \text{... (11·6)}$$

Usually it is so arranged that the maximum value of e_{gk} is small compared with the bias voltage E_{cc}. Hence this grid signal voltage e_{gk} may be considered as a slow variation of E_{cc}. This change e_{gk} in the grid voltage results in corresponding increments in the plate current and plate voltage, indicated respectively by i_p and e_p. The total instantaneous plate current i_b equals the no-signal plate current I_b plus the incremental or a.c. plate current i_p. Thus,

$$i_b=I_b+i_p \qquad \text{... (11·7)}$$

Similarly total instantaneous plate voltage e_b is the sum of the no-signal plate voltage E_b and incremental or a.c. plate voltage e_p. Thus

$$e_b=E_b+e_p \qquad \text{... (11·8)}$$

Instantaneous grid signal voltage e_{gk} may be for a sinusoidal form, expressed as :

$$e_{gk}=E_{gm}.\sin\omega t \qquad \text{... (11·9)}$$

where ω is the angular frequency $=2\pi f$, f being the signal frequency in cycles per second, and E_{gm} being the amplitude of the grid signal.

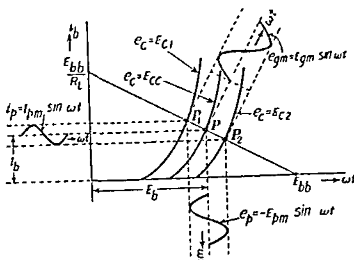


Fig. 11·6 Operation of an amplifier with sinusoidal grid signal and resistive load

Fig. 11·6 illustrates graphically the operation of the amplifier with sinusoidal grid signal. P is the point of operation with zero grid signal and with steady state grid bias of $E_c=E_{cc}$. The ordinate of this point P gives the steady (or d.c.) plate current I_b and the abscissa gives the d.c. plate voltage E_b. On application of sinusoidal grid signal of maximum value E_{gm}, the total grid voltage e_c varies between the limits E_{c1} and E_{c2} as shown in Fig. 11·6. As the a c. grid voltage e_{gk} executes the positive half cycle, the point of operation P moves towards the point P_1, with the result that the total plate current i_b becomes larger and at the same time the plate voltage l becomes smaller. The complete cycle of operation is shown in Fig 11·6 It is to be noted that the plate current cycle is in phase with the grid voltage cycle whereas the plate voltage cycle is 180 degrees out of phase. Hence the a.c. components of late current and plate voltage may be written as :

$$i_p=I_{pm}\sin\omega t \quad \text{... (11·10)}$$

and

$$e_p=E_{pm}.\sin(\omega t+180^\circ)$$

$$=-E_{pm}.\sin\omega t \quad \text{... (11·11)}$$

Such phase relations exist for a resistive load. For a load impedance which is not completely resistive, the a.c. plate current will no longer be in phase with the grid voltage and hence the incremental or a.c. plate voltage e_p will not b[illegible]ly 180 degrees out of phase with the a.c. grid voltage.

These a.c. or incremental current and voltages enable us to determine the amplification property of the amplifier. Since amplification or voltage gain is nothing but the ratio of a.c. output voltage to a.c. input voltage. Fig. 11·7 shows the waveforms and phase relations of a.c. grid voltage, a.c. plate current and a.c. plate voltage. These a.c. or incremental quantities are the variations about the zero signal or quiescent values.

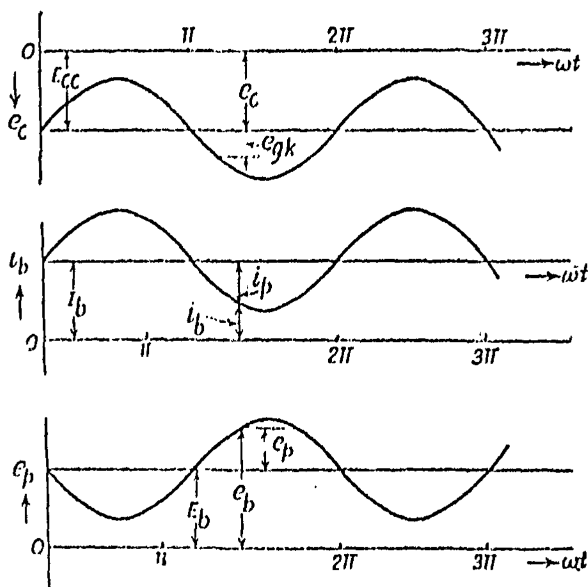


Fig. 11·7. Waveforms of a.c. grid signal, a.c. plate voltage and a.c. plate current.

It has been assumed that the point P moves along the load line and in so doing it comes across portions of the static characteristic curves which are linear, parallel and equidistant for equal increments of the grid bias. This condition is necessary in order that the waveform of a.c. plate voltage is an exact but enlarged reproduction of grid signal voltage. The three coefficients of the tube namely μ, g_m and r_p are constant only if the above conditions are satisfied. However, these conditions are never fully satisfied even over the small region of operation and hence the coefficients are not absolutely constant. As a result of this, the output voltage waveform is not an exact reproduction of the grid signal waveform. Deviations from these ideal conditions are, however, usually small and hence the distortion in the reproduced waveform is small.

Dynamic Transfer Characteristic. In Fig. 11·5, load line cuts the static plate characteristics for different values of grid voltage e_c in points which give the plate current for different values of grid

noted that it differs from the mutual characteristic which is a static characteristic *i.e.*, a characteristic pertaining to the tube alone. This dynamic characteristic, on the other hand, is a characteristic of the tube as well as the associated circuit and it enables us to obtain *directly* the *output* current for a given input signal. In Fig. 11·8,

exhibits marked non-linearity particularly for small values of plate current, the dynamic characteristic may be essentially linear if along the load line the plate characteristics are *essentially equidistant* for equal increments of grid voltage.

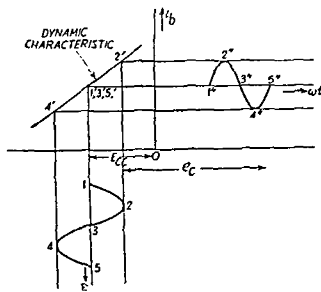


Fig. 11·8 Dynamic characteristic and its use for finding the output current waveform for a given input signal waveshape.

Example 1. *A triode amplifier uses resistance load. Keeping plate supply voltage constant, if the grid bias is changed from −4 to −6 volts, the no-signal plate current reduces from 5 to 3 milliamperes and no-signal plate voltage increases from 80 to 100 volts. Find the values of load resistance and plate supply voltage.*

Solution : $E_b = E_{bb} - I_b \,.\, R_L$

Substituting the values :

$$80 = E_{bb} - (5 \times 10^{-3}) \,.\, R_L \qquad \text{... (1)}$$

and

$$100 = E_{bb} - (3 \times 10^{-3}) \,.\, R_L \qquad \text{... (2)}$$

From (1) and (2) eliminating E_{bb},

$$80 + (5 \times 10^{-3}) \,.\, R_L = 100 + (3 \times 10^{-3}) \,.\, R_L$$

or

$$R_L \times (2 \times 10^{-3}) = 20.$$

Hence

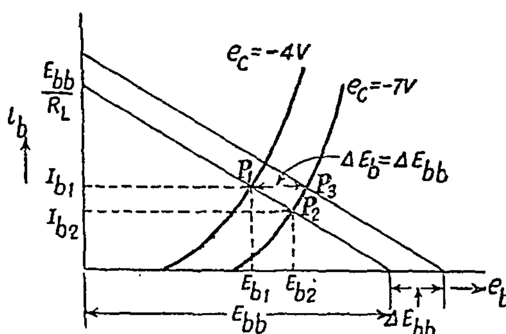
$$R_L = \frac{20}{2 \times 10^{-3}} = \mathbf{10{,}000\ ohms.}$$

From (1) substituting the value of R_L we get,

$$E_{bb} = 80 + (5 \times 10^{-3}) \,.\, 10^4 = \mathbf{130\ volts.}$$

Example 2. *In a triode amplifier with a grid bias of −4 volts, no-signal plate current and plate voltage are 7 mA and 150 volts respectively. Now if grid bias is changed to −7 volts, corresponding values of no-signal plate current and plate voltage are 4 mA and 210 volts respectively. Find the load resistance and plate supply voltage. If μ of the tube in the region of operation is 20, by what amount should the plate supply voltage be increased to bring the no-signal plate current I_b back to 7 mA ?*

Solution. In the figure below let P_1 and P_2 be the points of



operation for grid bias of −4 volts and −7 volts respectively with a plate supply voltage of E_{bb}.

Then

$$E_{b1} = E_{bb} - I_{b1} \,.\, R_L$$

$$E_{b2} = E_{bb} - I_{b2} \,.\, R_L$$

Substituting the values :

$$150 = E_{bb} - (7 \times 10^{-3}) \,.\, R_L \qquad \text{... (1)}$$

and

$$210 = E_{bb} - (4 \times 10^{-3}) \,.\, R_L \qquad \text{... (2)}$$

Eliminating E_{bb} from (1) and (2), we get

$$(210-150)=(7-4)\times10^{-3}\times R_L$$

Hence $$R_L=\frac{60 \text{ volts}}{3\times10^{-3} \text{ amp.}}=20{,}000 \text{ ohms.}$$

Substituting the value of R_L in (1) we get,

$$E_{bb}=150+7\times10^{-3}\times20\times10^{3}=290 \text{ volts.}$$

[illegible] of
[illegible] is
[illegible]

$$E_{b3}-E_{b1}=\triangle E_b-\triangle E_{bb}$$

$$\triangle E_c=-3 \text{ volts,}$$

But $$\mu=-\frac{\triangle E_b}{\triangle E_c}\Big/ I_b \text{ constant.}$$

Hence $$20=-\left(\frac{\triangle E_b}{-3}\right) \text{ or } \triangle E_b=\triangle E_{bb}=3\times20=60 \text{ volts.}$$

Example 3. *The plate characteristics of a triode are given by the expression*

$$i_b=8\times10^{-6}\,(e_b+15e_c)^{1.6} \text{ amperes.}$$

It is required to be operated at a plate voltage E_b of 200 volts and a grid bias E_c of -8 volts. (a) Calculate the dynamic plate resistance of the tube at the point of operation. (b) If the tube is to use a load resistance of 11 kilo-ohms, calculate the plate supply voltage necessary to get the specified operating point.

Solution : $$i_b=8\times10^{-6}(e_b+15e_c)^{1.6}$$

At the operating point, $e_c=E_{cc}=-8$ volts, so that

$$i_b=8\times10^{-6}(e_b-120)^{1.6} \qquad \dots (1)$$

Differentiating (1) with respect to e_b, we get

$$\frac{di_b}{de_b}=8\times10^{-6}\times1{\cdot}6\,(e_b-120)^{0.6}$$

At the operating point $e_b=200$ volts,

Hence $$\frac{di_b}{de_b}=12{\cdot}8\times10^{-6}\,(200-120)^{0.6}=12{\cdot}8\times10^{-6}\times15{\cdot}2$$

Dynamic plate resistance

$$r_p=\frac{de_b}{di_b}=\frac{10^6}{12{\cdot}8\times15{\cdot}2}=5635 \text{ ohms.}$$

At the specified points, plate current is given by

$$I_b=8\times10^{-6}\,(200-15\times8)^{1.6}$$
$$=8\times10^{-6}\times80^{1.6}=8{\cdot}874\times10^{-3} \text{ amperes}$$

Plate supply voltage is given by

$$E_{bb}=E_b+I_b.R_L=200+8{\cdot}874\times10^{-3}\times12\times10^3$$
$$=200+1.6{\cdot}5=306{\cdot}5 \text{ volts.}$$

Classification of amplifiers. Amplifiers may be classified in accordance with one of the following ways :—

I. **In accordance with the type of load.** According to this method of classification amplifiers may be classified as (*i*) Untuned amplifiers and (*ii*) Tuned amplifiers.

Untuned amplifiers may further be put into two categories (*a*) Audio frequency amplifiers and (*b*) video frequency amplifiers.

Audio frequency amplifiers, are used for amplifying a.c. signals having frequencies in the audio frequency range *i.e.*, upto about 15 kc/s. This is the frequency range in which signals are generated when we speak or when any sound instrument is played. Such amplifiers are, therefore, in general used for amplifying the output of a microphone responding to the sound signal.

Video frequency amplifiers. These are the amplifiers used for amplifying electrical signals extending in frequency upto a few Mc/s such as produced for transmission of vision or picture. For transmitting a picture, as required in Television, the picture or vision is dissected into a number of small elements and electrical signals generated corresponding to the light value of each element and then transmitted in proper sequence. A video amplifier used for amplifying such a vision signal must amplify all the frequency components equally.

Tuned amplifiers or Radio Frequency amplifiers. These are the amplifiers used for amplifying an electrical signal containing either a single radio frequency or narrow band of radio frequencies. By radio frequency we mean a frequency greater than 30 kc/s. Such Radio Frequency (abbreviated as R.F.) amplifiers invariably use a tuned circuit as the load and hence radio frequency amplifiers are often referred to as tuned amplifiers. This method of classification may thus be considered to be one based on the frequencies of signals to be amplified.

II. **Classification of amplifiers on the basis of the period of conduction of amplifier tube.** According to this method of classification, all amplifiers may be classified as A, AB, B or C amplifiers.

(*a*) **Class A amplifier** is one in which the grid bias and the a.c. grid voltage are so adjusted that the plate current flows for the complete a.c. cycle.

A subscript 1 is added to letter or letters of the class identification to indicate that the grid current does not flow during any part of the input cycle. Thus class A_1 amplifier is one in which the grid bias and a.c. grid signal amplitude are so adjusted that the plate current flows for the entire a.c. cycle while the grid current does not flow at any time. Such a condition is illustrated in Fig. 11·9.

A subscript 2 is added to indicate that the grid current does flow during some part of the input a c. cycle. Thus in a class A_2

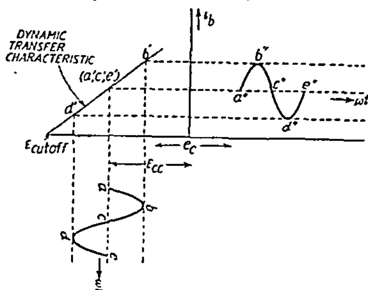


Fig. 11 9 Input voltage and output current waveforms in a class A_1 amplifier.

amplifier plate current flows throughout the a c. cycle while at the same time grid current also flows during some part of the a.c. cycle.

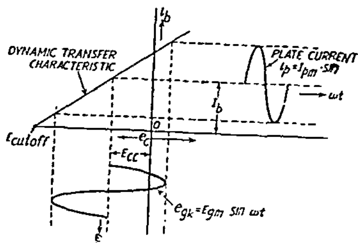


Fig. 11·10. Input voltage and output current waveforms in a class A_2 amplifier.

The operating conditions for A_2 amplifier are indicated in Fig. 11·10. During the shaded portion of the applied input signal total

instantaneous grid voltage is positive and hence grid current flows.

(*b*) **Class AB ampfilier** is one in which the grid bias and a.c. grid voltages are so adjusted that the plate current flows for appreciably greater than half but less than the entire cycle.

This type of amplifier may further be classified as AB_1 and AB_2 amplifiers depending upon whether the grid current does or does not flow for some period of the a.c. cycle. Fig. 11·11 shows the input grid voltage and output plate current waveforms in a class AB_1 amplifier. During the shaded part of the negative half cycle of the applied a.c. voltage, the total instantaneous grid voltage e_c becomes more negative than the cutoff voltage and hence plate current does not flow during this interval. During the positive half cycle of the applied a.c. voltage, the total instantaneous grid voltage e_c never becomes positive, and hence grid never flows. The plate current obviously flows for a period appreciably greater than half but less than the entire cycle. The shaded and dotted portion of the plate current a.c. waveform is removed. Obviously then the operation is class AB_1.

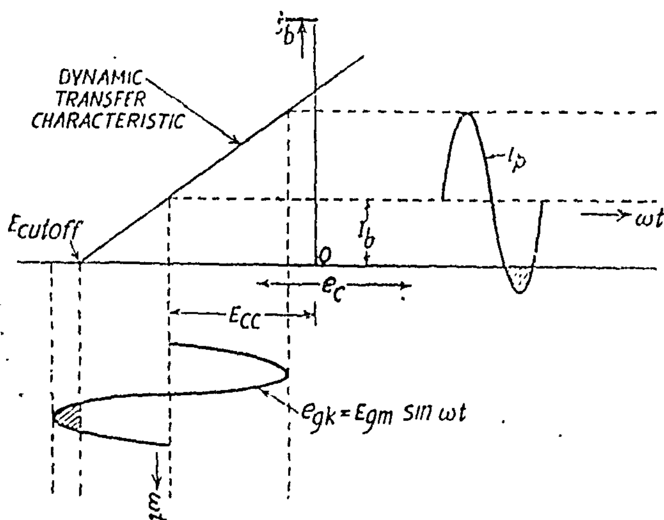


Fig. 11·11. Input voltage and output current waveforms in class AB_1 amplifier.

Fig. 11·12 shows the waveforms of input grid voltage and output plate current in a class AB_2 amplifier. It differs from AB_1 operation only in the fact that during a portion of the applied a.c. voltage cycle, total instantaneous grid voltage becomes positive and hence grid current flows for that period.

(c) Class B amplifier is one in which the grid bias is adjusted to be almost equal to cutoff voltage so that the plate current is zero

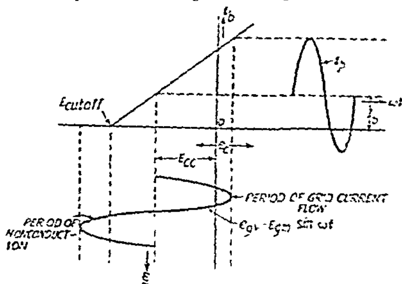


Fig. 11·12. Waveforms of grid voltage and output plate current in a class AB_2 amplifier.

with no-grid signal and on application of a.c. grid signal, plate current flows for half the a.c. cycle.

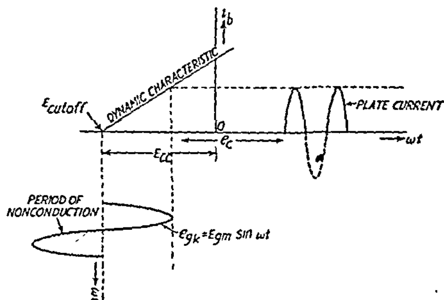


Fig. 11·13. Input signal voltage and output current waveforms in a class B_1 amplifier.

A class B amplifier may further be classified as B_1 and B_2 depending upon whether grid current does not or does flow for a portion of the applied signal cycle. Fig. 11·13 shows the waveforms of input grid voltage and output plate current in a class B_1 amplifier for sinusoidal input. Obviously the plate current flows for half the a.c. cycle. Further the grid bias and grid signal amplitude are so adjusted that the total instantaneous grid voltage never becomes positive and hence grid current never flows.

Fig. 11·14 shows the waveforms of grid voltage and output plate current in a class B_2 amplifier for sinusoidal signal. The operation is similar to that for class B_1 amplifier except that the grid signal amplitude is so adjusted that the total instantaneous grid voltage becomes positive for a part of the a.c. cycle and hence grid current does flow during that interval. The plate current in this case also flows for half the a.c. cycle.

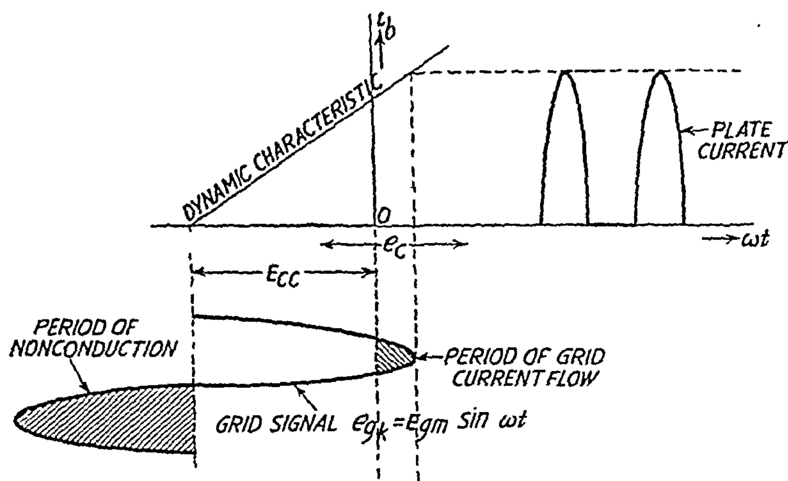


Fig. 11·14. Waveforms of input grid voltage and output plate current in a class B_2 amplifier.

(*d*) **Class C amplifier** is one in which the grid bias is sufficiently greater than the cutoff voltage with the result that the plate current is zero for no-signal condition and on application of grid signal, plate current flows for a period appreciably less than one-half of a.c. cycle as shown in Fig. 11·15.

Effect of grid current flow. In the above classification of amplifiers, subscript 2 is suffixed to the class identification *A, AB, B* or *C* to indicate that grid current flows for a part of the a.c. cycle. By grid current flow we mean an appreciable conduction grid current. This qualification is necessary to exclude firstly displacement grid current which always flows except at very low frequencies and

secondly very minute grid current which may flow even when the grid voltage is slightly negative with respect to the cathode. Thus appre-

Fig 11·15 Waveforms of input grid voltage and output plate current in a class *C* amplifier.

ciable grid current flows only when grid becomes positive with respect to the cathode and this grid current is generally avoided for the following reasons —

ıge drop across any element
ge actually applied to the
ecome prominent even at
es impedance is very large.
voltage, flowing through
a large inpedance may produce considerable voltage drop. This effect is of significance in audio frequency amplifiers and repeaters particularly those using small electron tubes.

(*ii*) Since total emission current from cathode consists of plate and grid currents, the presence of grid current reduces the plate current to a value below the emission current. This effect of plate current reduction becomes important in high power oscillators and amplifiers.

(*iii*) Grid current may cause overheating to the grid and in extreme cases may even damage the tube.

III. Classification of amplifiers depending on the number of

lassi-
and
mpli-
fier. Such an amplifier is suitable when small amplification is sufficient. A *cascade amplifier, on the other hand, consists of many* amplifiers connected in such a way that the output of one amplifier is fed to the input of the next. Each amplifier is then referred to as a stage of the overall cascade amplifier. Overa

where $\triangle i_b$, $\triangle e_c$, and $\triangle e_b$ are the increments of plate current, grid voltage and plate voltage respectively.

This equation (11·13) states that when a small change $\triangle e_c$ is introduced in the grid voltage, this itself changes the plate current by an amount $\frac{\partial i_b}{\partial e_c} . \triangle e_c$ which is multiplication of the grid voltage increment $\triangle e_c$ with $\frac{\partial i_b}{\partial e_c}$, the rate of change of plate current with grid voltage assuming all other quantities to be constant. But simultaneously this increment $\triangle e_c$ in the grid voltage causes an increment $\triangle e_b$ in the plate voltage. This increment $\triangle e_b$ in its turn causes a further change in the plate current amounting to $\frac{\partial i_b}{\partial e_b} . \triangle e_b$ which is the multiplication of the increment $\triangle e_b$ with $\frac{\partial i_b}{\partial e_b}$, the rate of change of plate current with plate voltage, keeping all other quantities constant.

If the increment $\triangle e_c$ in the grid voltage is sinusoidal, it may be replaced by $e_{gk} = E_{gm} . \sin \omega t$. Since the operation is linear class A_1, the corresponding increments in plate voltage and plate current namely $\triangle e_b$ and $\triangle i_b$ may also be replaced by the a.c. components e_p and i_p respectively. Further $\frac{\partial i_b}{\partial e_c}$, the rate of change of plate current with grid voltage keeping the plate voltage constant, is nothing but the coefficient mutual conductance g_m of the tube. Similarly $\frac{\partial i_b}{\partial e_b}$ is nothing but $\frac{1}{r_p}$ of the tube at the point of operation.

The equation (11·13) may then be written as :

$$i_p = g_m . e_{gk} + \frac{e_p}{r_p} \quad \text{... (11·14)}$$

or

$$e_p = - g_m . r_p . e_{gk} + i_p . r_p$$

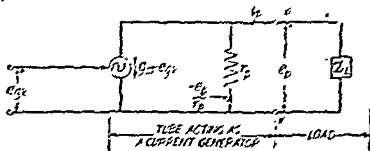
or

$$e_p = - \mu . e_{gk} + i_p . r_p \quad \text{... (11·15)}$$

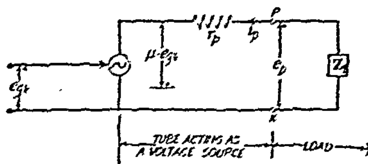
Current i_p, as given by equation (11·14) is the same as obtained from a constant current signal generator of current $g_m . e_{gk}$ with a source resistance of r_p in shunt with the generator. Hence the tube itself may be represented by a constant-current generator as indicated in Fig. 11·17 (*a*). The positive direction of e_{gk}, e_p, and i_p are also given in the figure. Current flowing upward through r_p is $-e_p/r_p$, negative sign appearing because of positive assigned direction of e_p. Because of no grid current flow, the input circuit is shown disconnected from the a.c. current-generator.

Output voltage e_p as given by Eq. (11·15) is, on the other hand, the same as obtained from a constant-voltage generator of voltage $\mu . e_{gk}$ with a source resistance of r_p as shown in Fig. 11·17 (*b*). With

the assigned positive directions of voltages μe_{gk} and e_p and current i_p, Eq. (11·15) is fully satisfied.



(a) Amplifier tube as a constant current generator.



(b) Amplifier tube as a constant voltage generator.

Fig. 11·17. A.C. equivalent form of an amplifier tube.

The a.c. equivalent circuits for electron tubes as given in Fig. 11·17 are valid only if the interelectrode capacitances in the tube are negligible. At high frequencies the effect of these capacitances become quite dominant and then the a.c. equivalent circuits must be modified. Further the load line is a straight line only for resistive load. If the load is complex the path of operation is an ellipse. This ellipse must lie over the region of static characteristics which are linear, parallel and equidistant for increments of the parameter, if linear amplification is to be obtained.

Voltage gain of a single stage untuned linear A_1 amplifier. Fig. 11·18 shows the basic circuit of a single stage untuned linear A_1 amplifier using load impedance Z_L which, in general, is complex.

To find the voltage amplification or voltage gain of the amplifier of Fig. 11·18, we first draw the a.c. equivalent circuit of this amplifier by (i) replacing the tube itself by either the equivalent current generator or voltage generator (ii) eliminating all d.c. voltage and current sources and (iii) considering only the a.c. or incremental values of currents and voltages. Fig. 11·19 shows the a.c. equivalent

where $\triangle i_b$, $\triangle e_c$, and $\triangle e_b$ are the increments of plate current, grid voltage and plate voltage respectively.

This equation (11·13) states that when a small change $\triangle e_c$ is introduced in the grid voltage, this itself changes the plate current by an amount $\frac{\partial i_b}{\partial e_c} . \triangle e_c$ which is multiplication of the grid voltage increment $\triangle e_c$ with $\frac{\partial i_b}{\partial e_c}$, the rate of change of plate current with grid voltage assuming all other quantities to be constant. But simultaneously this increment $\triangle e_c$ in the grid voltage causes an increment $\triangle e_b$ in the plate voltage. This increment $\triangle e_b$ in its turn causes a further change in the plate current amounting to $\frac{\partial i_b}{\partial e_b} . \triangle e_b$ which is the multiplication of the increment $\triangle e_b$ with $\frac{\partial i_b}{\partial e_b}$, the rate of change of plate current with plate voltage, keeping all other quantities constant.

If the increment $\triangle e_c$ in the grid voltage is sinusoidal, it may be replaced by $e_{gk} = E_{gm} . \sin \omega t$. Since the operation is linear class A_1, the corresponding increments in plate voltage and plate current namely $\triangle e_b$ and $\triangle i_b$ may also be replaced by the a.c. components e_p and i_p respectively. Further $\frac{\partial i_b}{\partial e_c}$, the rate of change of plate current with grid voltage keeping the plate voltage constant, is nothing but the coefficient mutual conductance g_m of the tube. Similarly $\frac{\partial i_b}{\partial e_b}$ is nothing but $\frac{1}{r_p}$ of the tube at the point of operation.

The equation (11·13) may then be written as :

$$i_p = g_m . e_{gk} + \frac{e_p}{r_p} \quad \dots (11 \cdot 14)$$

or

$$e_p = - g_m . r_p . e_{gk} + i_p . r_p$$

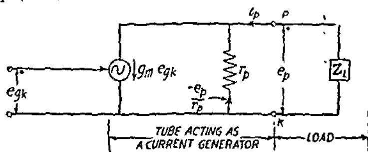
or

$$e_p = - \mu . e_{gk} + i_p . r_p \quad \dots (11 \cdot 15)$$

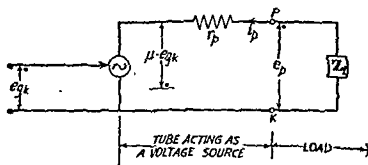
Current i_p, as given by equation (11·14) is the same as obtained from a constant current signal generator of current $g_m . e_{gk}$ with a source resistance of r_p in shunt with the generator. Hence the tube itself may be represented by a constant-current generator as indicated in Fig. 11·17 (*a*). The positive direction of e_{gk}, e_p, and i_p are also given in the figure. Current flowing upward through r_p is $-e_p/r_p$, negative sign appearing because of positive assigned direction of e_p. Because of no grid current flow, the input circuit is shown disconnected from the a.c. current-generator.

Output voltage e_p as given by Eq. (11·15) is, on the other hand, the same as obtained from a constant-voltage generator of voltage $\mu . e_{gk}$ with a source resistance of r_p as shown in Fig. 11·17 (*b*). With

the assigned positive directions of voltages $\mu.e_{gk}$ and e_p and current i_p, Eq. (11·15) is fully satisfied.



(a) Amplifier tube as a constant current generator.



(b) Amplifier tube as a constant voltage generator.

Fig. 11·17. A.C. equivalent form of an amplifier tube.

The a.c. equivalent circuits for electron tubes as given in Fig. 11·17 are valid only if the interelectrode capacitances in the tube are negligible. At high frequencies the effect of these capacitances become quite dominant and then the a.c. equivalent circuits must be modified. Further the load line is a straight line only for resistive load. If the load is complex the path of operation is an ellipse. This ellipse must lie over the region of static characteristics which are linear, parallel and equidistant for increments of the parameter, if linear amplification is to be obtained.

Voltage gain of a single stage untuned linear A_1 amplifier. Fig. 11·18 shows the basic circuit of a single stage untuned linear A_1 amplifier using load impedance Z_L which, in general, is complex.

To find the voltage amplification or voltage gain of the amplifier of Fig. 11·18, we first draw the a.c. equivalent circuit of this amplifier by (*i*) replacing the tube itself by either the equivalent current generator or voltage generator (*ii*) eliminating all d.c. voltage and current sources and (*iii*) considering only the a.c. or incremental values of currents and voltages. Fig. 11·19 shows the a.c. equivalent

circuit of the amplifier of Fig. 11·18 using voltage generator form for the tube.

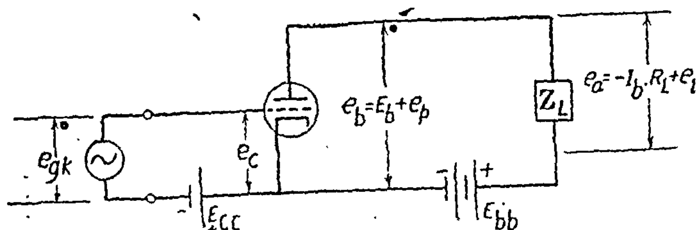


Fig. 11·18. Basic circuit of a single stage untuned class A_1 amplifier.

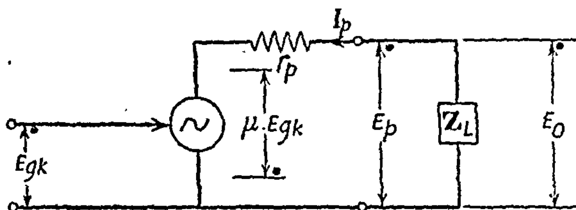


Fig. 11·19. A.C. equivalent circuit of amplifier of Fig. 11·18.

Let the grid signal voltage applied to the amplifier be sinusoidal and given by the equation :

$$e_{gk} = \sqrt{2}.E_{gk}.\sin \omega t \qquad \dots (11·16)$$

where e_{gk} is the instantaneous value and E_{gk} is the r.m.s. or effective value of the sinusoidal signal voltage of angular frequency ω radians/sec. Then if the amplifier is linear, all other voltages and currents will also be sinusoidal and of the same frequency. In that case, instead of using the instantaneous values, such as e_{gk}, e_p, e_L, i_p etc., we may use the corresponding effective or r.m.s. values E_{gk}, E_p, E_L, I_p etc. With this modification, we may say that the grid signal is of r.m.s. value E_{gk} and the equivalent voltage generator generates a voltage μE_{gk} and has a series resistance r_p as shown in Fig. 11·19. The total impedance in the plate circuit is then $r_p + Z_L$ so that effective value of a.c. plate current is given by :

$$\mathbf{I}_p = \frac{\mu \mathbf{E}_{gk}}{r_p + \mathbf{Z}_L} \qquad \dots (11·17)$$

This current $\mathbf{I}_p$ flowing through the load impedance $\mathbf{Z}_L$ produces the output voltage $\mathbf{E}_o$. Obviously $\mathbf{E}_0$ is the same as the r.m.s. plate voltage $\mathbf{E}_p$. $\mathbf{E}_0$ is given by :

$$\mathbf{E}_0 = -\mathbf{I}_p.\mathbf{Z}_L \qquad \dots (11·18)$$

Substituting the value of I_p from Eq. (11·17) into Eq. (11·18), we get :

$$E_o = -\mu\, E_{gk} \cdot \frac{Z_L}{r_p + Z_L} \quad \ldots (11\cdot19)$$

Complex voltage amplification (or gain), of an amplifier is defined as the ratio of the complex value of the output voltage to the complex value of the input voltage. Thus from Eq. (11·19),

$$\text{Complex voltage gain } A = \frac{E_o}{E_{gk}} = -\mu \frac{Z_L}{r_p + Z_L} \quad \ldots (11\cdot20)$$

This complex voltage gain, often abbreviated simply as voltage gain, may be put as,

$$A = A\angle\theta \quad \ldots (11\cdot21)$$

where A is the magnitude of complex voltage gain A *i.e.*, simply the ratio of the amplitudes or r.m.s. values of the output voltage and input signal voltage, and θ is the phase angle of the complex voltage gain.

A study of Eq. (11·20) reveals that this phase angle θ has two components (*i*) constant 180° phase shift corresponding to the minus sign in Eq. (11·20) and (*ii*) additional phase angle due to the complex nature of load impedance Z_L. Thus if Z_L is purely resistive, this additional phase angle is zero

Three types of load impedance Z_L will be considered :

CASE (I) : Z_L is a complex inductive impedance.

The a.c. equivalent circuit is given in Fig. 11·20.

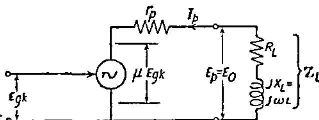


Fig. 11·20. A C. equivalent circuit of a single stage amplifier using complex inductive load

$$\text{Then } Z_L = R_L + j X_L \quad \ldots (11\cdot22)$$

$$\text{where } X_L = \omega_L \quad \ldots (11\cdot23)$$

$$\text{Hence } I_p = \frac{\mu\, E_{gk}}{r_p + R_L + j\omega_L} \quad \ldots (11\cdot24)$$

$$\text{magnitude } I_p = \frac{\mu\, E_{gk}}{\sqrt{(r_p + R_L)^2 + \omega^2 L^2}} \quad \ldots (11\cdot25)$$

and phase angle α of plate current is given by :

$$\alpha=-\tan^{-1}\frac{\omega_L}{r_p+R_L} \quad \text{... (11·26)}$$

Further plate voltage E_p=output voltage $E_0=-I_p.Z_L$

$$=-\mu E_{gk}.\frac{Z_L}{r_p+Z_L}$$

$$=-\mu E_{gk}.\frac{R_L+j\omega_L}{(r_p+r_L)+j\omega_L} \quad \text{... (11·27)}$$

The magnitude of the output voltage is given by :

$$E_o=\mu E_{gk}.\frac{\sqrt{R_L^2+\omega^2L^2}}{\sqrt{(r_p+R_L)^2+\omega^2L^2}} \quad \text{... (11·28)}$$

and phase angle $\varphi=180°+\theta+\alpha$

where θ=phase angle of load$=\tan^{-1}\frac{\omega L}{R_L}$ ... (11·29)

and α=phase angle of plate current

$$I_p=-\tan^{-1}\frac{\omega L}{r_p+R_L}$$

Hence phase angle of output voltage may be written as :

$$\varphi=180°+\tan^{-1}\frac{\omega L}{R_L}-\tan^{-1}\frac{\omega L}{r_p+R_L} \quad \text{... (11·30)}$$

Complex voltage gain $A=\frac{E_0}{E_{gk}}=-\mu\frac{Z_L}{r_p+Z_L}$

$$=-\mu\frac{R_L+j\omega L}{(r_p+R_L)+j\omega L} \quad \text{... (11·31)}$$

Magnitude of voltage is given by :

$$A=\mu\frac{\sqrt{R_L^2+\omega^2L^2}}{\sqrt{(r_p+R_L)^2+\omega^2L^2}} \quad \text{... (11·32)}$$

and phase angle φ of the complex voltage gain is given by :

$$\varphi=180°+\theta+\alpha=180°+\tan^{-1}\frac{\omega L}{R_L}-\tan^{-1}\frac{\omega L}{r_p+R_L} \quad \text{... (11·33)}$$

This is the same as the phase angle of the output voltage E_0.

The vector diagram or phasor diagram for sinusoidally varying quantities in the amplifier of Fig. 11·20 for complex inductive load is shown in Fig. 11·21. The input grid voltage E_{gk} is used as the reference vector. Since Z_L is inductive, a.c. plate current I_p lags behind the driving voltage μE_{gk}. The phase angle of I_p with respect to μE_{gk} is given by Eq. (11·26). The voltage drop $I_p(R_L+r_p)$ across the series resistances (r_p+R_L) is in phase with I_p while the voltage drop $jI_p.X_L$ across induc-

tance L leads $\mathbf{I}_p$ by 90°. These two components namely $\mathbf{I}_p\,(R_L+r_p)$ and $j\mathbf{I}_p \cdot X_L$ added vectorially equal to the total driving voltage $\mu\mathbf{E}_{gk}$ as shown in phasor diagram of Fig. 11·21. The components $\mathbf{I}_p.R_L$ and $j\mathbf{I}_p.X_L$ added vectorially constitute the voltage drop $\mathbf{I}_p.\mathbf{Z}_L$ across the load impedance as shown in the phasor diagram.
… an angle θ as given by Eqn.
…ge with respect to reference
… in phasor diagram The a.c.
output voltage, however, is equal to $-\mathbf{I}_p.\mathbf{Z}_L$ and it is shown as a
… The phase angle φ of this out-
… (11·27). The phase angle of
… the same as that of the output voltage $\mathbf{E}_0$.

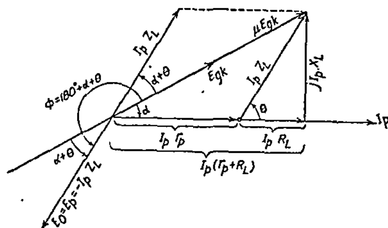


Fig. 11·21. Vector diagram or phasor diagram for sinusoidally varying a.c components in a single stage amplifier using inductive load

CASE II : Pure resistance load

In this case, $\mathbf{Z}_L = R_L + j.0$ ... (11·34)

and $\mathbf{I}_p = \dfrac{\mu\,\mathbf{E}_{gk}}{r_p + R_L}$ ... (11·35)

$\mathbf{I}_p$ is in phase with $\mu\,\mathbf{E}_{gk}$.

Hence $\mathbf{E}_0 = -\mathbf{I}_p.R_L = -\,\mu\mathbf{E}_{gk}.\dfrac{R_L}{r_p + R_L}$ ... (11·36)

and voltage gain $\mathbf{A} = \dfrac{\mathbf{E}_0}{\mathbf{E}_{gk}} = -\mu.\dfrac{R_L}{r_p + R^L}$ ... (11·37)

The phase angle of output voltage $\mathbf{E}_0$ and also that of the voltage gain $\mathbf{A}$ is a fixed quantity 180°. Thus we conclude that if we

exclude all reactive components from amplifier circuit, the phase shift produced by one stage of amplifier is always 180° irrespective of frequency.

CASE III : Z_L *a pure inductance*

In this case $Z_L = 0 + j X_L = 0 + j.\omega L$ ... (11·38)

and $$I_p = \frac{\mu E_{gk}}{r_p + j\omega L} \quad \text{... (11·39)}$$

Phase angle α of I_p is given by,

$$\alpha = -\tan^{-1} \frac{\omega L}{r_p} \quad \text{... (11·40)}$$

and magnitude of I_p is given by

$$I_p = \frac{-\mu E_{gk}}{\sqrt{r_p^2 + \omega^2 L^2}} \quad \text{... (11·41)}$$

The output voltage E_0 is given by

$$E_0 = -I_p \,.\, Z_L = -\frac{\mu E_{gk}.j\omega L}{r_p + j\omega L} = \frac{-\mu E_{gk}}{1 + \dfrac{r_p}{j\omega L}} \quad \text{... (11·42)}$$

$$= \frac{\mu E_{gk}}{\sqrt{1 + (r_p/\omega L)^2}} \Big|270° - \tan^{-1}\frac{\omega L}{r_p} \quad \text{... (11·43)}$$

i.e., the magnitude is $\dfrac{\mu E_{gk}}{\sqrt{1 + \left(\dfrac{r_p}{\omega L}\right)^2}}$ and the phase angle is

$$270° - \tan^{-1} \frac{\omega L}{r_p}.$$

Voltage gain $$A = \frac{E_o}{E_{gk}} = \frac{-\mu . j\omega_L}{r_p + j\omega_L} \quad \text{... (11·44)}$$

$$= \frac{\mu}{\sqrt{1 + \left(\dfrac{r_p}{\omega L}\right)^2}} \Big|270° - \tan^{-1}\frac{\omega_L}{r_p} \quad \text{... (11·45)}$$

From Eq. 11·31 it may be seen that, for a given tube, as the ratio $\dfrac{Z_L}{r_p}$ is increased by increasing the value of Z_L, the magnitude of the voltage gain of the amplifier increases. Fig. 11·22 shows the nature of the variation of magnitude A with ratio Z_L/r_p for the three cases of complex inductive load, pure inductance load and pure resistance load.

It may be seen from Fig. 11·22 that for the same value of ratio Z_L/r_p, pure inductive load produces greater voltage gain because the magnitude of the denominator in equation (11·44) is $\sqrt{r_p^2+\omega^2L^2}$ *i.e.*, r_p and ωL are vectorially added with a phase difference of 90° whereas the magnitude of the denominator in Eq. (11·37) for pure resistive load is simply (r_p+R_L) *i.e.*, r_p and R_L are scalarly added. Stated alternatively, to produce the same voltage gain, the impedance of pure inductance load required is much smaller than the impedance of a pure resistive load. Another advantage obtained by the use of pure inductance load is that there is no d.c voltage drop in it and hence to provide the same d.c plate voltage E_b, the requisite plate supply

pure resistance load.

and expensive and

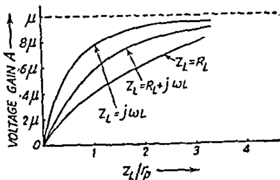


Fig. 11·22. Voltage of a single stage untuned amplifier as a function of ratio $\frac{Z_L}{r_p}$.

Curves of Fig. 11·23 show the nature of variation of phase angle of voltage gain with the ratio Z_L/r_p for pure resistive load, pure inductance load and for complex-inductive load. With resistance load, the phase angle has a constant value of 180° as seen from Eq (11·37). With pure inductive load, value of phase angle ϕ is 270° for Z_L/r_p equal to zero and then it reduces approaching the value of 180° for larger values of Z_L/r_p, in accordance with equation (11·44). For complex inductive load, the curve lies in-between the two curves of pure inductance load and pure resistance load and it follows the Eq. (11·33). The value at Z_L/r_p equal to zero, depends upon ı

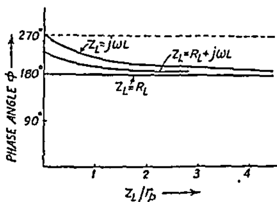


Fig 11·23. Variation of phase angle of voltage gain with the ratio Z_L/r_p

of ωL and R_L and their relative variations.

It is of interest to know the variation of voltage gain with frequency. Fig. 11·24 shows the nature of these curves in the cases of pure resistance load, pure inductance load, and complex inductive load. For pure resistance load, the voltage gain is independent of frequency and its magnitude is $\mu \cdot \frac{R_L}{r_p+R_L}$ as given by Eq. (11·37). For pure inductive load, voltage gain increases with frequency in accordance with Eq. (11·45). For complex inductive load, the voltage gain increases with frequency in accordance with Eq. (11·32). Obviously the minimum value of voltage gain in this case is $\mu \cdot \frac{R_L}{r_p+R_L}$ and occurs at zero frequency.

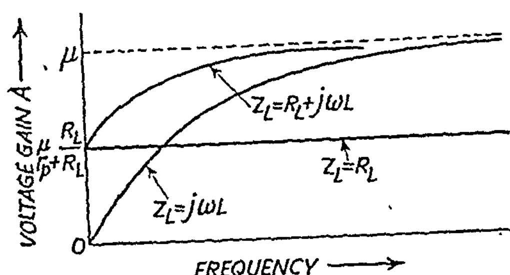


Fig. 11·24. Variation of voltage gain with frequency in a single stage amplifier.

Voltage gain of a single stage amplifier using pentode (or tetrode). The dynamic plate resistance r_p of a pentode or a tetrode is very large, usually of the order of a meg-ohm for receiving tubes. Hence in Eq. (11·20) for voltage gain, $\mathbf{Z}_L$ may be neglected compared with r_p so that the expression for voltage gain of a single stage amplifier using pentode becomes :

$$\mathbf{A}=-\mu \cdot \frac{\mathbf{Z}_L}{r_p}$$

or

$$\mathbf{A}=-g_m \cdot \mathbf{Z}_L \qquad \dots (11·45)$$

Thus with a pentode, the voltage gain may be increased simply by increasing the load impedance.

Since plate resistance r_p is very large for a pentode, it is convenient to replace pentode by a constant current generator generating a current $g_m \cdot E_{gk}$ with such a large shunt impedance r_p that it may be omitted. This current generator when connected to the load impedance $\mathbf{Z}_L$, produces output voltage equal to $-g_m E_{gk} \mathbf{Z}_L$ and hence voltage gain $\omega -g_m.\mathbf{Z}_L$. This expression is the same as (11·45), obtained by using voltage generator equivalent circuit for the tube.

Example 4. *A linear A_1 amplifier uses a vacuum triode having amplification factor of 20 and dynamic plate resistance r_p of 8000 ohms. The load in the plate circuit is a resistance of 12000 ohms. Calculate the voltage gain of the amplifier.*

Solution : The gain of a triode amplifier is given by

$$A=-\mu\frac{Z_L}{r_p+Z_L}$$

Substituting values of μ, r_p and Z_L,

$$A=-\frac{20\times12000}{8000+12000}=-12.$$

Example 5. *A linear A_1 triode amplifier has a voltage gain of −9 with a load resistance of 6 kilo-ohms and gain of −12 with a load resistance of 12 kilo-ohms. Calculate the amplification factor and plate resistance of the triode.*

Solution : For resistive load, voltage gain is given by :

$$A=-\frac{\mu R_L}{r_p+R_L}$$

Let load resistance R_L and plate resistance r_p be expressed in kilo-ohms.

Then $$-9=-\frac{\mu\times6}{r_p+6} \quad \ldots (1)$$

and $$-12=-\frac{\mu\times12}{r_p+12} \quad \ldots (2)$$

Combining (1) and (2) we get

$$18(r_p+6)=12(r_p+12)$$

or $$3(r_p+6)=2(r_p+12)$$

Hence $$r_p=24-18=6 \text{ kilo-ohms.}$$

Substituting the value of r_p in (1), we get

$$-9=\frac{-\mu\times6}{6+6}$$

Hence $$\mu=18.$$

Example 6. *A linear A_1 amplifier uses a triode having amplification μ of 10 and plate resistance r_p of 6000 ohms. The load in the plate circuit consist of a resistance of 10,000 ohms in series with an inductor of 1·5 henry. If the input is a sinusoidal voltage of effective value 2 volts and frequency 1000 c/s, find the amplitudes and phase angles of alternating plate current, plate voltage and complex voltage gain.*

Solution : Effective value of a.c. plate current is given by

$$\mathbf{I}_p=\frac{\mu\mathbf{E}_{gk}}{r_p+Z_L}=\frac{\mu\mathbf{E}_{gk}}{(r_p+R_L)+j\omega L}$$

Its magnitude $$I_p=\frac{\mu E_{gk}}{\sqrt{(r_p+R_L)^2+\omega^2L^2}}$$

Example 10. *An amplifier on using a valve V_1 having amplification factor of 30 and plate resistance of 50 kilo-ohms has the same gain as on using another valve V_2 having amplification factor of 15 and plate resistance of 8 kilo-ohms. Find the value of load resistance if such a condition is to be obtained.*

Solution. $$A = -\frac{30 \times R_L}{50000 + R_L}$$

Also $$A = -\frac{15 \times R_L}{8000 + R_L}$$

Hence $$\frac{30}{50000 + R_L} = \frac{15}{8000 + R_L}$$

or $2R_L + 16000 = R_L + 50000$

Hence $R_L = 50000 - 16000 = 34{,}000$ ohms.

Example 11. *The characteristics of a vacuum triode are given by*

$$i_b = 13 \times 10^{-6}[e_b + 16e_c]^{\frac{3}{2}}$$

This value is used in an amplifier with an anode load of 5 kilo-ohms. Find (a) the plate supply voltage to get the no-signal voltage E_b = 180 volts for a grid bias of -5 volts and (b) the voltage gain.

Solution. $$i_b = 13 \times 10^{-6}[e_b + 16e_c]^{\frac{3}{2}} \quad \ldots (1)$$

For $E_b = 180$ volts and $E_c = -5$ volts,

$$I_b = 13 \times 10^{-6}[180 - 80]^{\frac{3}{2}} \text{ amp.}$$
$$= 13 \times 10^{-6} \times 1000 \text{ amp.}$$
$$= 13 \text{ milliamp.}$$

Hence $E_{bb} = E_b + I_b.R_L = 180 + (13 \times 10^{-3}) \times 5 \times 10^3$ volts

$$= 180 + 65 = 245 \text{ volts}$$

Differentiating Eq. (1) with respect to e_c, we get

$$\frac{di_b}{de_c} = 13 \times 10^{-6} \times \tfrac{3}{2} \times \left(e_b + 16e_c\right)^{\frac{1}{2}} \times \left(\frac{de_b}{de_c} + 16\right)$$

At the point of operation $e_b = E_b = 180$ volts and

$e_c = E_c = -5$ volts, so that

$$\frac{di_b}{de_c} = 13 \times 10^{-6} \times 1{\cdot}5\left(180 - 16 \times 5\right)^{\frac{1}{2}}\left(\frac{de_b}{de_c} + 16\right)$$

But $\quad -di_b \times R_L = de_b$ and

$$\frac{de_b}{de_c} = \text{voltage gain A,}$$

and hence $-\frac{A}{R_L} = 19.5\times10^{-6}\times10\times(A+16)$

$R_L = 5000$ ohms.

so that $-A = 19.5\times10^{-6}\times10\times5000\,(A+16)$

$= 0.975\,(A+16)$

Hence $A = -\frac{16\times0.975}{1.975} = -7.9.$

Interelectrode capacitances in a triode. Fig. 11·25 shows the three interelectrode capacitances in a triode. C_{gk} denotes the capacitance between grid and cathode, C_{gp} is the capacitance between grid and plate, and C_{pk} is the capacitance between plate and cathode. These interelectrode capacitances are insignificant at low frequencies but they effect the operation of the tube at high frequencies. Although these capacitances are distributed throughout the length of the electrodes and exist inside the tube, for convenience, these are shown as lumped capacitances external to the tube and connected between the electrode terminals.

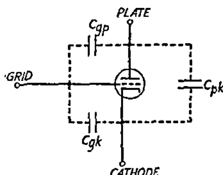
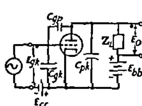
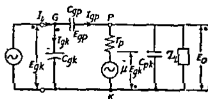


Fig. 11 25. Three inter electrode capacitances in a triode.

Fig. 11·26 (*a*) gives schematic diagram of a triode amplifier including the three inter-electrode capacitances. Fig. 11·26 (*b*) shows the a.c. equivalent circuit of this triode amplifier including the three inter electrode capacitances.





(*a*) Schematic triode amplifier with inter-electrode capacitances. (*b*) A.c. equivalent circuit of the triode amplifier.

Fig. 11·26. Schematic triode amplifier circuit and its a.c. equivalent circuit considering the three inter-electrode capacitances.

These inter-electrode capacitances have two effects on the performance of an amplifier using a triode : (*i*) modification of c plex voltage gain A and (*ii*) modification of input admitta These two effects are considered below.

Example 10. *An amplifier on using a valve V_1 having amplification factor of 30 and plate resistance of 50 kilo-ohms has the same gain as on using another valve V_2 having amplification factor of 15 and plate resistance of 8 kilo-ohms. Find the value of load resistance if such a condition is to be obtained.*

Solution. $$A = -\frac{30\times R_L}{50000+R_L}$$

Also $$A = -\frac{15\times R_L}{8000+R_L}$$

Hence $$\frac{30}{50000+R_L} = \frac{15}{8000+R_L}$$

or $2R_L+16000 = R_L+50000$

Hence $R_L = 50000-16000=34{,}000$ ohms.

Example 11. *The characteristics of a vacuum triode are given by*

$$i_b=13\times 10^{-6}[e_b+16e_c]^{\frac{3}{2}}$$

This value is used in an amplifier with an anode load of 5 kilo-ohms. Find (a) the plate supply voltage to get the no-signal voltage E_b = 180 volts for a grid bias of —5 volts and (b) the voltage gain.

Solution. $$i_b = 13\times 10^{-6}[e_b+16e_c]^{\frac{3}{2}} \quad \ldots (1)$$

For E_b = 180 volts and E_c = —5 volts,

$$I_b = 13\times 10^{-6}[180-80]^{\frac{3}{2}}\text{amp.}$$
$$= 13\times 10^{-6}\times 1000 \text{ amp.}$$
$$= 13 \text{ milliamp.}$$

Hence $E_{bb}=E_b+I_b.R_L = 180+(13\times 10^{-3})\times 5\times 10^3$ volts

$= 180+65 = 245$ volts

Differentiating Eq. (1) with respect to e_c, we get

$$\frac{di_b}{de_c} = 13\times 10^{-6}\times \tfrac{3}{2} \times \left(e_b+16e_c\right)^{\frac{1}{2}} \times\left(\frac{de_b}{de_c}+16\right)$$

At the point of operation $e_b=E_b=180$ volts and

$e_c=E_c=-5$ volts, so that

$$\frac{di_b}{de_c}= 13\times 10^{-6}\times 1{\cdot}5\left(180-16\times 5\right)^{\frac{1}{2}}\left(\frac{de_b}{de_c}+16\right)$$

But $-\,di_b\times R_L = de_b$ and

$$\frac{de_b}{de_c} = \text{voltage gain A,}$$

and hence $-\dfrac{A}{R_L} = 19{\cdot}5 \times 10^{-6} \times 10 \times (A+16)$

$$R_L = 5000 \text{ ohms.}$$

so that $\quad -A = 19{\cdot}5 \times 10^{-6} \times 10 \times 5000\ (A+16)$

$$= 0{\cdot}975\ (A+16)$$

Hence $\quad A = -\dfrac{16 \times 0{\cdot}975}{1{\cdot}975} = -7{\cdot}9.$

Interelectrode capacitances in a triode. Fig. 11·25 shows the three interelectrode capacitances in a triode. C_{gk} denotes the capacitance between grid and cathode, C_{gp} is the capacitance between grid and plate, and C_{pk} is the capacitance between plate and cathode. These interelectrode capacitances are insignificant at low frequencies but they effect the operation of the tube at high frequencies. Although these capacitances are distributed throughout the length of the electrodes and exist inside the tube, for convenience, these are shown as lumped capacitances external to the tube and connected between the electrode terminals.

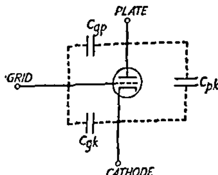
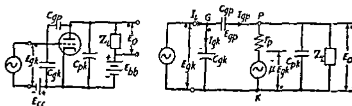


Fig 11 25 Three inter-electrode capacitances in a triode.

Fig. 11·26 (*a*) gives schematic diagram of a triode amplifier including the three inter-electrode capacitances. Fig. 11·26 (*b*) shows the a.c. equivalent circuit of this triode amplifier including the three inter electrode capacitances



(*a*) Schematic triode amplifier with inter-electrode capacitances. (*b*) A.c. equivalent circuit of the triode amplifier.

Fig. 11·26. Schematic triode amplifier circuit and its a c. equivalent circuit considering the three inter-electrode capacitances.

These inter-electrode capacitances have two effects on the performance of an amplifier using a triode : (*i*) modification of complex voltage gain A and (*ii*) modification of input admittance. These two effects are considered below.

Voltage gain of a triode amplifier considering the inter-electrode capacitances. With reference to the a.c. equivalent circuit of Fig. 11·26 (*b*), the voltage gain of the amplifier may be readily found by application of Millman Theorem between points P and K. Four shunt branches then existing are :

(*i*) load impedance with zero potential (*ii*) capacitance C_{pk} with zero potential (*iii*) potential $-\mu E_{gk}$ in series with r_p and (*iv*) potential E_{gk} in series with capacitance C_{gp}. Capacitance C_{gk} does not appear since it is across the source potential E_{gk}. Application of Millman Theorem then yields

$$E_o = \frac{-\mu E_{gk}.Y_p + E_{gk}.Y_{gp}}{Y_l + Y_{pk} + Y_p + Y_{gp}} \quad \ldots (11·46)$$

where $Y_l = \frac{1}{Z_l}$ is admittance of load impedance Z_l

$Y_{pk} = j\omega C_{pk}$ is admittance of C_{pk}

$Y_p = \frac{1}{r_p}$ is admittance of r_p

and $Y_{gp} = j\omega C_{gp}$ is admittance of C_{gp}.

Hence complex voltage gain is given by

$$A = \frac{E_o}{E_{gk}} = \frac{Y_{gp} - \mu.Y_p}{Y_l + Y_{pk} + Y_p + Y_{gp}} \quad \ldots (11·47)$$

or

$$A = \frac{j\omega C_{gp} - g_m}{\frac{1}{Z_l} + j\omega C_{pk} + \frac{1}{r_p} + j\omega C_{gp}} \quad \ldots (11·48)$$

It may be noted that if C_{gp} and C_{pk} are neglected, Eq. (11·48) reduces to Eq. (11·20) obtained by using conventional method and neglecting inter-electrode capacitances.

Assumptions made in the above analysis are : (*i*) no conduction or leakage currents exist between various tube terminals and (*ii*) interwiring and stray capacitances have to be neglected. In practice these may be added in parallel with C_{pk}, C_{gk} and C_{gp}.

Input admittance of a triode amplifier considering the inter-electrode capacitances. In the amplifier circuit of Fig. 11·26 (*a*), grid is kept sufficiently negative so that no conduction grid current flows but because of the presence of these inter-electrode capacitances displacement current flows through C_{gk}. Further, owing to the presence of C_{gp} grid is no longer isolated from the plate circuit. For any input a.c. signal voltage at G, amplified a.c. voltage appears at P and hence larger a.c. voltage difference appears across C_{gk}. Currents through C_{gp} and C_{gk} are supplied from the signal source. This results in input admittance of the amplifier. This admittance depends upon C_{gk}, C_{gp} and also the gain of the amplifier. Hereunder we derive an expression for the input admittance of the amplifier.

Let I_{gk} and I_{gp} be the effective values of the current through C_{gk} and C_{gp}. E_{gp} is grid-to-plate voltage.

Then,

$$\mathbf{I}_{gk} = \mathbf{E}_{gk}.\mathbf{Y}_{gk} \qquad \text{... (11·49)}$$

and

$$\mathbf{I}_{gp} = \mathbf{E}_{gp}.\mathbf{Y}_{gp} \qquad \text{... (11·50)}$$

But

$$\mathbf{E}_{gp} = \mathbf{E}_{gk} - \mathbf{E}_o = \mathbf{E}_{gk} - \mathbf{A}.\mathbf{E}_{gk} = \mathbf{E}_{gk}(1-\mathbf{A}) \qquad \text{... (11·51)}$$

where A is the complex voltage gain.

Total input current is,

$$\mathbf{I}_i = \mathbf{I}_{gk} + \mathbf{I}_{gp} = \mathbf{E}_{gk}[\mathbf{Y}_{gk} + \mathbf{Y}_{gp}(1-\mathbf{A})] \qquad \text{... (11·52)}$$

The input admittance is then given by

$$\mathbf{Y}_i = \frac{\mathbf{I}_i}{\mathbf{E}_{gk}} = \mathbf{Y}_{gk} + \mathbf{Y}_{gp}(1-\mathbf{A}) \qquad \text{... (11·53)}$$

Voltage gain A may be put as,

$$\mathbf{A} = A_r + jA_i \qquad \text{... (11·54)}$$

where A_r and A_i are the real and imaginary components of complex voltage gain A.

Hence $\mathbf{Y}_i = G_i + jB_i = j\omega C_{gk} + j\omega C_{gp}(1-A_r) + \omega C_{gp} A_i$ ... (11·55)

So that input shunt conductance $G_i = \omega C_{gp} A_i$ .. (11·56)

and input shunt susceptance $= \omega[C_{gk} + C_{gp}(1-A_r)]$ ... (11·57)

Input admittance as given by Eq. (11·53) or (11·55) has resulted [illegible]

[illegible]

Presence of large effective input capacitance is usually undesirable and in such cases because of this Miller effect, even a small [illegible] ...some special applica- [illegible] ...e input capacitance is [illegible] between plate and grid.

In accordance with Eq (11·55), a c. equivalent input circuit of an amplifier may be represented by a shunt resistance equal to reciprocal of G_i, in parallel with capacitance equal to $C_{gk} + C_{gp}(1-A_r)$ as shown in Fig. 11·27.

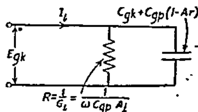


Fig. 11 27. a c. equivalent input circuit of a triode ampli

It may be noted that, in general, both A_r and A_i are functions of frequency and hence input shunt resistance and input shunt capacitance are both functions of frequency. The complex voltage gain A may be calculated for different types of loads for a given tube

constants and Y_i calculated therefrom. This is done for two types of load below :

Case I : *Pure Resistive Load* : The gain of amplifier may be calculated by application of Millman theorem and Eq. (11·46) may be used but in general, if the frequency is low, not much error is caused by neglecting the reactances of C_{gp} and C_{pk}. Voltage gain is then given by

$$A=-\mu.\frac{R_L}{r_p+R_L} \qquad \ldots (11·58)$$

where R_L is the load resistance.

From Eq. (11·58), $A_r=-\mu\frac{R_L}{r_p+R_L}$ and $A_i=0$ ... (11·59)

Hence

$$Y_i=j\omega\left[C_{gk}+C_{gp}\left(1+\frac{\mu R_L}{r_p+R_L}\right)\right] \qquad \ldots (11·60)$$

so that input shunt capacitance

$$C_i=C_{gk}+C_{gp}\left(1+\frac{\mu R_L}{r_p+R_L}\right) \qquad \ldots (11·61)$$

and input shunt conductance $G_i=0$. ... (11·62)

Case II : *Complex Load* : Load is a combination of a resistance and a reactance. If C_{gk} and C_{gp} cannot be neglected in calculation of voltage gain, the calculation of voltage gain becomes quite involved but Millman Theorem may be applied to get the value of voltage gain readily. In practice, however, reactance of C_{gp} may be neglected without causing much error. C_{pk} may be merged with the load impedance to give the overall effective load impedance Z which may be expressed in the form,

$$Z=R+jX \qquad \ldots (11·63)$$

Then the voltage gain

$$A=\frac{E_o}{E_{gk}}=-\mu\frac{Z}{r_p+Z}$$

$$=-\mu\frac{R+jX}{(r_p+R)+jX} \qquad \ldots (11·64)$$

or $$A_r+jA_i=-\mu.\frac{R+jX}{(r_p+R)+jX}$$

$$=-\mu\frac{R(r_p+R)+X^2}{(r_p+R)^2+X^2}-j\mu\frac{r_p.X}{(r_p+R)^2+X^2} \qquad \ldots (11·65)$$

Hence $$A_r=-\mu\frac{R(r_p+R)+X^2}{(r_p+R)^2+X^2} \qquad \ldots (11·66)$$

and $$A_i = -\mu \frac{r_p.X}{(r_p+R)^2+X^2} \qquad \text{... (11·67)}$$

Substituting the values of A_r and A_i from equations (11·66) and (11·67) into equations (11·56) and (11·57) respectively, we get

$$G_i = -\mu\omega C_{gp} \frac{r_p.X}{(r_p+R)^2+X^2} \qquad \text{... (11·68)}$$

and $$B_i = \omega\left[C_{gk}+C_{gp}\left(1+\mu\frac{R(r_p+R)+X^2}{(r_p+R)^2+X^2}\right)\right] \qquad \text{... (11·69)}$$

Complex load impedance Z including C_{pk} may be either inductive or capacitive so that when expressed in the form (11·63), the component X may be either positive or negative.

If the overall load impedance Z (including actual load impedance Z_L, C_{pk} and any other shunt impedance) is inductive, *i.e.*, X in Eq. (11·63) is positive, then input shunt conductance in accordance with Eq. (11·68) is negative. On the other hand, if overall load Z is capacitive *i.e.*, reactance X is negative, then input shunt conductance G_i is positive.

Thus for overall inductive plate load, input conductance is negative and hence input resistive is negative. Since a positive ... ower. ... er, at ... this means that power is fed back from the output plate circuit to the ... The voltage ... magnitude ... to get an ... he fact that

If, therefore, the amplifier is to amplify stably with little tendency of oscillation, the element C_{gp} which provides the feedback path or results in negative input resistance, must be eliminated as in tetrodes and pentodes. Alternatively the effect of C_{gp} must be balanced out ... the effect of ... used as an ... provided.

If the above discussion, effect of C_{gp} has been neglected in calculating the complex voltage gain A. When this effect of C_{gp} on voltage gain is considered, it is found that with inductive load the input conductance remains negative for a range of inductive reactance, but for values on either side, the input shunt conductance G_i becomes positive.

Inter-electrode capacitances in a tetrode :

In a tetrode, screen grid is inserted between the control grid (grido no. 1) and the plate. The screen grid is usuall

as to almost completely surround the plate and thereby reduce the electrostatic coupling between the control grid and plate. Time reduces the value of C_{gp} by the factors of the order of one thousand. Typical value of C_{g1p} (control grid-to-plate capacitance) in a tetrode or pentode is only 0·01 $\mu\mu F$. The screen grid mesh is, however, sufficiently coarse as to allow most of the electrons travelling from cathode to plate to pass through it.

Fig. 11·28 shows that the basic circuit of a single stage untuned amplifier using a tetrode. Screen grid g_2 is usually fed from the plate supply source E_{bb} through a dropping resistance R_d. The d.c. screen-grid voltage E_{c2} is equal to supply voltage minus the d.c. voltage drop in dropping resistance R_d. By-pass condenser C_1 is connected from screen grid to cathode or ground, in order to by-pass the a.c. components so that the screen grid is at a.c. ground potential. Screen grid current through R_d then has no a.c. component.

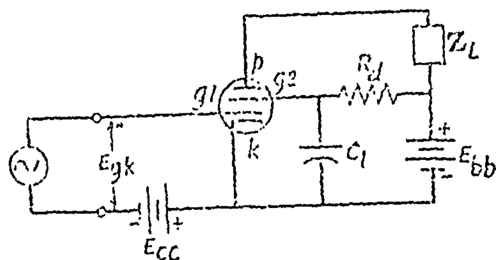


Fig. 11·28. Basic circuit of a single stage tetrode amplifier.

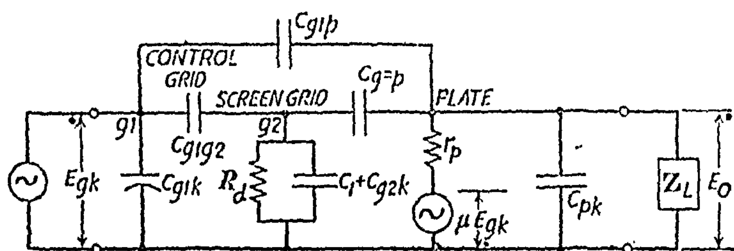
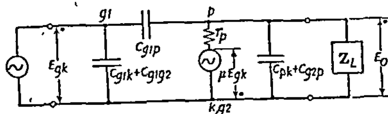


Fig. 11·29. A.c. equivalent circuit of the tetrode amplifier of Fig. 11·28 including the inter-electrode capacitances.

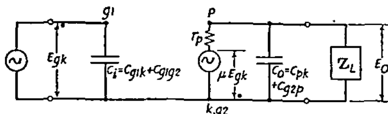
Fig. 11·29 shows the a.c. equivalent circuit of the tetrode amplifier of Fig. 11·28 including the six inter-electrode capacitances between the four electrodes of the tetrode.

In general, however, bypass condenser C_1 is so large that its reactance is negligibly small. This effectively puts an a.c. short circuit across C_{g2k}. This short circuit puts C_{g1g2} in parallel with C_{g1k} and C_{g2p} in parallel with C_{pk}. The a.c. equivalent circuit of Fig. 11·29 may then be redrawn as shown in Fig. 11·30 (*a*). This a.c. equivalent circuit of Fig. 11·30 is similar to that of the triode amplifier as drawn in Fig. 11·26 (*b*). But because of the shielding action of the screen grid, capacitance C_{g1p} becomes so small that it

may be neglected as shown in Fig. 11·30 (*b*). No feedback path from plate to control grid then exists. For successful elimination of feedback path, electrostatic screening should be provided between the control grid and plate outside the tube as well in order to avoid any electrostatic coupling between the various connecting wires in the plate and grid circuits.



(*a*) Modified a c equivalent circuit of a tetrode amplifier assuming screen grid at zero a.c potential



(*b*) Ideal a.c. equivalent circuit of a tetrode amplifier neglecting control grid-to-plate capacitance.

Fig. 11·30. A C. equivalent circuits of a tetrode amplifier.

The input admittance B_i of the tube from circuit of Fig. 11·30 (*b*) is,

$$B_i = j\omega C_i = j\omega[C_{g1k}+C_{g1g2}]. \qquad \text{... (11·71)}$$

Obviously input shunt conductance is zero and input shunt capacitance is given by,

$$C_i = C_{g1k}+C_{g1g2}. \qquad \text{... (11·71)}$$

Strictly speaking, to this value of input capacitance C_i as given by Eq. (11·71) we must add C_{g1p} $(1-A_v)$ to get the total ... tremely small, ... input capaci- ... ng the gain of ... d as a part of the load impedance Z_L.

In the above analysis, stray and wiring capacitances outside the tube have been neglected. These may increase the values of input shunt capacitance C_i and output shunt capacitance C_o. Further these may provide plate-to-control grid coupling unless care is taken to shield and widely separate the plate and grid circuits.

Inter-electrode Capacitances in a pentode.—Pentode when used in an amplifier has the advantage over tetrode of having no secondary emission effects at the plate. This is achieved by the presence of a suppressor grid placed between the screen grid and the plate and maintained at zero potential by being connected to the cathode. Five electrodes in a pentode result in ten inter-electrode capacitances. Out of these, C_{g3k}, *i.e.*, the capacitance between suppressor grid and cathode is absent since suppressor grid is directly connected to the cathode. The remaining nine are shown in the a.c. equivalent ircuit of the pentode amplifier in Fig. 11·31.

Fig. 11.31. A.C. equivalent circuit of pentode amplifier including all inter-electrode capacitances.

Since value of bypass condenser C_1 is usually very large, its reactance is almost zero and hence screen grid is at zero a.c. potential. Also value of control grid-to-plate capacitance C_{g1p} is negligibly small. Hence equivalent circuit of Fig. 11·31 reduces to one given in Fig. 11·32.

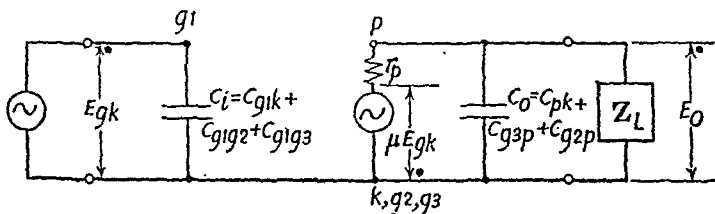


Fig. 11·32. Ideal a c. equivalent circuit of pentode amplifier.

From a.c. equivalent circuit of Fig. 11·32, the input admittance is,

$$Y_i = j\omega C_i = j\omega\,[C_{g1k} + E_{g1g2} + C_{g1g3}]. \qquad \ldots (11\cdot72)$$

Input conductance is zero and input shunt capacitance is given by,

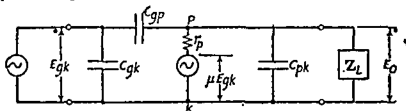
$$C_i = C_{g1k} + C_{g1g2} + C_{g1g3}. \qquad \ldots (11\cdot73)$$

Similarly effective output shunt capacitance is

$$C_o = C_{pk} + C_{g2p} + C_{g3p}. \qquad \ldots (11\cdot74)$$

Example 12. *Calculate the complex voltage gain of a triode untuned amplifier at an angular frequency of 10^6 radians per second given that : amplification factor of the tube is 20, plate resistance is 10,000 ohms, C_{gp} is $4\mu\mu F$, $C_{gk}=4\ \mu\mu F$, $C_{pk}=3\mu\mu F$ and load impedance is a resistance of 10,000 ohms.*

Solution. A.C. equivalent circuit of the amplifier at high frequencies is given below :



Applying Millman Theorem between points P and k, we get,

$$E_o = \frac{-\mu E_{gk}\dfrac{1}{r_p} + E_{gk} \cdot j\omega C_{gp}}{\dfrac{1}{Z_L}+\dfrac{1}{r_p}+j\omega C_{pk}+j\omega C_{gp}}\ .$$

Hence voltage gain

$$A = \frac{E_o}{E_{gk}} = \frac{-g_m+j\omega C_{gp}}{\dfrac{1}{Z_L}+\dfrac{1}{r_p}+j\omega(C_{pk}+C_{gp})}$$

$$= \frac{-2\times10^{-3}+j\times10^6\times4\times10^{-12}}{10^{-4}+10^{-4}+j\times10^6(4+4)\times10^{-12}}$$

$$= -\left[\frac{500+j}{50+2j}\right] = -[10-3\ 8$$

$$= \sqrt{(10)^2+(.38)^2}\ \underline{180^\circ-\tan^{-1}.038}$$

$$= 10{\cdot}007\ \underline{|177^\circ\ 48'}.$$

Ex[illegible] 13 [illegible] *stage amplifier uses a* [illegible] *plate resistance r_p of* [illegible] *uF. At the operating* [illegible] *$Z_L=20000+J\,20000$. The wiring and stray capacitances increase the effective values of the inter-electrode capacitances to twice the specified values. Calculate the magnitude and phase angle of the voltage gain.*

Solution. Effective values of inter-electrode capacitances are $C_{gk}=C_{gp}=4\mu\mu F$ and $C_{pk}=2\mu\mu F$. The gain of this amplifier by application of Millman Theorem becomes,

$$A = \frac{Y_{gp}-g_m}{Y_L+Y_{pk}+Y_p+Y_{gp}}$$

$$= \frac{j\omega C_{gp} - g_m}{\frac{1}{Z_L} + j\omega C_{pk} + \frac{1}{r_p} + j\omega C_{gp}}$$

Substituting the values,

$$A = \frac{-\frac{40}{20000} + j\,2\times10^6\times4\times10^{-12}}{\frac{1}{20000(1+j)} + \frac{1}{20000} + j\,2\times10^6\,(4+2)\times10^{-12}}$$

$$= -\,[25{\cdot}9 + 4{\cdot}383]$$

Magnitude of $A = \sqrt{(25{\cdot}9)^2 + (4{\cdot}383)^2} = 26{\cdot}27.$

Phase angle $\phi = 180 + \tan^{-1}\frac{4{\cdot}383}{25{\cdot}9}$

$$= 180^\circ + 9^\circ\,36' = 189^\circ\,36'.$$

Example 14. *A class A_1 amplifier uses a high μ triode having $\mu = 80$, $r_p = 50$ kilo-ohms, $C_{gp} = C_{gk} = 2\mu\mu F$. A signal of frequency $\frac{80{,}000}{2\pi}$ c/s is applied at the input. Calculate the input shunt conductance, input shunt capacitance and input shunt admittance of the amplifier if the load including C_{pk} in shunt is (a) resistance of 200 kilo-ohms and (b) a pure inductive reactance of 200 kiloohms and (c) complex inductive impedance of $10^5 + j10^5$ ohms.*

Solution Case (a). Voltage gain $A = \frac{-\mu.\,R_L}{r_p + R_L}$

$$= \frac{-80\times200\times10^3}{(50+200)\times10^3} = -64.$$

Input admittance is given by,

$$Y_i = j\omega\, C_{gk} + j\omega\, C_{gp}\,(1-A)$$

$$= j\times80\times10^3.\,[C_{gk} + C_{gp}\,(1+64)].$$

Obviously input conductance is zero and effective input shunt capacitance is given by,

$$C_i = C_{gk} + C_{gp}\,(1+64)$$

$$= 2 + 2\,(65) = 132\ \mu\mu F.$$

Effective input admittance $Y_i = \omega\,[C_{gk} + C_{gp}\,(1-A)]$

$$= 80000\times132\times10^{-12}\text{ mho.}$$

$$= 10{\cdot}56\times10^{-6}\text{ mho.}$$

Case (b). Load impedance $Z_l = 0 + j\,200000$ ohms.

Hence voltage gain $A = -\frac{\mu.Z_l}{r_p + Z_L} = -\frac{80\times(j\,200000)}{50000 + j\,200000}$

$$= \frac{-j\,320}{1+4j} = -\,[75{\cdot}3 + j\,18{\cdot}8].$$

Effective input admittance is given by,

$Y_i = j\omega C_{gk} + j\omega C_{gp}[1-A_r-jA_i]$

$= j\omega[C_{gk} + C_{gp}(1-A_r)] + \omega C_{gp} A_i.$

Hence effective input capacitance $C_i = C_{gk}+C_{gp}(1-A_r)$

$= [2+2(1+75.3)]\ \mu\mu F.$

$= 154.6\ \mu\mu F.$

Effective input conductance $G_i = \omega C_{gp}.A_i$

$= -(80\times10^3)\times2\times10^{-12}\times18.8$

$= -3\times10^{-6}$ mho.

Effective input admittance $Y_i = -(3\times10^{-6}) + j80\times10^3\times154.6\times10^{-12}$ mho

$= [-3+j12.37]10$

Hence magnitude of $Y_i = \sqrt{(3)^2+(12.37)^2}\times10^{-6}$ m

$= 12.7\times10^{-6}$ mho

Case (c). $Z_L = 10^5 + j10^5$.

Hence $A = \dfrac{-\mu Z_L}{r_p+Z_L} = \dfrac{-80(10^5+j10^5)}{(50\times10^3)+(10^5+j10^5)}$

$= -[61.5 + j12.3].$

Effective input admittance is

$Y_i = j\omega C_{gk} + j\omega C_{gp}(1-A_r-jA_i).$

Hence effective input capacitance $C_i = C_{gk} + C_{gp}(1-A_r)$

$= [2+2(1+61.5)]\ \mu\mu F$

$= 127\ \mu\mu F.$

Effective input conductance $G_i = \omega C_{gp} A_i$

$= 80\times10^3\times2\times10^{-12}\times(-12.3)$

$= -1.968\times10^{-6}$ mho.

Effective input admittance

$Y_i = -1.968\times10^{-6} + j80\times10^3(127\times10^{-12})$

$= (-1.968 + j10.16)\times10^{-6}.$

Hence magnitude of $Y_i = \sqrt{(1.968)^2 + (10.16^2)}\times10^{-6}$ mho.

$= 10.35\times10^{-6}$ mho.

...lifier uses
100 ohms,
impedance
Angular
...effective

input conductance, effective input susceptance and effective input admittance. Draw the equivalent input circuit. Assume wiring and stray capacitances to be negligible.

Solution. By application of Millman theorem to the a.c. equivalent circuit of the amplifier,

$$A=\frac{-g_m+Y_{gp}}{Y_l+Y_{pk}+Y_p+Y_{gp}}=\frac{-g_m+j\omega C_{gp}}{\frac{1}{Z_l}+j\omega C_{pk}+\frac{1}{r_p}+j\omega C_{gp}}.$$

Substituting the values,

$$A=\frac{-\frac{20}{8000}+j\,10^6\times4\times10^{-12}}{\frac{1}{20000}+\frac{1}{8000}+j\times10^6\,(4+10)\times10^{-12}}$$

$$=-14{\cdot}3+{\cdot}1166j.$$

Input admittance $Y_i=j\omega\,C_{gk}+j\omega\,C_{gp}\,[1-A_r-jA_i]$.

Input capacitance $C_i=C_{gk}+C_{gp}\,(1-A_r)$

$$=[4+4\,(1+14{\cdot}3)]\,\mu\mu F$$
$$=65{\cdot}2\,\mu\mu F.$$

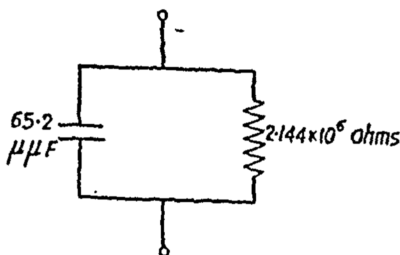
Input shunt conductance $G_i=\omega\,C_{gp}\,A_i$

$$=10^6\times4\times10^{-12}\times{\cdot}1166\text{ mho.}$$
$$={\cdot}4664\times10^{-6}\text{ mho.}$$

Input shunt resistance $=\frac{1}{G_i}=\frac{1}{{\cdot}4664\times10^{-6}}$ ohms.

$$=2{\cdot}144\times10^6\text{ ohms.}$$

Input equivalent circuit then is as below



Input shunt admittance $Y_i={\cdot}4664\times10^{-6}+j\times10^6\times55{\cdot}2\times10^{-12}$

$$=[{\cdot}4664+j\;65{\cdot}2]\times10^{-6}$$

Magnitude of input admittance $=\sqrt{({\cdot}04664)^2+(65{\cdot}2)^2}\times10^{-6}.$

$$\simeq 65{\cdot}2\times10^{-6}\text{ mho.}$$

Example 16 . . . . *nts is used in a linear* . . . $\mu\mu F$, $C_{gp}=4\mu\mu F$ *and* . . . $_{gk}$ *and shunt wiring and* . . . *e that wiring and stray capacitances increase the values of inter-electrode capacitances two-fold. If the frequency of input signal is* 50×10^3 *radians/sec., calculate (a) effective input capacitance (b) effective input conductance and (c) effective input admittance. Assume that the effect of* C_{gp} *on voltage gain is negligible.*

Solution. Neglecting the effect of C_{gp}, the voltage gain of the amplifier is given by,

$$A=\frac{-\mu Z}{r_p+Z}$$

where Z is the overall load impedance including shunt capacitances in the plate circuit.

Then $$A=\frac{-40\times(40{,}000+j\,10{,}000)}{(20{,}000+40{,}000)+j10{,}000}=-[27+2\cdot16j].$$

Effective $C_{gk}=2\times5=10\mu\mu F$

and $C_{gp}=2\times4=8\mu\mu F.$

Effective Input capacitance $C_i=C_{gk}+C_{gp}(1-A_r)$

$=10+8(1+27)=234\mu\mu F.$

Effective Input conductance $G_i=\omega C_{gp}A_i$

$=50\times10^3\times8\times10^{-12}(-2\cdot16)=-0\cdot864\times10^{-6}$ mho.

Effective input admittance $Y_i=G_i+j\omega C_i$

$=-0\cdot864+j50\times10^3$

$\times234\times10^{-12}$ mho.

$=(-0\cdot864+11\cdot7j)\times10^{-6}$ mho

Magnitude of input admittance $=\sqrt{(0\cdot864)^2+(11\cdot7)^2}\times10^{-6}$ mho

$=11\cdot73\times10^{-6}$ mho.

Method of biasing the control grid

There are three common methods of biasing negatively the control grid of an electron tube.

(A) Fixed Bias. A battery may be used for biasing the control grid as shown in Fig. 11·1. It is usually cumbersome, however, to install and maintain a constant voltage battery in the circuit. To overcome this trouble, control grid bias voltage may be obtained from the same d.c. voltage source as the plate supply voltage by use of appropriate potential divider as shown in the pentode amplifier circuit of Fig. 11·33.

input conductance, effective input susceptance and effective input admittance. Draw the equivalent input circuit. Assume wiring and stray capacitances to be negligible.

Solution. By application of Millman theorem to the a.c. equivalent circuit of the amplifier,

$$A=\frac{-g_m+Y_{gp}}{Y_l+Y_{pk}+Y_p+Y_{gp}}=\frac{-g_m+j\omega C_{gp}}{\frac{1}{Z_l}+j\omega C_{pk}+\frac{1}{r_p}+j\omega C_{gp}}.$$

Substituting the values,

$$A=\frac{-\frac{20}{8000}+j\,10^6\times4\times10^{-12}}{\frac{1}{20000}+\frac{1}{8000}+j\times10^6\,(4+10)\times10^{-12}}$$

$$=-14{\cdot}3+{\cdot}1166j.$$

Input admittance $Y_i=j\omega\,C_{gk}+j\omega\,C_{gp}\,[1-A_r-jA_i]$.

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$$=[4+4\,(1+14{\cdot}3)]\,\mu\mu F$$
$$=65{\cdot}2\,\mu\mu F.$$

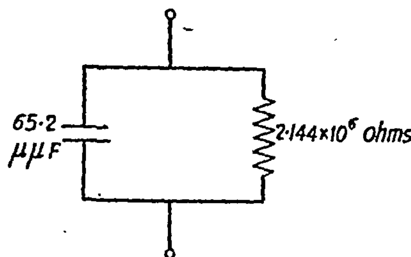
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Example 16. *A* [illegible] *used in a linear* A_1 *an* [illegible] $C_{gp}=4\mu\mu F$ *and* $C_{gk}=$ [illegible] *shunt wiring and stray* [illegible] *wiring and stray capacitances increase the values of inter-electrode capacitances two-fold. If the frequency of input signal is* 50×10^3 *radians/sec., calculate (a) effective input capacitance (b) effective input conductance and (c) effective input admittance. Assume that the effect of* C_{gp} *on voltage gain is negligible.*

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$$=(-0\cdot864+11\cdot7j)\times10^{-6}\text{ mho}$$

Magnitude of input admittance $=\sqrt{(0\cdot864)^2+(11\cdot7)^2}\times10^{-6}$ mho

$$=11\cdot73\times10^{-6}\text{ mho.}$$

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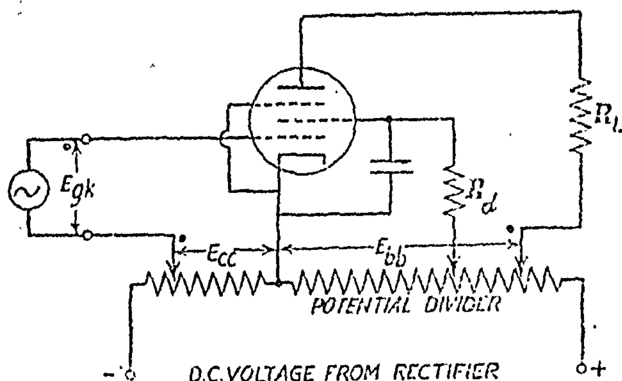


Fig. 11·33. Method of providing plate & screen grid supply voltages and control grid bias from a potential divider in the output of a rectifier.

(B) Cathode Bias or Self Bias.—This is a more common method of providing the grid bias. This consists in connecting a resistor in the cathode circuit as shown in Fig. 11·34. This has the advantage that no power supply is necessary for biasing purpose. Fig. 11·34 shows the basic circuit of a triode amplifier using this self bias.

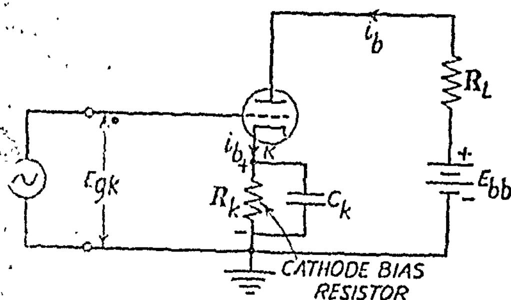


Fig. 11·34. Amplifier using self bias or Cathode bias.

The instantaneous total plate current i_b consists of two components: (*i*) the d.c. plate current I_b and (*ii*) the a.c. component of plate current i_p. This total plate current i_b on reaching the cathode terminal K, encounters a parallel combination of resistor R_k and large value condenser C_k. The d.c. component of plate current flows through the resistor R_k since it cannot flow through the condenser C_k. On the other hand, the a.c. component i_p of the total plate current takes the low impedance path through the high value condenser C_k. Almost the entire a.c. component of plate current flows through the condenser provided that the capacity of condenser C_k is so large that its reactance at the operating frequency is much smaller than resistance R_k. The d.c. plate current I_b flowing through R_k produces a d.c. voltage drop across it with positive polarity at the cathode end and negative at the lower end, *i.e.*, the grid end. This makes the grid

negative with respect to cathode by a d.c. voltage equal to $I_b.R_k$.

Condenser C_k is used simply to provide a low impedance shunt path to a.c. component of plate current. In the absence of this

R_k-C_k combination in the cathode lead provides a simple and economic method of obtaining the grid bias. High capacitance, low-voltage electrolytic capacitors are used for this purpose These are quite small physically. If the tube used is a pentode, the screen-grid current also produces a d c voltage drop, across R_k and increases the grid bias. Hence in selecting the value of R_k, for a required value of grid bias, both the d c plate current and the screen-grid current must be considered. The use of this self bias is restricted to class A and AB amplifiers.

(C) **Grid Leak Bias.** Sometimes grid-leak bias is used. The circuit consists of a condenser C in series with the grid and a grid-leak resistor R_g connected from grid to ground as shown in Fig. 11·35.

During part of the positive half cycle of the applied a c. grid signal, the total grid voltage e_c almost reaches zero potential or may even become positive, with the result that grid current may flow charging the series condenser in the polarity shown. During the remaining portion of the applied signal cycle the grid current does not flow with the result that the condenser C discharges through the grid leak resistor R_g. Since R_g is usually

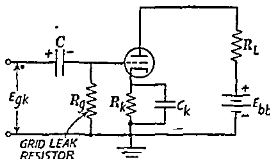


Fig. 11·35. Method of providing grid leak bias in an amplifier.

kept large, of the order of 250 kilo-ohm or higher, the time constant of discharge is usually very large so that after a few cycles of alternate charge and discharge, an almost constant d.c. voltage appears across the condenser in the polarity shown making grid negative with respect to the cathode. This provides the necessary grid leak bias. Usually grid leak bias is used in conjunction with fixed bias or self bias.

Zero-signal operation of an amplifier with cathode bias resistor.

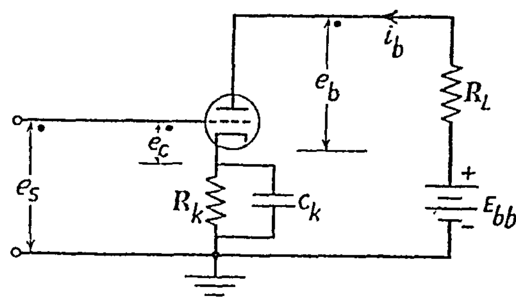
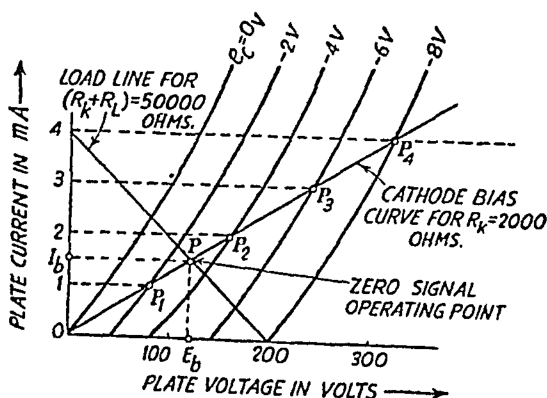


Fig. 11·36. (*a*) Amplifier Circuit.

When cathode bias is utilized, it is not possible to determine the zero-signal condition directly. Complication comes in because of the fact that the plate current and grid bias voltages are mutually dependent.

Total impedance in the plate circuit is (R_L+R_k) and hence plate current is given by the relation



(*b*) Cathode bias curve.

Fig. 11·36. Graphical determination of zero-signal operating point in a triode amplifier using cathode-bias resistor.

$$E_{bb}=i_b(R_L+R_k)+e_b \qquad \dots (11{\cdot}75)$$

or

$$i_b=\frac{E_{bb}-e_b}{(R_L+R_k)}. \qquad \dots (11{\cdot}76)$$

Equation (11·76) when plotted provides the load line shown in Fig. 11·36 (*b*). This load line is similar to that obtained with the fixed bias but with the difference that the slope of the load line in this case corresponds to (R_L+R_k). The zero signal operation point must lie somewhere on this line.

Another relation governing the plate current i_b is :

$$i_b=f(e_c,\ e_b) \qquad \dots (11{\cdot}77)$$

But for zero signal condition,

$$e_c=-i_b.R_L\ . \qquad \dots (11{\cdot}78)$$

We are required to find a curve satisfying Equations (11·77) and (11·78) simultaneously. There is only one point of each plate characteristic representing Eq. (11·77) which simultaneously satisfies the Eq. (11·78).

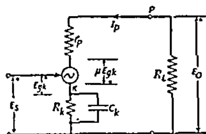
Let R_k=2,000 ohms. Then for i_b=1 mA, $e_c=-i_b\,R_L=-2$ volts corresponding to point P_1 in Fig. 11·36 (*b*). Similarly point P_2 satisfies Equations (11·77) and (11·78) simultaneously for i_b=2mA and so on. Points P_1, P_2, P_3 etc. are joined by a curve, which may be

t on this curve satisfies
This curve intersect
then the zero-signal
operating point.

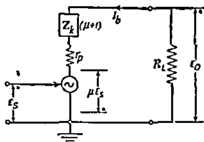
In a pentode, the situation is somewhat complicated by the pre-

screen-grid currents. Hence in Eq. (11 78) this total cathode current should be used instead of plate current i_b. For a given value of plate current i_b, screen-grid current i_{c2} may be found by remembering that in the region of plate characteristics beyond the knees, the values of plate current and screen-grid current are almost constant and bears a definite ratio depending upon the construction of the screen-grid. The ratio of screen grid current to total emission current for negative control-grid operation in the saturation region is a constant independent of grid bias and represents the fraction of the total emission current intercepted by the wires of the screen grid and hence dependent only on the fineness of the screen grid. Thus once we know this ratio, for any value of plate current in the saturation region, we may find the corresponding value of screen grid current. The cathode bias curve obtained for a pentode is almost horizontal. Point of intersection of this curve with load line for (R_L+R_k) yields the zero-signal conditions.

Voltage gain of an amplifier using cathode impedance It has been seen that R_k-C_k combination is generally used in the cathode

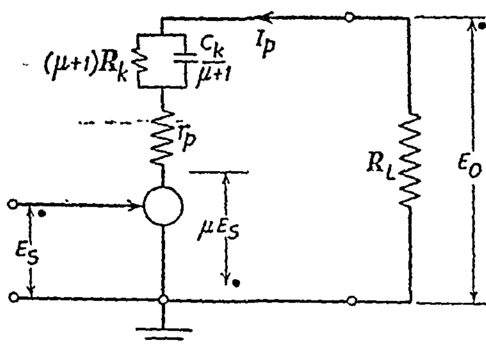


(*a*) Simple a c. equivalent circuit.



(*b*) Modified a c. equivalent circuit for a general cathode impedance Z_k.

lead to provide the self bias or cathode bias. Value of condenser C_k is chosen so high that it offers almost zero impedance at the lowest frequency of the signal to be amplified. In practice, however, often condenser C_k fails to offer negligible impedance at the lowest frequency contained in the signal voltage to be amplified. Hence this $R_k - C_k$ combination must be included in the a.c. equivalent circuit of the amplifier. Fig. 11·37 (*a*) shows the a.c. equivalent circuit of a linear A_1 voltage amplifier using resistance load R_L and cathode load impedance $\mathbf{Z}_k$ which is a parallel combination of resistance R_k and condenser C_k. Input is assumed to be sinusoidal so that effective values are shown in the equivalent circuit. This circuit of Fig. 11·37 (*a*) has been obtained by replacing the tube by voltage generator $\mu \mathbf{E}_{gk}$ and internal impedance r_p.



(*c*) A.C. equivalent circuit when Z_k is a parallel combination R_k and C_k.

Fig. 11·37. A.C. equivalent circuits for linear A_1 amplifier using cathode impedance.

Let $\mathbf{E}_s$ be the effective value of the sinusoidal voltage applied at the grid. Then, plate current is given by

$$\mathbf{I}_p = \frac{\mu \, \mathbf{E}_{gk}}{r_p + \mathbf{Z}_k + R_L}. \qquad \dots (11·79)$$

Here $\mathbf{Z}_k$ is the impedance of the parallel combination of R_k and C_k but it may, in general, be any complex impedance.

But $\qquad \mathbf{E}_{gk} = \mathbf{E}_s - \mathbf{I}_p . \mathbf{Z}_k. \qquad \dots (11·80)$

Substituting the value of $\mathbf{E}_{gk}$ from Eq. (11·80) into Eq. (11·79), we get :

$$\mathbf{I}_p = \frac{\mu[\mathbf{E}_s - \mathbf{I}_p . \mathbf{Z}_k]}{r_p + \mathbf{Z}_k + R_L}. \qquad \dots (11·81)$$

Rearranging Eq. (11·79), we get

$$\mathbf{I}_p = \frac{\mu \, \mathbf{E}_s}{r_p + (\mu+1) \, \mathbf{Z}_k + R_L}. \qquad \dots (11·82)$$

The presence of impedance Z_k in the cathode circuit results in a feedback into grid circuit which modifies the value of grid-to-cathode voltage E_{gk} as given by Eq. (11·80). The ultimate result of this feedback is to yield an expression for plate current as given

observed that the effect of an impedance in the cathode circuit is the same as that of a much larger impedance $(\mu+1)Z_k$ in the plate circuit.

If the cathode impedance Z_k is a parallel combination of a resistance R_k and a condenser C_k, the impedance $(\mu+1)Z_k$ in the plate circuit may be put as a parallel combination of a resistance $(\mu+1)R_k$ and a condenser $\frac{C_k}{\mu+1}$ in the plate circuit as shown in the a.c. equivalent circuit of Fig. 11·37 (c). This equivalence is true since the impedance of $(\mu+1)R_k$ and $\frac{C_k}{\mu+1}$ in parallel is $(\mu+1)Z_k$ where Z_k is the impedance of R_k and C_k in parallel.

From the a.c. equivalent circuit of Fig. 11·37 (b), output voltage E_o is given by

$$E_o=-I_p.R_L=\frac{-\mu\, E_s R_L}{r_p+R_L+(\mu+1)Z_k}. \quad \text{... (11·83)}$$

Hence complex voltage gain is given by

$$A=\frac{E_o}{E_s}=\frac{-\mu\, R_L}{r_p+R_L+(\mu+1)Z_k}. \quad \text{... (11·84)}$$

From Eq. (11·84), it is seen that the voltage gain obtainable using cathode-bias is smaller than the value $\frac{-\mu\, R_L}{r_p+R_L}$ obtainable with fixed bias. This occurs because the impedance Z_k in the cathode does not offer negligible impedance at the low frequencies and thus provides a negative feedback. At high frequencies Z_k is almost zero and no reduction in voltage gain is caused but as the frequency of applied signal is lowered or the value of capacitance C_k is reduced, the impedance Z_k increases and hence voltage gain correspondingly reduces.

Example 17. *A linear A_1 amplifier uses a triode with load resistance of 24,000 ohms and a cathode bias circuit consisting of a parallel combination of R_k and C_k. The value of R_k is so chosen that ... zero-signal plate current ... 10 volts. The amplification ... resistance is 16,000 ohms. Condenser C_k is 10 micro-farad. Calculate (i) value of R_k (ii) zero signal plate voltage and (iii) magnitude of voltage gain at frequencies of 2, 20 and 2000 c/s.*

Solution. (*i*) $-R_k.I_b$=grid bias.

Hence $-R_k(10\times10^{-3})=-10$ volts

or $$R_k=\frac{10}{10\times10^{-3}}=1000 \text{ ohms.}$$

(*ii*) Zero signal plate current is given by

$$E_b=E_{bb}-I_b(R_L+R_k)$$
$$=350-10\times10^{-3}(24{,}000+1{,}000)$$
$$=350-250=100 \text{ volts.}$$

(*iii*) Complex voltage gain is given by

$$A=\frac{-\mu R_L}{r_p+R_L+(\mu+1)Z_k}=\frac{-\mu\frac{R_L}{Z_k}}{\frac{r_p+R_L}{Z_k}+(\mu+1)}$$

Z_k is given by,

$$\frac{1}{Z_k}=\frac{1}{R_k}+j\omega C_k=\frac{1+j\omega C_kR_k}{R_k}.$$

At frequency of 2 c/s,

$$\frac{1}{Z_k}=\frac{1+j\,2\pi\times2\times10\times10^{-6}\times1000}{1000}$$
$$=10^{-3}[1+j\times\cdot1257].$$

Hence $$A=\frac{-30\times24{,}000\times10^{-3}(1+j\times\cdot1257)}{40{,}000\times10^{-3}(1+j\times\cdot1257)+30+1}$$
$$=-\frac{720(1+\cdot1257j)}{71+j5\cdot028}.$$

Magnitude $$A=\frac{720\sqrt{(1)^2+(\cdot1257)^2}}{\sqrt{(71)^2+(5\cdot028)^2}}=10\cdot82.$$

At frequency of 20 c/s,

$$\frac{1}{Z_k}=10^{-3}\ [1+1\cdot257j].$$

Hence $$A=\frac{-30\times24{,}000\times10^{-3}[1+j\times1\cdot257]}{40{,}000\times10^{-3}[1+j\times1\cdot257]+(30+1)}$$
$$=\frac{-720(1+1\cdot257j)}{71+50\cdot28j}.$$

Magnitude $$A=\frac{720\sqrt{(1)^2+(1\cdot257)^3}}{\sqrt{(71)^2+(50\cdot28)^2}}=13\cdot31.$$

At frequency of 2000 c/s,

$$\frac{1}{Z_k}=10^{-3}[1+j\times125\cdot7]$$

Hence
$$A=\frac{-30\times 24{,}000\times 10^{-3}[1+j\times 125\cdot 7]}{40{,}000\times 10^{-3}[1+j\times 125\cdot 7]+31}$$

$$\text{Magnitude } A=\frac{720\sqrt{(1)^2+(125\cdot 7)^2}}{\sqrt{(71)^2+(5028)^2}}=18.$$

Distortions in amplifiers. In an ideal amplifier, the reproduced output is an enlarged version of the input wave without any change in waveshape. But in all practical amplifiers, there always exists some difference in the waveshapes of the amplified output wave and applied input wave. Distortion is said to be present in such amplifiers. This distortion may be caused either by the amplifier tube itself or by the associated circuit or both. An important and common type of distortion is the so-called "nonlinear distortion" consisting in a monlinear relation between the instantaneous values of the input voltage and the amplified output current. Such a relation is caused generally by the nonlinearity of the tube characteristics.

It is more convenient, however, to express the response of an amplifier for a periodic input waveform. If the input waveform is periodic, it may be expressed in terms of its Fourier components of different amplitudes and phase angles as below :—

$$e_i=E_{m1}\sin(\omega t+\phi_1)+E_{m2}\sin(2\omega t+\phi_2)+ \quad \ldots (11\cdot 85)$$

where E_{m1}, E_{m2}, etc. are the amplitudes of fundamental, second etc. harmonic components, and ϕ_1, ϕ_2, etc. are the corresponding phase angles.

If the amplifier is ideal, amplitudes of all the Fourier terms get
it voltage gain A while the phase angles of
increased by a quantity proportional to their
an ideal amplifier, the amplified output wave-
l in the form,

$$\omega t+\psi+\phi_1)+AE_{m2}\sin(2\omega t+2\psi+\phi_2)$$
$$+AE_{m3}\sin(3\omega t+3\psi+\phi_3)+ \quad \ldots (11\cdot 86)$$

. Then Eq. (11 86) may be written as

$$(\omega t'+\phi_1)+AE_{m2}\sin(2\omega t'+\phi_2)$$
$$+AE_{m3}\sin(3\omega t'+\phi_3) \quad \ldots (11\cdot 87)$$

·87), we see that the amplified output e_o has
pe as the applied input e_i. Only differ-
ncreased magnitude and time delay of wave-
erval ψ/ω. Thus if the phase shift of different
ents is proportional to frequency, a time delay
waveshape distortion occurs.

se of the amplifier deviates from this ideal condi-
f different types may occur. The different types of
ay occur either separately or simultaneously are :

(*i*) amplitude distortion (*ii*) frequency distortion and (*iii*) phase distortion.

Amplitude distortion. In this case, the voltage gain A of the amplifier varies with the amplitude of the input wave, *i.e.*, the amplifier output waveform is nonlinearly related with the applied input waveform. Thus if the input amplitude is doubled, the output is not doubled. This, therefore, is a form of nonlinear distortion. With reference to the periodic input of the form given by Eq. (11·85), since the amplitudes E_{m1}, E_{m2} for different Fourier components are different, the voltage gain for these terms will be different. The output waveform will then differ from the input waveform. Next take the simple case of a sinusoidal input voltage. Let the output-input characteristic of the amplifier be as shown by the solid curve in Fig. 11·38. Then the amplified output waveshape

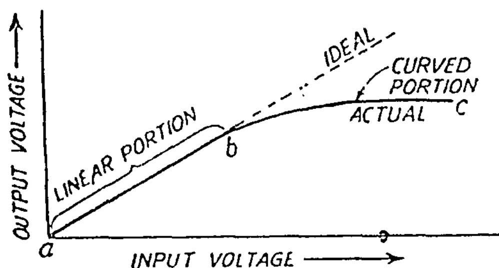


Fig. 11·38. Output-input characteristic of an ideal and practical amplifier.

differs from the input sinusoidal waveshape as shown in Fig. 11·39.

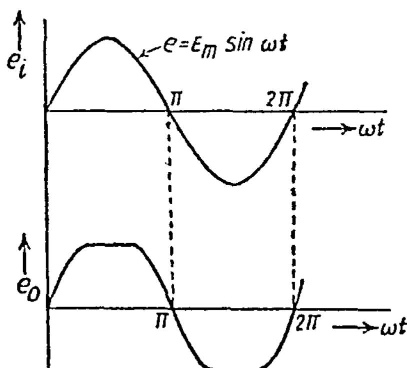


Fig. 11·39. Waveforms of input e_i and output e_o in an amplifier having amplitude distortion.

If the amplitude of the input signal lies between points a and b in Fig. 11·38, no nonlinear distortion takes place but if the amplitude lies beyond point b, nonlinear distortion occurs and then the output waveform differs from the input waveform as shown in Fig. 11·39. On analysis of waveform of output voltage e_o in Fig. 11·39, it will be found to be consisting of the fundamental component plus low amplitude harmonic components as well. Thus nonlinear distortion always results in generation of harmonics.

(*ii*) **Frequency distortion.** Frequency distortion is one in which the voltage gain varies with frequency of applied input voltage. Thus with reference to the input voltage as given by Eq. (11·85), even if the amplitudes E_{m1}, E_{m2} etc. of different frequency terms may be equal, the voltage gain for each term may be different with the result that reproduced waveform differs from the applied input waveform.

Frequency distortion is primarily caused by the presence of reactive elements in the amplifier circuit. In general, both amplitude and frequency distortions may be present simultaneously and may cause the output waveform to differ materially from the input waveform.

(*iii*) **Phase distortion.** It has been seen that in an ideal amplifier, the phase shift should be either zero or proportional to frequency. If an amplifier has phase shift not proportional to frequency, phase distortion results. Thus in a distortion free ampli-

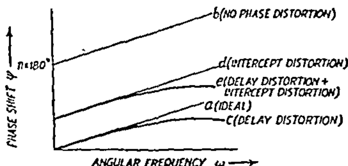


Fig. 11·40. Delay and intercept distortions in an amplifier.

fier the time delay introduced for all the frequency components has the same value of ψ/ω where ψ is the phase shift at frequency ω. The waveform is then not changed. This condition is, however, not essential for preserving the waveform. Thus if each frequency component is shifted in phase by an integral multiple of 180 degrees, the polarity of the overall waveform of the signal may be reserved but the waveform is not changed. Reversal of polarity

is usually not taken as a distortion. Thus with reference to Eq. (11·86), the phase when plotted against frequency is as shown by curve "*a*" of Fig. 11·40 for the case of an ideal amplifier. Curve "*b*" also exhibits no phase distortion since in addition to a phase shift proportional to frequency there exists only an additional phase shift equal to integral multiple of 180° for all frequency terms. Curve "*c*" indicates one type of phase distortion called the "delay distortion" in which case the slope $d\psi/dw$ of phase shift curve is not constant so that different frequency components are delayed unequally. Curve "*d*" exhibits the second type of phase distortion called the "intercept distortion" in which case the phase shift curve has an intercept on the phase-shift axis other than zero or an integral multiple of 180 degrees. Curve "*e*" exhibits both delay distortion and intercept distortion simultaneously.

In general intercept distortion is neglected and delay distortion alone is taken care of. Even delay distortion is not taken as a serious drawback because it is not perceptible to the ear. It is however, duly considered in all systems which use waveshapes for their operation such as Television or fascimile. Delay distortion, just like frequency distortion, results from the frequency characteristics of the circuit associated with the amplifier tube. In all cases where an almost ideal phase characteristic is desired, special phase equalising networks are used.

Determination of waveform distortion due to nonlinearty of amplifier tube characteristics.

Approximate graphical method for a triode. It has been seen that waveform distortion takes place when the tube characteristics are not linear, parallel and equidistant for equal increments of the parameter. We propose to establish methods of determining this waveform distortion. For the purpose of this analysis, we ignore the effects of inter-electrode and stray capacitances. The dynamic characteristic then enables us to determine the waveform a.c. plate current i_p for applied a.c. grid voltage e_{gk}. This dynamic transfer characteristic includes the combined effects of tube and load. For resistive load the load line on the plate characteristics is a straight line but the corresponding dynamic transfer characteristic is a slightly curved line. The amount of curvature of dynamic transfer characteristic then depends on the nonlinearity of tube characteristics. Situation is further complicated if the load is reactive or complex as when the output voltage across a resistive load is capacitively coupled to the next stage or when a transformer coupling is used. But even with a pure resistance load, the dynamic transfer characteristic is slightly non-linear and hence results in waveform distortion. The waveform distortion results in generation of harmonics. Hereunder we will discuss an approximate graphical of method of determining the percentage harmonic distortion in the case of a triode.

Let the nonlinear dynamic transfer characteristic be represented by an infinite power series and for the approximate analysis, only first two terms of the power series may be considered. Thus

$$i_p = a_1 e_{gk} + a_2 e_{gk}^2 \qquad \text{... (11·88)}$$

where a_1 and a_2 are constants depending on the tube characteristics and the load.

Let the applied signal be sinusoidal and expressed by the relation

$$e_{gk} = E_{gm} \cdot \cos \omega t. \qquad \text{... (11·89)}$$

Substituting the values of e_{gk} from Eq. (11·89) into Eq (11·88). we get

$$i_p = a_1 E_{gm} \cos \omega t + a_2 (E_{gm} \cos \omega t)^2 \qquad \text{... (11·90)}$$

$$= a_1 E_{gm} \cos \omega t + a_2 E_{gm}^2 . \tfrac{1}{2} (1 + \cos 2\omega t) \qquad \text{... (11·91)}$$

or $$i_p = \tfrac{1}{2} a_2 E_{gm}^2 + a_1 E_{gm} \cos \omega t + \tfrac{1}{2} a_2 E_{gm}^2 \cos 2 \omega t. \qquad \text{(11·92)}$$

In Eq. (11·92), the term $\frac{1}{2}a_2 E_{gm}^2$ is a constant term whereas $a_1 E_{gm} \cos \omega t$ is the fundamental frequency term and $\frac{1}{2}a_2 E_{gm} \cos 2\omega t$ is the second harmonic term. Such is the case when the first two terms in the power series are used, *i.e.*, when the dynamic transfer characteristic is expressed by the quadratic expression of Eq. (11·88). It is to be noted that the constant term and the second harmonic term have resulted entirely due to nonlinearity of dynamic transfer characteristic *i.e.*, due to the term $a_2 e_{gk}^2$ in Eq. (11·88). Further, it is seen that the constant term is equal to the amplitude of the second harmonic. Let I_{po}, I_{p1m} and I_{p2m} represent respectively the constant term, amplitude of fundamental and second harmonic plate current. Then Eq. (11·92) may be written as,

$$I_p = I_{po} + I_{p1m} \cos \omega t + I_{p2m} \cos 2\omega t. \qquad \text{... (11·93)}$$

This i_p is superimposed on the zero-signal plate current I_b so that the total instantaneous plate current i_b may be written as

$$i_b = I_b + I_{po} + I_{p1m} \cos \omega t + I_{p2m} \cos 2\omega t. \qquad \text{... (11·94)}$$

These four components of plate current i_b are plotted in Fig. 11·41. Corresponding plate characteristics with load line for resistive load is also indicated. From Eq. (11·94), as also from the curves in Fig. 11·41, it may be seen that i_b is maximum at $\omega t = 0$ and minimum at $\omega t = \pi$ radians, so that from Eq. (11·94) we may write

$$i_{b\,max} = I_b + I_{po} + I_{p1m} + I_{p2m} \qquad \text{... (11·95)}$$

and $$i_{b\,min} = I_b + I_{po} - I_{p1m} + I_{p2m}. \qquad \text{... (11·}$$

From Eqns. (11·95) and (11·96), we get

$$I_{p1m} = \frac{i_{b\,max} - i_{b\,min}}{2}. \quad \dots (11\cdot97)$$

Noting that $I_{po} = I_{p2m}$, we get

$$I_{p2m} = I_{po} = \tfrac{1}{2}\left[\frac{i_{b\,max} + i_{b\,min}}{2} - I_b\right]. \quad \dots (11\cdot98)$$

In order to find the values of I_{p1m} and I_{p2m}, it is only necessary to know $i_{b\,max}$, $i_{b\,min}$ and I_b. These three quantities may be found graphically from the plate characteristics by drawing the load line for the given value of plate supply voltage E_{bb} and R_L, and noting the values of plate current i_b for E_{cc}, $(E_{cc}+E_{gm})$ and $(E_{cc}-E_{gm})$ as indicated in Fig. 11·41.

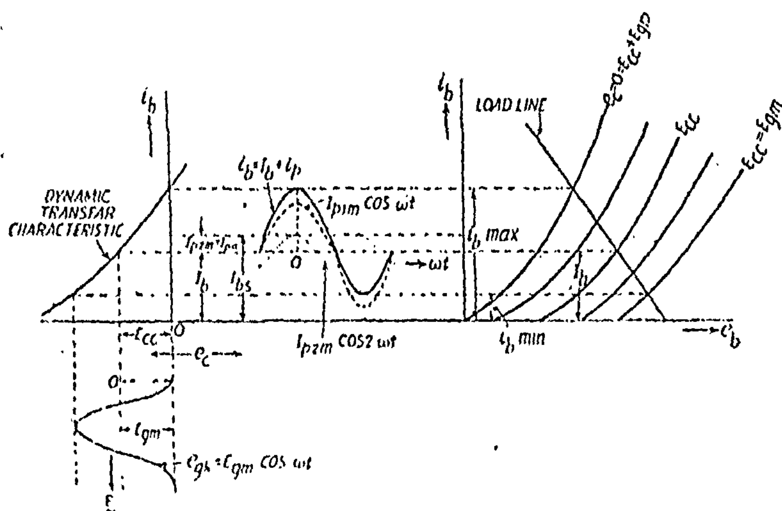


Fig. 11·41. Waveform distortion due to nonlinearity of triode characteristics.

For resistive load similar expressions may be obtained for plate voltage wave remembering that plate voltage e_b is related to the plate current i_b by the relation,

$$e_b = E_{bb} - i_b.R_L\,. \quad \dots (11\cdot99)$$

From Eqns. (11·97) and (11·98), percentage second harmonic for plate current i_b or plate voltage e_b is given by

Percent second harmonic

$$= \frac{I_{p2m}}{I_{p1m}} \times 100 = \frac{\frac{1}{2}(i_{b\,max} + i_{b\,min}) - I_b}{(i_{b\,max} - i_{b\,min})} \times 100. \quad \dots (11\cdot100)$$

It may be further noted from Eq. (11·94) that on application [illegible] to [illegible] gain [illegible] ent [illegible] be [illegible] ent I_{po} has resulted due to waveform assynetry or stated alternatively, due to partial rectification in the plate circuit, this component I_{po} is often referred to as "rectified component of plate current". "The average plate current with signal, symbolized by I_{bs} is then given by

$$I_{bs} = I_b + I_{po} \qquad \text{... (11·101)}$$

Fischer-Hinnen Method (or Selected Ordinates Method) of determining harmonic distortion. This is a more general method of harmonic analysis compared with the approximate method decribbed. This method may be applied equally well for triode, pentode *or any other tube and enable harmonics of any order to be deter*mined. Calculations, however, are quite involved if harmonics higher than third are to be evaluated.

For a periodic input voltage, each cycle of plate current is identical so that plate current may be represented by a Fourier series of the form

$$i_p = A_o + A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + \ldots$$
$$+ B_1 \sin \omega t + B_2 \sin 2\omega t + B_3 \sin 3\omega t + \ \ldots \qquad (11·102)$$

But if we chose the origin for ωt at a point where the plate current is maximum, then for a pure sinusoidal input and 'resistive load, the a c. plate current i_p is symmetrical about' the origin, so that the curve is an even function representable by a series containing only cosine terms as below :—

$$i_p = I_{po} + I_{p1m} \text{ cost } \omega t + I_{p2m} \cos 2\omega t + \ldots \ \ldots \qquad (11·103)$$

or $$i_b = I_b + i_p = I_b + I_{po} + I_{p1m} \cos \omega t + I_{p2m} \cos 2\omega t + \ \ldots \qquad (11·104)$$

$$= I_{bs} + I_{p1m} \cos \omega t + I_{p2m} \cos 2\omega t + \ldots \qquad \ldots \ (11·104a)$$

Let it be assumed that the plate current contains upto nth significant harmonics only. Then in accordance with this Fischer-Hinnen method, half-cycle period is divided into n equal intervals giving in all $(n+1)$ ordinates at the ends of these intervals. Then the values of these $(n+1)$ ordinates and $(n+1)$ corresponding values of ωt when substituted in Eq. (11·104a) yield $(n+1)$ simultaneous equation which may be solved to get the harmonic amplitudes.

In what follows, we use this Fischer-Hinnen method for detei mination of harmonics upto 3rd. Similar method may be followe for fourth or higher harmonics. Fig. 11·42 shows the pla characteristics for a pentode and also the load line. The input g

voltage e_{gk} is assumed to be sinusoidal and given by relation

$$e_{gk} = E_{gm} \cos \omega t \quad \dots (11.105)$$

The half cycle of e_{gk} is divided into three equal intervals of $\pi/3$ radians starting from zero and the corresponding angles at the ends of these intervals are 0, $\pi/3$, $\frac{2\pi}{3}$ and π radians, so that the corresponding values of e_{gk} from Eq. (11.105) are E_{gm}, $\frac{E_{gm}}{2}$, $-\frac{E_{gm}}{2}$ and $-E_{gm}$ respectively. The corresponding points on the load line are P_0, P_1, P_2 and P_π respectively. Let the corresponding values of total plate current be indicated by $i_{b\,max}$, i_1, i_2 and $i_{b\,min}$ respectively as shown in Fig. 11.42. Substituting these values of angles and corresponding ordinate in Eq. (11.104 *a*) we get

$$i_{b\,max} = I_{bs} + I_{p1m} + I_{p2m} + I_{p3m} \quad \dots (11.106)$$

$$i_1 = I_{bs} + \tfrac{1}{2} I_{p1m} - \tfrac{1}{2} I_{p2m} - I_{p3m} \quad \dots (11.107)$$

$$i_2 = I_{bs} - \tfrac{1}{2} I_{p1m} - \tfrac{1}{2} I_{p2m} + I_{p3m} \quad \dots (11.108)$$

and

$$i_{b\,min} = I_{bs} - I_{p1m} + I_{p2m} - I_{p3m} \quad \dots (11.109)$$

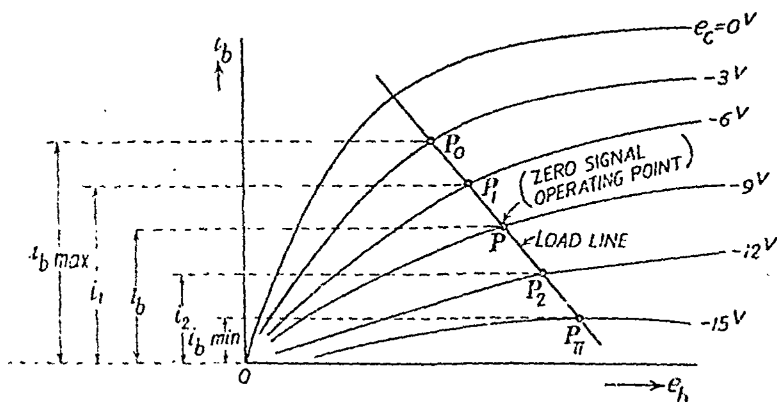


Fig. 11.42. Determination of harmonic generation in a pentode amplifier using Fischer-Hinnen Method.

Solution of Eqns. (11.106) to (11.109) yields,

$$I_{bs} = \frac{(i_{bmax} + i_{bmin}) + 2(i_1 + i_2)}{6} \quad \dots (11.110)$$

$$I_{p1m} = \frac{(i_{bmax} - i_{bmin}) + (i_1 - i_2)}{3} \quad \dots (11.111)$$

$$I_{p2m} = \frac{(i_{bmax} + i_{bmin}) - (i_1 + i_2)}{3} \quad \dots (11.112)$$

and

$$I_{p3m} = \frac{(i_{bmax} - i_{bmin}) - 2(i_1 - i_2)}{6} \quad \dots (11.113)$$

From these equations we get,

Per cent Second Harmonic

$$H_2=\frac{I_{p2m}}{I_{p1m}}\times 100=\frac{(i_{bmax}+i_{bmin})-(i_1+i_2)}{(i_{bmax}-i_{bmin})+(i_1-i_2)}\times 100 \quad \text{... (11·114)}$$

and Per cent Third Harmonic

$$H_3=\frac{I_{p3m}}{I_{p1m}}\times 100=\frac{(i_{bmax}-i_{bmin})-2(i_1-i_2)}{2[(i_{bmax}-i_{bmin})+(i_1-i_2)]}\times 100 \quad \text{... (11·115)}$$

Per cent total harmonic considering upto nth harmonics is then given by,

Per cent Total Harmonic

$$H=\sqrt{H_2^2+H_3^2+H_4^2+...+H_n^2} \quad \text{... (11·116)}$$

The amount of harmonic generation in an amplifier for sound reproduction is a function of the sensitivity of the ear to waveform distortion. The maximum percentage harmonic usually permitted for sound reproduction is ten per cent. For high quality reproduction the maximum harmonic generation is limited to only five per cent.

EXERCISES

1. In a vacuum triode amplifier using resistance load, as the grid bias is increased from -5 to -7 volts, the no-signal plate current decreases from 8 to 5 milliamperes and no-signal plate voltage increases from 100 to 160 volts. Calculate the load resistance and the plate supply voltage.

2. A vacuum triode amplifier with resistance load has no-signal plate current of 10 mA and no-signal plate voltage of 120 volts, for a grid bias of -5 volts. On changing the grid bias to -8 volts, values of no-signal plate current and plate voltage become 6 mA and 200 volts respectively. Calculate the plate supply voltage and load resistance. If now the plate supply voltage is required to be increased by 45 volts to bring the no-signal plate current back to original value of 10 mA, calculate the value of amplification factor of the tube.

3. The characteristics of a high vacuum triode are given by the expression

$$i_b=6\times 10^{-6}(e_b+20\ e_c)^{1.8}\ \text{amperes}$$

where e_b and e_c are the plate and grid voltages in volts. It is required to be operated with grid bias voltage of -6 volts and no-signal plate voltage of 180 volts. (a) Calculate the dynamic plate resistance of the tube at the point of operation. (b) If the tube is used with a load resistance of 8 kilo-ohms, calculate the plate supply voltage necessary to get the specified operating point.

4. Obtain a.c. equivalent circuits for a triode for use in a linear A_1 amplifier. State the assumptions made and limitations of such an equivalence.

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The half cycle of e_{gk} is divided into three equal intervals of $\pi/3$ radians starting from zero and the corresponding angles at the ends of these intervals are 0, $\pi/3$, $\frac{2\pi}{3}$ and π radians, so that the corresponding values of e_{gk} from Eq. (11·105) are E_{gm}, $\frac{E_{gm}}{2}$, $-\frac{E_{gm}}{2}$ and $-E_{gm}$ respectively. The corresponding points on the load line are P_0, P_1, P_2 and P_π respectively. Let the corresponding values of total plate current be indicated by $i_{b\ max}$, i_1, i_2 and $i_{b\ min}$ respectively as shown in Fig. 11·42. Substituting these values of angles and corresponding ordinate in Eq. (11·104 *a*) we get

$$i_{b\ max} = I_{bs} + I_{p1m} + I_{p2m} + I_{p3m} \qquad \dots (11\cdot106)$$

$$i_1 = I_{bs} + \tfrac{1}{2}I_{p1m} - \tfrac{1}{2}I_{p2m} - I_{p3m} \qquad \dots (11\cdot107)$$

$$i_2 = I_{bs} - \tfrac{1}{2}I_{p1m} - \tfrac{1}{2}I_{p2m} + I_{p3m} \qquad \dots (11\cdot108)$$

and

$$i_{b\ min} = I_{bs} - I_{p1m} + I_{p2m} - I_{p3m} \qquad \dots (11\cdot109)$$

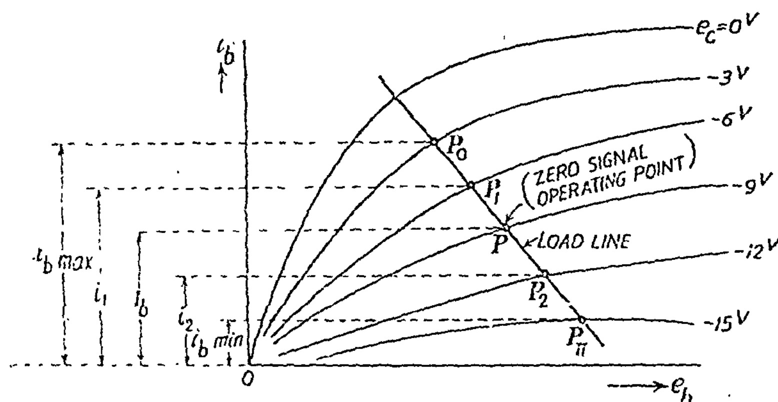


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$$I_{p1m} = \frac{(i_{bmax} - i_{bmin}) + (i_1 - i_2)}{3} \qquad \dots (11\cdot111)$$

$$I_{p2m} = \frac{(i_{bmax} + i_{bmin}) - (i_1 + i_2)}{3} \qquad \dots (11\cdot112)$$

and

$$I_{p3m} = \frac{(i_{bmax} - i_{bmin}) - 2(i_1 - i_2)}{6} \qquad \dots (11\cdot113)$$

From these equations we get,

Per cent Second Harmonic

$$H_2 = \frac{I_{p2m}}{I_{p1m}} \times 100 = \frac{(I_{bmax}+I_{bmin})-(I_1+I_2)}{(I_{bmax}-I_{bmin})+(I_1-I_2)} \times 100 \quad \ldots (11{\cdot}114)$$

and Per cent Third Harmonic

$$H_3 = \frac{I_{p3m}}{I_{p1m}} \times 100 = \frac{(I_{bmax}-I_{bmin})-2(I_1-I_2)}{2[(I_{bmax}-I_{bmin})+(I_1-I_2)]} \times 100 \quad \ldots (11{\cdot}115)$$

Per cent total harmonic considering upto nth harmonics is then given by,

Per cent Total Harmonic

$$H = \sqrt{H_2^2 + H_3^2 + H_4^2 + \ldots + H_n^2} \quad \ldots (11{\cdot}116)$$

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2. A vacuum triode amplifier with resistance load has no-signal plate current of 10 mA and no-signal plate voltage of 120 volts, for a grid bias of -5 volts. On changing the grid bias to -8 volts, values of no-signal plate current and plate voltage become 6 mA and 200 volts respectively. Calculate the plate supply voltage and load resistance. If now the plate supply voltage is required to be increased by 45 volts to bring the no-signal plate current back to original value of 10 mA, calculate the value of amplification factor of the tube.

3. The characteristics of a high vacuum triode are given by the expression

$$i_b = 6 \times 10^{-6}(e_b + 20 e_c)^{1{\cdot}5} \text{ amperes}$$

where e_b and e_c are the plate and grid voltages in volts. It is required to be operated with grid bias voltage of -6 volts and no-signal plate voltage of 180 volts. (a) Calculate the dynamic plate resistance of the tube at the point of operation. (b) If the tube is used with a load resistance of 8 kilo-ohms, calculate the plate supply voltage necessary to get the specified operating point.

4. Obtain a.c. equivalent circuits for a triode for use in a linear A_1 amplifier. State the assumptions made and limitations of such an equivalence.

5. Draw the voltage source a.c. equivalent circuit for a linear A_1 amplifier using a triode. Hence derive expressions for a.c. plate current, a.c. plate voltage and complex voltage gain. If the load impedance Z_L is a series combination of a resistance R and inductance L, derive expressions for the magnitude and phase angle of voltage gain.

6. A linear A_1 amplifier using triode has a voltage gain of -10 and -12 with load resistances of 8 and 12 kilo-ohms respectively. Calculate the amplification factor and a.c. plate resistance of the tube.

7. A linear class A_1 amplifier uses a triode which has amplification factor of 18 and dynamic plate resistance of 9000 ohms. Load impedance is a series combination of a resistance R of 5000 ohms and inductance of 1 henry. Calculate the magnitude and phase angle of the complex voltage gain at input signal angular frequencies of 5000 and 10000 radians/second.

8. A untuned triode voltage amplifier uses a resistance load of 50 kilo-ohm. The constants of the triode tube are : $\mu=60$, $r_p=30$ kilo-ohms, $C_{pk}=1{\cdot}0\mu\mu F$, $C_{gp}=C_{gk}=3\mu\mu F$. The input is a voltage $2\sqrt{2}\sin 10^6t$. Neglecting the stray and wiring capacitances, calculate the r.m.s. output voltage in magnitude and phase.

9. Triode used in a class A_1 voltage amplifier has $\mu=20$, $r_p=8000$ ohms, $C_{gp}=3\mu\mu F$, $C_{gk}=2\mu\mu F$ and $C_{pk}=2\mu\mu F$. The frequency of applied sinusoidal signal is 1 Mc/s. The load impedance is a series combination of a resistance of 10000 ohms and inductance of 0·1 henry. The wiring and stray capacitances increase the value of interelectrode capacitance to thrice their specified values. Calculate the magnitude and phase of the voltage gain.

10. The following data pertain to the triode used in a linear A_1 amplifier : $\mu=60$, $r_p=30{,}000$ ohms, $C_{gk}=C_{gp}=3\mu\mu F$, $C_{pk}=2\mu\mu F$. The stray and wiring capacitances increase the effective values of interelectrode capacitances three times. The load impedance including C_{pk} and stray and wiring capacitances is 20,000 ohms resistive. If the input signal frequency is 10,000 c/s, calculate (a) effective input shunt capacitance (b) effective input shunt conductance and (c) effective input shunt admittance. Neglect the effect of C_{gp} on complex voltage gain.

11. A single stage vacuum tube A_1 amplifier employs a triode having the following particulars : $\mu=20$, $r_p=10{,}000$ ohms, $C_{gk}=C_{gp}=8\mu\mu F$, $C_{pk}=10\mu\mu F$. The stray and wiring capacitances effectively double the interelectrode capacitances. The signal frequency is $\dfrac{100\times10^3}{2\pi}$ cycles/sec. The load impedance is 20000 ohms. Calculate the effective input shunt capacitance, conductance and admittance. Neglect the effect of C_{gp} and C_{pk} on the voltage gain. Repeat the calculations if an inductance of reactance 2000 ohms is added in series with the load resistance.

12. A linear A_1 amplifier uses a triode having amplification factor of 20 and plate resistance of 12000 ohms. Load resistance is 14000 ohms. There is cathode bias impedance consisting of parallel combination of resistance R_k=500 ohms and condenser C_k of 16 micro-farad. If the zero-signal bias developed across R_k is −10 volts and zero signal plate voltage is 120 volts, find the zero-signal [illegible]

[illegible]

[illegible]

of cathode impedance.

CHAPTER XII

UNTUNED A_1 CASCADE AMPLIFIERS

The voltage gain of a single stage amplifier is always less than the amplification factor of the tube used. Often, however, it is required to have voltage gains far greeding the amplification factor of any tube. In such cases, we use a number of amplifier stages in cascade, *i.e.*, a number of amplifiers so connected that the output of one is fed to the input of the next. By this means we may achieve any amount of amplification subject, of course, to the practical limitations. Such an amplifier is called a cascade amplifier and each constituent amplifier is termed as a "Stage" of this cascade amplifier may consist of two, three or more stages depending upon the overall gain requirement. Each stage of a cascade amplifier may use either a single tube or two tubes in pushpull or in parallel.

In a class A_1 amplifier, plate current flows for the entire cycle and hence plate efficiency is low. Further grid current does not flow and hence no power is fed to the grid circuit of any stage from the previous stage. Low plate efficiency is not as in low power level operation such as we come across in voltage amplifiers. Hence in a cascade A_1 amplifier, as the stages but the last, are voltage amplifiers. Waveform distortion is kept low in class A_1 operation. The design of the last stage depends upon the purpose of the amplifier. It may supply power with appreciable voltage and current as may be required to drive a loudspeaker. Alternatively the last stage or output stage may be used to supply voltage with little current such as is required to feed voltage to deflection plates of a cathode ray oscilloscope. This constitutes a high impedance load. Lastly output stage may supply current with little power such as may be used to feed current to an indicating device or a low-impedance cable.

The network used for coupling the output of one stage to the input of the next may assume a number of configurations and accordingly cascade amplifiers may be classified as :—

(A) Direct coupled or D.C. Amplifiers.

(B) Resistance-Capacitance coupled Amplifiers (abbreviated as R. C. coupled amplifiers).

(C) Inductance-Capacitance coupled Amplifiers ; and

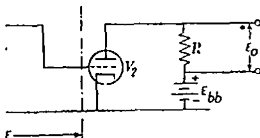
(D) Transformer Coupled Amplifiers.

(A) DIRECT COUPLED AMPLIFIERS

In a direct coupled amplifier coupling from one stage to the next is made either directly or through a battery so that there exists

a path from plate of one stage to the grid of the next stage. Hence this amplifier is also sometimes referred to as direct current amplifier. The abbreviation D. C. amplifier may thus stand for either direct-coupled amplifier or direct current amplifier without making any difference. D. C. amplifiers are useful whenever a d.c. voltage or time-average component of signal is required to be amplified through several stages. On the other hand, only a.c. component or time-varying component is amplified in other types of cascade amplifier.

Fig. 12·1 shows the basic circuit of a direct-coupled cascade amplifier. The circuit shows only two stages but any number of the same fashion. For the purpose of stage of cascade amplifier is sufficient. mplifier is usually taken from the grid of e to the grid of amplifier tube in the next circuit of Fig. 12·1 is done by a battery of end towards the grid of the second stage ed to the plate of the first stage. This is that the grid of the second tube does not ct to its cathode.



. of basic direct-coupled amplifier.

e incremental equivalent circuit for the s in the voltages and currents.

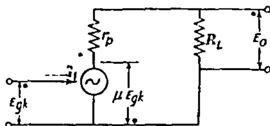


Fig. 12·2. Incremental equivalent circuit of d. c. amplifier of Fig. 12·1.

From the equivalent circuit of Fig. 12·2,

$$\text{a. c. output voltage } E_0 = -I_p.R_L = \frac{\mu E_{gk}}{r_p + R_L} \times R_L \quad \ldots (12\cdot1)$$

Hence voltage again $A = \dfrac{-\mu R_L}{r_p + R_L}$. ... (12·2)

If there are n stages in cascade, then overall voltage gain of the amplifier is given by,

$$A = A_1 \cdot A_2 \ldots\ldots\ldots\ldots A_n \qquad \ldots (12·3)$$

where A_1, A_2 etc., are the complex voltage gains of the n stages.

The d.c. amplifier of Fig. 12·1 has the disadvantage of requiring several batteries. Further in order to provide negative bias on the grid of any stage, corresponding coupling battery must have quite large voltage. Magnitude of its voltage is, in general, of the order of half the plate supply voltage. Installation and maintenance of these batteries is expensive and tedious. Further these batteries cause considerable stray capacitance to cathode. This stray capacitance adds to the input capacitance C_i and reduces the gain of the amplifier at high frequencies and deteriorates the transient response of the amplifier *i.e.*, the response of the amplifier to sudden changes in the input.

All the above mentioned troubles are obviated by using a single battery for different stages of amplifiers. One circuit, commonly used for this purpose, is shown in Fig. 12·3. It makes use of a potential divider to provide inter-stage coupling. Either two separate batteries may be used to provide plate supply voltages and grid bias voltages for different stages or alternatively a single battery with top at a suitable point on the battery may be used as shown in Fig. 12·3. In first amplifier stage a potential divider consisting of resistors R_1 and R'_1 is conencted from the plate to the negative terminal of bias voltage E_{cc}. By proper selection of

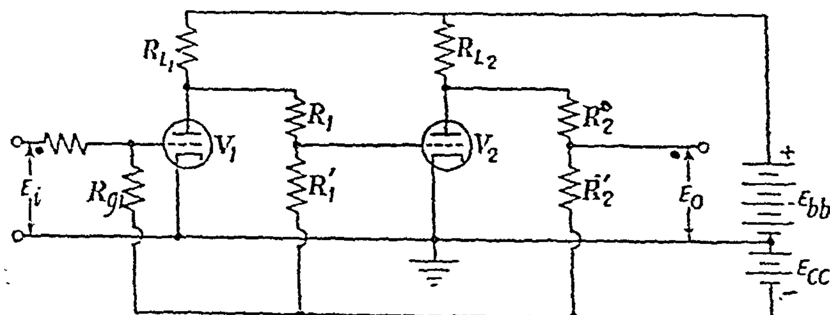


Fig. 12·3. D. C. amplifier using potential divider coupling.

the values of resistors R_1 and R_1', the grid of second tube may be given suitable bias with respect to its cathode. The main drawback of this potential divider coupling is that this potential-divider comes in shunt with the load resistance and thereby reduces the voltage amplification. Further because of this potential divider only a part (equal to R_1'/R_1) of the a.c. output voltage of the first stage is fed

to the input of the second stage and thus the effective voltage amplification is further reduced. To reduce loss of voltage amplification due to this potential divider, the ratio R_1/R'_1 must be small but in order to establish suitable negative bias on the grid of the tube in the second stage, the ratio R_1/R'_1 has to be quite large. Thus there is bound to be loss of voltage amplification due to the presence of this potential divider.

The main advantages of d.c. amplifiers are :—(i) Voltage gain is more or less constant over a wide range of frequencies and (ii) it responds to even d.c. voltages.

The main drawback of d.c. amplifier is its instability of operation caused basically by the direct coupling. This occurs because of slow variation of tube parameters and battery voltages with time. These variations occuring say in the first stage get amplified in subsequent stages so that even a small change in the [illegible] g point of the last stage to [illegible] n may result. This insta- [illegible] in the number of stages and hence if many stages are to be used, special means, such as the use of balanced circuits or special modulation methods, should be adopted. In practice it is usually not possible to use more than two or three stages of d.c. amplifier without excessive instability.

(B) Resistance Capacitance Coupled Amplifier

Resistance-capacitanced coupled amplifier, abbreviated as R. C. coupled amplifier, is the most commonly used cascade amplifier. It makes use of a resistance load and a condenser for coupling the output of one stage to the input of the next and hence the name resistance-capacitance coupled amplifier is given to it. Fig. 12·4 shows the circuit of two stages of R. C. coupled amplifier. R_L is the load resistance and C_c is the coupling condenser.

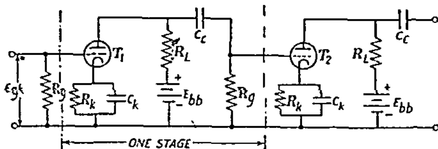


Fig. 12·4. Circuit diagram of two stage R. C. coupled triode amplifier.

[illegible] 'd of tube [illegible] Fig. 12·4. [illegible] significant

The coupling condenser C_c serves two purposes :—(*i*) to couple the a.c. output of one stage to the input of the next stage and to block the direct current passage from plate of one stage to grid of the next stage so that the grid of the latter stage does not become positive with respect to its cathode. The R. C. coupled amplifier thus differs from the D.C. amplifier in the fact that it can be used for amplification of a.c. voltages and currents only at the same time, since there is no direct coupling, the basic draw back of d.c. amplifier namely instability of operation, is eliminated The value of coupling condenser C_c is kept high so that its reactance even at the lowest operating frequency is much smaller than resistance of the grid leak resistor R_g: This will ensure that of the total output voltage developed across the load resistance a large fraction is available across the grid leak resistor R_g to fed to the input of the next stage.

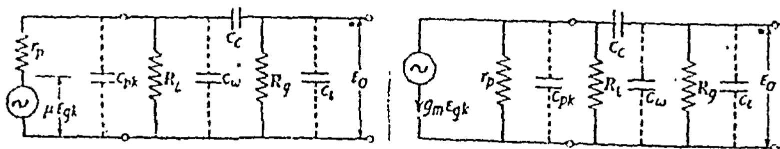
Parallel combination of resistor R_k and condenser C_k in the cathode circuit provides the self-bias. The bypass condenser to has a large capacity, of the order of 25 or 50 micro-farads so that it may effectively bypass a.c. component of plate current even at the lowest frequency of signal used.

The grid leak resistor R_g serves two purposes :—(*i*) it provides a d.c. path from cathode to grid to enable the cathode bias voltage to reach the grid (*ii*) in case grid current flows this resistor R_g in conjunction with the coupling condenser C_c provides the grid leak bias.

R. C. coupled amplifier is popularly used for the following reasons :—(*i*) it is stable in operation since in contradiction with d.c. amplifier it does not respond to slow variations of tube parameters and biasing voltage (*ii*) it may be designed and constructed for a good frequency response and (*iii*) it is comparatively inexpensive because both the load resistance R_L and coupling condenser C_c are simple and cheap elements.

The leakage resistance of coupling condenser C_c must be high so that no d.c. current flows from plate of one tube to the grid of the next tube. Hence electrolytic condensers can not be used for this purpose since they have large leakage current condenser with paper or mica as dielectric should be used for this purpose.

Analysis of R. C. Coupled Amplifier :—In order to find the voltage gain of R. C. coupled amplifier, we first draw the a.c.



(*a*) Voltage generator from. (*b*) Current generator from.
Fig. 12·5. A. C. equivalent circuit of one stage of R. C. coupled amplifier.

equivalent circuit of one stage of amplifier. Fig. 12·5 shows the a.c. equivalent circuit of one stage of R. C. coupled amplifier using equivalent voltage generator form in Fig. 12·5 (*a*) and current generator form in Fig. 12·5 (*b*).

A. C. equivvlent circuit of Fig. 12·5 shows in addition to the usual elements R_L, C_c and R_g, three additional small capacitances C_{pk}, C_w and C_i. C_{pk} represents plate-to-cathode capacitance of the amplifier tube. C_w represents the stray and wiring capacitance. C_i is the input capacitance of the next stage. Actually the input admittance of the second stage has conductance component in addi

assume the input conductance to be negligible and further the input capacitance may be assumed to be constant and independent of frequency. All these shunt capacitances may be grouped together into a single shu [illegible] This shunt capacitan [illegible] gain of the amplifier [illegible] at such high frequencies.

General expression for voltage gain of this amplifier may be obtained by using the a.c. equivalent circuit of Fig. 12·5. This general expression for voltage gain applicable for complete frequency range may be obtained by using one of the following methods :— (*i*) by using Millman Theorem and (*ii*) by using straight forward junction solution. These two general methods are described in the next article. In this article, however, we take up a method that yields expressions applicable to portions of audio frequency range. The entire audio frequency range is divided into three arbitrary frequency bands designated as low, middle and high frequency ranges. Three expressions for voltage gain applicable to these three frequency bands are derived hereunder. No attempt is made in this article to obtain a general expression for voltage applicable to all the three frequency ranges

(*I*) Middle Frequency Range :—Middle frequency range also called the mid-band, is so chosen that in this entire frequency range (*i*). frequency is so low that the reactance of shunting capacitance C_s is large compared with the resistance of parallel combination of r_p, R_L and R_g and hence may be neglected, (*ii*) frequency is high enough to make the reactance of coupling condenser C_c negligible

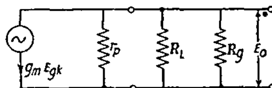


Fig. 12·6. A.C. equivalent circuit of R.C. coupled amplifier in the middle frequency range.

small compared with the resistance R_g of grid leak resistor. The incremental or a.c. equivalent circuit of the amplifier for the mid-band may then be drawn as in Fig. 12·6 using the equivalent current generator for the amplifier tu be.

Let R_{plg} represent the resistance of r_p, R_L and R_g in parallel, so that

$$\frac{1}{R_{plg}}=\frac{1}{r_p}+\frac{1}{R_L}+\frac{1}{R_g} \quad \dots (12·4)$$

Current gm. E_{gk} flowing through R_{plg} produces the output voltage E_o. Hence,

$$E_o=-g_m E_{gk} . R_{plg} \quad \dots (12·5)$$

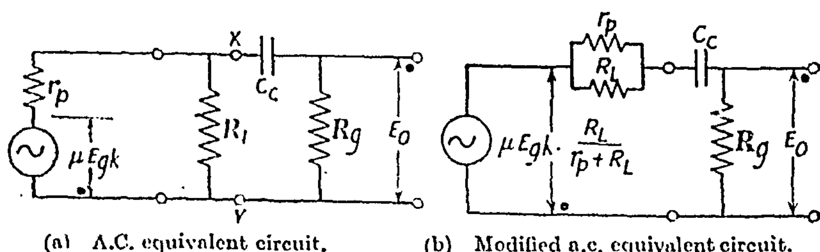
Hence complex voltage gain of the R.C. coupled amplifier in mid-band is given by,

$$A_m=\frac{E_o}{E_{gk}}=-\frac{g_m E_{gk} . R_{plg}}{E_{gk}}=-g_m . R_{plg} \quad \dots (12·6)$$

Magnitude $A_m=g_m . R_{plg}$ ...(12·6a)

and phase angle $\phi=180$ degrees ...(12·6b)

(II) Low Frequency Range :—In the low frequency range, *i.e.* at frequencies be low the middle frequency range, the frequency is so low that reactance of shunt capacitance C_s is much larger than resistance R_{plg} and hence may be neglected. The reactance of the coupling condenser C_c, is, however, sufficiently large and cannot be neglected in comparison with resistance R_g. The a.c. equivalent circuit of R.C. coupled amplifier in the low frequency range is then given in Fig. 12·7 (a). Amplifier tube is replaced by voltage generator since this yields results more easily.



(a) A.C. equivalent circuit. (b) Modified a.c. equivalent circuit.

Fig. 12·7. A.C. equivalent circuit of R.C. coupled amplifier in the low frequency range.

In order to simplify calculations, we apply Thevenin's Theorem to the portion to the left of points X and Y in Fig. 12·7 (a) to get equivalent voltage generator and its source impedance. Let E_{eq} be the open circuit voltage of the equivalent generator and let R_{eq} be the corresponding equivalent source impedance. Then in accordance with Thevenin's Theorem, open circuit voltage E_{eq} is that voltage

which we find across the terminals X-Y considering only the circuit to the left of points X-Y. Then

$$E_{sg}=\mu E_{gk} \cdot \frac{R_L}{R_L+r_p} \quad \ldots (12\cdot7)$$

The equivalent source impedance R_{sg} is the impedance found looking into the terminals X-Y after having replaced the energy sources by the source impedances. Then we get,

$$R_{sg}=\frac{r_p \cdot R_L}{r_p+R_L} \quad \ldots (12\cdot8)$$

i.e., R_{sg} is parallel combination of r_p and R_L.

The modified a.c. equivalent circuit so obtained is shown in Fig. 12 7 (*b*). Let R'_{sg} represent the total impedance in this modified equivalent circuit. Then,

$$R'_{sg}=R_{sg}+R_g=\frac{r_p \cdot R_L}{r_p+R_L}+R_g=\frac{r_pR_L+r_pR_g+R_L R_g}{(r_p+R_L)} \quad \ldots (12\cdot9)$$

Output a.c. voltage is then given by,

$$E_o=-\frac{E_{sg} R_g}{R'_{sg}-JX_c}=\frac{-\mu E_{gk}\frac{R_L}{r_p+R_L} R_g}{R'_{sg}-JX_c} \quad \ldots (12\cdot10)$$

where $\quad X_c=\text{reactance of } C_c=\frac{1}{\omega C_c} \quad \ldots (12\cdot11)$

Hence complex voltage gain in low frequency range is given by,

$$A_L=\frac{E_o}{E_{gk}}=-\mu \cdot \frac{R_L}{r_p+R_L} \times \frac{R_g}{R_{sg}'-JX_c} \quad \ldots (12\cdot12)$$

$$=-gm. \frac{r_p \cdot R_L}{r_p+R_L} \cdot \frac{R_g/R_{sg}'}{1-J\frac{X_c}{R_{sg}'}}$$

$$=-gm. \frac{r_p \cdot R_L \cdot R_g}{(r_p+R_L)} \times \frac{(r_p+R_L)}{(r_pR_L+r_pR_g+R_L R_g)} \times \frac{1}{1-J\frac{X_c}{R_{sg}'}}$$

$$=-gm. \frac{r_p \, R_L \cdot R_g}{(r_pR_L+r_pR_g+R_L R_g)} \cdot \frac{1}{1-J\frac{X_c}{R_{sg}'}}$$

or $$A_L=\frac{-gm. R_{plg}}{1-J\frac{X_c}{R_{sg}'}} \quad \ldots (12\cdot13)$$

or $$A_L=\frac{A_m}{1-J\frac{X_c}{R_{sg}'}} \quad \ldots (12\cdot14)$$

Magnitude $A_L = \dfrac{gm\ R_{plg}}{\sqrt{1+\left(\dfrac{X_c}{R_e{'}_q}\right)^2}} = \dfrac{A_m}{\sqrt{1+\left(\dfrac{X_c}{R_e{'}_q}\right)^2}}$ ... (12·14a)

and phase angle $\phi = 180° + \tan^{-1} \dfrac{X_c}{R_e{'}_q}$ .... (12·14b)

Let there be a frequency f_1 such that reactance X_c equal $R_e{'}_q$, then

$$f_1 = \frac{1}{2\pi R_e{'}_q C_c} \qquad \text{... (12·15)}$$

Then at frequency f_1, voltage gain $A_{f1} = \dfrac{A_m}{1-J}$

or $A_{f1} = \dfrac{A_m}{\sqrt{2}} = 0{\cdot}707\ A_m$. ... (12·16)

If we apply to the input of the amplifier, a voltage of fixed amplitude and variable frequency, then at this frequency f_1 the square of the output voltage is half of the corresponding value obtainable in middle frequency range. If this output voltage is fed to a fixed resistance load, the output power at this frequency f_1 will be half of that in the mid-band. This frequency f_1 is, therefore, termed as "half-power frequency" or "cross-over frequency" and since this frequency lies in at the boundary of low frequency range, it may be called the "lower half power frequency" or "lower cross-over frequency".

Substituting the value of $R_e{'}_q$ in terms of f_1, the expression for voltage gain in the low frequency range becomes,

$$A_L = \frac{A_m}{1-J\dfrac{f_1}{f}} \qquad \text{... (12·17)}$$

$$= \frac{-gm.\ R_{plg.}}{1-J\dfrac{f_1}{f}} \qquad \text{... (12·17a)}$$

Magnitude of A_L is given by,

$$A_L = \frac{A_m}{\sqrt{1+\left(\dfrac{f_1}{f}\right)^2}} = \frac{gm.\ R_{plg}}{\sqrt{1+\left(\dfrac{f_1}{f}\right)^2}} \qquad \text{... (12·17b)}$$

and phase angle of A_L is given by,

$$\phi = 180° + \tan^{-1} \frac{f_1}{f} \qquad \text{... (12·17c)}$$

(III) High Frequency Range :—In the high frequency range, *i.e.* at frequencies above the middle frequency range, the reactance of the coupling condenser C_c is very small compared with the resistance R_g and hence may be neglected. On the other hand reactance of total shunt capacitance C_s is sufficiently small and cannot be neglected in shunt with resistance R_{plg}. This reactance of shunt capacitance C_s will then cause the voltage gain to fall at high frequencies. Higher the frequency, larger will be the reduction in voltage gain. The a.c. equivalent circuit of R-C. coupled amplifier in the high frequency range then reduces to the form given Fig. 12·8 (*a*).

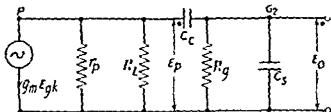


Fig. 12·8(*a*) A.C. equivalent circuit of R C coupled amplifier in the high frequency range

Resistances r_p, R_L and R_g may be combined to form R_{plg}. The a.c. equivalent circuit for high frequency range then gets modified to the form shown in Fig. 12·8 (*b*).

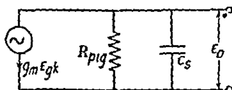


Fig. 12·8(*b*). Modified a.c equivalent circuit of R.C. coupled amplifier in high frequency range

Let Z represent the impedance of the parallel combination of R_{plg} and C_s. Then

$$Z = \frac{R_{plg}\times(-J\,X_s)}{R_{plg}-J\,X_s} = \frac{R_{plg}}{1+J\,\dfrac{R_{plg}}{X_s}} \qquad (12{\cdot}18)$$

where $\quad X_s =$ reactance of $C_s = \dfrac{1}{2\pi f\,C_s}$. .. (12·19)

$$\text{Output voltage } E_o = gm.\,E_{gk}\,Z = -\frac{gm\;E_{gk}\,R_{plg}}{1+J\,\dfrac{R_{plg}}{X_s}} \qquad \ldots (12{\cdot}20)$$

Hence complex voltage gain in high frequency range is given by,

$$A_h = \frac{-gm\,R_{plg}}{1+J\,\dfrac{R_{plg}}{X_s}} = -\frac{A_m}{1+J\,\dfrac{R_{plg}}{X_s}} \qquad \ldots (12{\cdot}21)$$

$$\text{Magnitude } A_h = \frac{A_m}{\sqrt{1+\left(\frac{R_{plg}}{X_s}\right)^2}} = \frac{gm.\ R_{plg}}{\sqrt{1+\left(\frac{R_{plg}}{X_s}\right)^2}} \quad \ldots (12{\cdot}21a)$$

and phase angle of A_h is given by

$$\phi = 180° - \tan^{-1} \frac{R_{plg}}{X_s}. \quad \ldots (12{\cdot}21b)$$

Again we may choose a frequency f_2 such that

$$R_{plg} = X_s \text{ so that } f_2 = \frac{1}{2\pi\ R_{plg}\ C_s}. \quad \ldots (12{\cdot}22)$$

At this frequency f_2 voltage gain is

$$A_{f_2} = \frac{A_m}{\sqrt{2}}. \quad \ldots (12{\cdot}23)$$

This frequency f_2 is called the "upper half power frequency" or "upper cross-over frequency".

Substituting the value of f_2 in the expression for voltage gain, we get,

$$A_h = \frac{A_m}{1+J\ \frac{f}{f_2}} \quad \ldots (12{\cdot}24)$$

$$= \frac{-gm\ .\ R_{plg}}{1+J\ (f/f_2)}. \quad \ldots (12{\cdot}24a)$$

Numerical value of A_h is given by

$$A_h = \frac{A_m}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}} = \frac{gm.\ R_{plg}}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}} \quad \ldots (12{\cdot}24b)$$

and phase angle of A_h is given by

$$\phi = 180° - \tan^{-1} \frac{f}{f_2}. \quad \ldots (12{\cdot}24c)$$

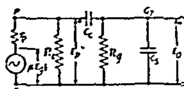
The "geometric mean frequency" or "mid-band frequency" denoted by f_0 is defined by the relation,

$$f_0 = \sqrt{f_1\, f_2}. \quad \ldots (12{\cdot}25)$$

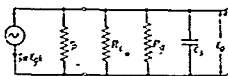
This frequency lies in the middle of the mid-frequency range and has maximum value of voltage gain.

General expression for voltage gain of R.C. coupled amplifier obtained by using Millman Theorem :—A.C. equivalent circuits of Fig. 12·5 are redrawn in Fig. 12·9 after grouping all shunt capacitances into a single shunt capacitance C_s placed across R_g. This

approximation of [illegible] R_g does not [illegible] respectively [illegible] G_2 represents grid of next tube.



(a) A.C. equivalent circuit using voltage source.



(b) A.C. equivalent circuit using current source.

Fig. 12·9. A.C. equivalent circuits of R.C. coupled amplifier.

Application of Millman Theorem between points G_2 and K in circuit of Fig. 12·9 (a) gives,

$$E_o = \frac{E_p \cdot Y_c}{Y_c + Y_g + Y_s} \quad \text{... (12·26)}$$

where Y_c = admittance of $C_c = J\omega C_c$

Y_g = admittance of $R_g = \dfrac{1}{R_g}$

Y_s = admittance of $C_s = J\omega C_s$.

Application of Millman Theorem between points P and K yields,

$$E_p = \frac{-\mu E_{gk} Y_p + E_o Y_c}{Y_p + Y_L + Y_c} \quad \text{... (12·27)}$$

where Y_p = admittance of $r_p = \dfrac{1}{r_p}$

and Y_L = admittance of $R_L = \dfrac{1}{R_L}$

Substituting the value of E_p from Eq. (12·27) into Eq. (12·26), we get,

$$E_o = \frac{\dfrac{-\mu E_{gk} Y_p + E_o Y_c}{Y_p + Y_L + Y_c} \cdot Y_c}{Y_c + Y_g + Y_s} \quad \text{... (12·28)}$$

Rearranging Eq. (12·28), we get

$$E_o = -E_{gk} \frac{\mu Y_p Y_c}{(Y_c + Y_g + Y_s)(Y_p + Y_L) - Y_c(Y_g + Y_s)} \quad \text{... (12·29)}$$

Hence complex voltage gain is given by,

$$A = \frac{E_o}{E_{gk}} = \frac{-\mu Y_p Y_c}{(Y_c + Y_g + Y_s)(Y_p + Y_L) - Y_c(Y_g + Y_s)} \quad \text{... (12·30)}$$

General Expression for voltage gain of R.C. coupled amplifier obtained by using junction solution :— With reference to the a.c. equivalent circuit of Fig. 12·9 (*b*), application of Kirchhoff current law to node *P* yields,

$$+gm\ E_{gk}+Y_p\ .\ E_p+Y_L\ .\ E_p+Y_c(E_p-E_o)=0 \quad \ldots (12·31)$$

or

$$(Y_p+Y_L+Y_c)E_p-Y_c\ .\ E_o=-gm\ .\ E_{gk}. \quad \ldots (12·32)$$

Applying Kirchhoff current law to node G_2, we get,

$$E_o\ (Y_g+Y_s)-Y_c(E_p-E_o)=0 \quad \ldots (12·33)$$

or

$$E_o\ (Y_g+Y_s+Y_c)=Y_c\ .\ E_p$$

or

$$E_p=\frac{E_o\ (Y_g+Y_s+Y_c)}{Y_c}. \quad \ldots (12·34)$$

Substituting the value of E_p from Eq. (12·34) into Eq. (12·32), we get,

$$E_o\left[\frac{(Y_g+Y_s+Y_c)(Y_p+Y_L+Y_c)}{Y_c}-Y_c\right]=-gm.\ E_{gk}. \quad \ldots (12·35)$$

Hence voltage gain is given by,

$$A=\frac{E_o}{E_{gk}}=\frac{-\mu\ Y_p\ Y_c}{(Y_g+Y_s+Y_c)(Y_p+Y_L+Y_c)-Y_c^2} \quad \ldots (12·36)$$

or

$$A=\frac{-\mu\ Y_p\ Y_c}{(Y_g+Y_s+Y_c)(Y_p+Y_L)+Y_c(Y_g+Y_s)} \quad \ldots (12·37)$$

Equation (12·37) is the same Eq. (12·30) obtained by using man Theorem.

Frequency response of R.C. coupled amplifier :— Fig. 12·10 gives typical frequency response curve of one stage of R.C. coupled amplifier. Magnitude of voltage gain is plotted against frequeney of signal. Since a large frequency range is involved it is convenient to use a logarithmic scale for frequency. The voltage gain is plotted along the vertical axis either on a linear scale or a logarithmic scale. Sometimes instead of using a logarithmic scale for voltage gain, the

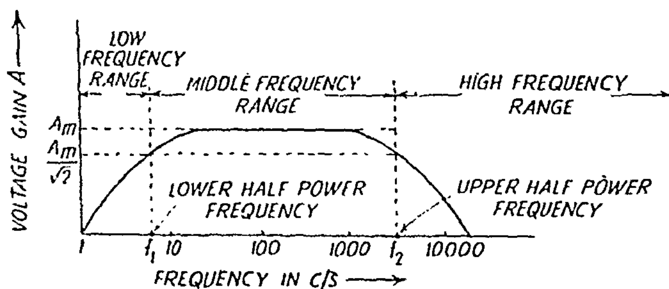


Fig. 12·10. Frequency response curve of R.C. coupled Amplifier.

same result is achieved by using a linear scale, but plotting voltage gain in decibels. Voltage gain expressed in decibels is given by,

$$A_{db}=20\ \log_{10}\frac{E_o}{E_i} = 20\ \log_{10} A \quad \text{... (12 38)}$$

In Fig. 12·10 linear scale is used for voltage gain.

Sometimes it is preferred to plot the relative voltage A/A_m in decibels along the vertical axis. This is shown in Fig. 12·11. The half power frequencies are then obtained when relative voltage gain is $-3db$, *i.e.* actual voltage gain is $3db$ below the mid-band gain A_m.

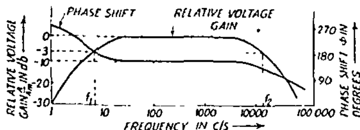


Fig. 12·11. Relative voltage gain curve and phase characteristic of R C. coupled amplifier.

The voltage gain is almost constant in the middle frequency range. In the low frequency range the voltage gain progressively falls as the frequency is lowered because of the presence of coupling condenser C_c. Gain may be considered to be almost constant upto lower cross-over frequency f_1, *i.e.* mid-band may be considered to be extending on the low frequency side upto f_1 and cross-over from mid-band to low frequency range takes place at this cross-over frequency and hence the name cross-over frequency is given to it. It is desired that the mid-band be as large as possible, *i.e.* f_1 be as small as possible and f_2 be as large as possible From E_q (12 15) we see that if f_1 is to be reduced either $R'_e\left(=R_e+\frac{r_p R_L}{r_p+R_L}\right)$ or cou- ... a given tube R'_e may be ... But R_e cannot be increased ... to reduce f_1, the coupling condenser C_c must be increased. In practice, however, an upper limit for C_c is reached, since for very large value of capacity C_c, capacity to earth of this coupling condenser increases and high frequency response suffers accordingly.

In the high frequency range, the valtage gain falls because of the presence of the shunt capacitance C_s. If the mid-band is to be increased, as is required in broad-band amplifiers, f_2 must be increased. In accordance with E_q (12 22), f_2 may be increased by reducing either

may be almost in phase with the input. In such a case, even a slight amount of feedback from last stage to first stage may cause unstable operation of the amplifier. To avoid such a possibility various stages of cascade amplifier must be properly arranged preferably in a straight line and decoupling filters must be used in all plate circuits if a common B supply is used.

Gain-bandwidth product of R. C. coupled amplifier :— In broad-band amplifiers, the general effort is to have a large frequency band between the upper and lower half power frequencies. This bandwidth (f_2-f_1) is referred to as "half power bandwidth" or "$3-db$ bandwidth." In general f_1 is very small so that f_2 alone represents the half-power bandwidth. Let this bandwidth be indicated by B. Then in accordance with Eq. (12·22) B is given by approximate relation,

$$B=\frac{1}{2\pi\, R_{plg}C_s} \qquad \text{... (12·46)}$$

An attempt to increase bandwidth B by reducing R_{plg} reduces the mid-band gain. Hence the product of mid-band voltage gain $A_m \times$ Bandwidth B may be considered to be an index of the quality of the amplifier.

Mid-band gain is given by,

$$A_m=g_m \,.\, R_{plg}, \qquad \text{(12·47)}$$

Hence gain bandwidth product

$$f_0=A_m \,.\, B=\frac{g_m}{2\pi\, C_s}=\frac{g_m}{2\pi(C_{pk}+C_w+C_i)}. \qquad \text{... (12·48)}$$

Eq. (12·48) shows that the gain-bandwidth product of the R. C. coupled amplifier is a constant, independent of circuit parameters R_L, R_g and C_c. Thus for a given tube and circuit configuration, if the bandwidth is increased, the gain is proportionately reduced. To have a large gain-bandwidth product, tubes having large value of g_m and low inter electrode capacitances should be used. C_w may be reduced by suitable layout of the components. But inspite of all care it cannot be reduced below a minimum value. However, let us assume for the purpose of assessing the quality of the tube alone, that the stray and wiring capacitance C_w has been reduced to zero. Then the minimum shunt capacitance is the sum of the output capacitance C_{pk} of the amplifier tube and input shunt capacitance C_i of the next stage. Let us call this sum C_t. Then we may say that the gain-bandwidth product under this condition is given by,

$$f_t=\frac{g_m}{2\pi\,(C_{pk}+C_i)}=\frac{g_m}{2\pi\, C_t} \qquad \text{... (12·49)}$$

Then since g_m and C_t are both functions of the tube, the gain-bandwidth product f_t is the "figure of merit" of the tube alone. Special pentodes have been designed which give high value of g_m

and low inter electrode capacitances. 6AK5 has a figure of merit of roughly 117 MC/S which value is reduced due to socket and wiring capacitance to a value of 55 MC/S i.e., $f_0=55$ MC/S. The corresponding value of f_0 for 6AC7 is 50MC/S. Thus using say 6AC7, under ideal practical conditions if the bandwidth is as high as 5 MC/S, the gain obtainable per stage is still ten.

Effect of cascading on the bandwidth :—Let us assume for simplicity that all the stages are identical. Let n be the number of such stages. Obviously then the overall mid-band gain is given by,

$$A_n=(A_m)^n \quad \ldots (12\cdot50)$$

where A_m is the gain of each stage in the middle frequency range.

If there is no reduction in half-power band width, then the effect of cascading will be to increase the overall gain-bandwidth product in the same ratio as the gain itself. But such is not the case since the bandwidth reduces as the number of stages in cascade increase. Hereunder we will find expression for half power frequencies f_{1n} and f_{2n}, when n stages have been cascaded.

In the low frequency range, the relative gain using n stages in cascade is given by,

$$\left(\frac{A_L}{A_m}\right)^n=\frac{1}{\left[1+\left(\frac{f_1}{f}\right)^2\right]^{\frac{n}{2}}} \quad \ldots (12\cdot51)$$

Let f_{1n} indicate the lower half power frequency of the cascade amplifier having n stages. At this frequency

$$\left[\frac{A_L}{A_m}\right]^n=\frac{1}{\sqrt{2}}, \text{ so that}$$

$$\frac{1}{\sqrt{2}}=\frac{1}{\left[1+\left(\frac{f_1}{f_{1n}}\right)^2\right]^{\frac{n}{2}}} \quad \ldots (12\cdot52)$$

or

$$1+\left(\frac{f_1}{f_{1n}}\right)^2=2^{\frac{1}{n}}$$

or

$$\frac{f_1}{f_{1n}}=\sqrt{2^{\frac{1}{n}}-1} \quad \ldots (12\cdot53)$$

In the high frequency range, the relative voltage gain of n-stage cascade amplifier is given by,

$$\left(\frac{A_h}{A_m}\right)^n=\frac{1}{\left[1+\left(\frac{f}{f_2}\right)^2\right]^{\frac{n}{2}}} \quad \ldots (12\cdot54)$$

Solution. $$\frac{1}{R_{plg}}=\frac{1}{r_p}+\frac{1}{R_L}+\frac{1}{R_g}$$

$$=\frac{1}{10^4}+\frac{1}{2\times10^4}+\frac{1}{100\times10^4}$$

$$=(1+0{\cdot}5+{\cdot}01)\times10^{-4}$$

$$=1{\cdot}51\times10^{-4}\text{ mho.}$$

Hence $$R_{plg}=\frac{10^4}{1{\cdot}51}\text{ ohms.}$$

Mid-band gain is then given by,

$$A_m=-g_m.R_{plg}=-\frac{15}{10^4}\times\frac{10^4}{1{\cdot}51}=-9{\cdot}98.$$

Lower half power frequency f_1 is given by,

$$f_1=\frac{1}{2\pi\,C_c\,R'_{eq}}$$

where $$R'_{eq}=R_g+\frac{r_p\,.\,R_L}{r_p+R_L}$$

$$=10^6+\frac{10^4\times2\times10^4}{(1+2)\times10^3}\text{ ohms.}$$

$$=100{\cdot}66\times10^4\text{ ohms.}$$

Hence $$f_1=\frac{1}{2\pi\times0{\cdot}01\times10^{-6}\times100{\cdot}66\times10^4}$$

$$=15{\cdot}82\text{ cycles/sec.}$$

Upper half power frequency is given by,

$$f_2=\frac{1}{2\pi\,C_e\,R_{plg}}=\frac{1{\cdot}51}{2\pi\times100\times10^{-12}\times10^4}$$

$$=238\times10^3\text{ cycles/sec.}$$

To get lower half power frequency of 10 c/s., value of requisite coupling condenser is given by,

$$C_c'=\frac{1}{2\pi f\,R'_{eq}}=\frac{1}{2\pi\times10\times100{\cdot}66\times10^4}\text{ Farad}$$

$$=0{\cdot}0158\times10^{-6}\text{ Farad.}$$

Example 3. *A resistance-capacitance coupled A_1 amplifier uses triode having amplification factor of 50 and dynamic plate resistance of 14000 ohms. The load resistance is 100 kilo-ohms and the grid leak resistance of the next stage is 800 kilo-ohms. The total shunt capacitance is 200 μμ F. Calculate for one stage of this amplifier (i) mid-band gain, (ii] values to coupling condenser to get lower half power frequencies of 3, 10 and 20 c/s. (iii) upper half power frequency and (iv) for coupling condenser of 0·05 μF, voltage gain at frequencies of 5,500 and 50,000 cycles/sec.*

Solution. $\frac{1}{R_{pts}}=\frac{1}{r_p}+\frac{1}{R_L}+\frac{1}{R_s}$

$$=\frac{1}{14\times10^3}+\frac{1}{100\times10^3}+\frac{1}{800\times10^3}$$

$$=0\cdot8268\times10^{-4} \text{ mho.}$$

Hence $\quad R_{pts}=\frac{10^4}{0\cdot8268}$ ohms.

(i) Mid-band gain

$$A_m=-g_m.R_{pts}=-\frac{50}{14000}\times\frac{10^4}{\cdot8268}=-43\cdot19$$

(ii) Coupling condenser is given by,

$$C_s=\frac{1}{2\pi f_1 R'_{eq}}$$

where $\quad R'_{eq}=R_s+\frac{r_p\cdot R_L}{r_p+R_L}$

$$=(800\times10^3)+\frac{14\ 10^3\times100\times10^3}{(14+100)\times10^3}$$

$$=812\cdot3\times10^3 \text{ ohms.}$$

In order to get lower half power frequency f_1 of 3 c/s.,

$$C_s=\frac{1}{2\pi\times3\times812\cdot3\times10^3}$$

$$=0\cdot0654\times10^{-6} \text{ Farad}$$

In order to get $f_1=10$ c/s.,

$$C_s=\frac{1}{2\pi\times10\times812\cdot3\times10^3}$$

$$=0\cdot1962\times10^{-6} \text{ Farad.}$$

In order to get $f_1=20$ c/s.,

$$C_s=\frac{1}{2\pi\times20\times812\cdot3\times10^3}$$

$$=0\cdot0981\times10^{-6} \text{ Farad}$$

(iii) Upper half power frequency f_2 is given by,

$$f_2=\frac{1}{2\pi C_s R_{pts}}=\frac{0\cdot8268}{2\pi\times200\times10^{-12}\times10^4}$$

$$=65\cdot8\times10^3 \text{ c/s.}$$

(iv) For $C_s=0\cdot05\ \mu F$, lower half power frequency is given by,

$$f_1=\frac{1}{2\pi C_s R'_{eq}}=\frac{1}{2\pi\times0\cdot05\times10^{-6}\times812\cdot3\times10^3}$$

$$=3\cdot92 \text{ cycles/sec.}$$

Solution. $\frac{1}{R_{plg}}=\frac{1}{r_p}+\frac{1}{R_L}+\frac{1}{R_g}$

$$=\frac{1}{10^4}+\frac{1}{2\times10^4}+\frac{1}{100\times10^4}$$

$$=(1+0{\cdot}5+{\cdot}01)\times10^{-4}$$

$$=1{\cdot}51\times10^{-4}\text{ mho.}$$

Hence $\quad R_{plg}=\frac{10^4}{1{\cdot}51}$ ohms.

Mid-band gain is then given by,

$$A_m=-g_m.R_{plg}=-\frac{15}{10^4}\times\frac{10^4}{1{\cdot}51}=-9{\cdot}98.$$

Lower half power frequency f_1 is given by,

$$f_1=\frac{1}{2\pi\,C_c\,R'_{eq}}$$

where $\quad R'_{eq}=R_g+\frac{r_p\,.\,R_L}{r_p+R_L}$

$$=10^6+\frac{10^4\times2\times10^4}{(1+2)\times10^3}\text{ ohms.}$$

$$=100{\cdot}66\times10^4\text{ ohms.}$$

Hence $\quad f_1=\frac{1}{2\pi\times0{\cdot}01\times10^{-6}\times100{\cdot}66\times10^4}$

$$=15{\cdot}82\text{ cycles/sec.}$$

Upper half power frequency is given by,

$$f_2=\frac{1}{2\pi\,C_s\,R_{plg}}=\frac{1{\cdot}51}{2\pi\times100\times10^{-12}\times10^4}$$

$$=238\times10^3\text{ cycles/sec.}$$

To get lower half power frequency of 10 c/s., value of requisite coupling condenser is given by,

$$C_c'=\frac{1}{2\pi f\,R'_{eq}}=\frac{1}{2\pi\times10\times100{\cdot}66\times10^4}\text{ Farad}$$

$$=0{\cdot}0158\times10^{-6}\text{ Farad.}$$

Example 3. *A resistance-capacitance coupled A_1 amplifier uses triode having amplification factor of 50 and dynamic plate resistance of 14000 ohms. The load resistance is 100 kilo-ohms and the grid leak resistance of the next stage is 800 kilo-ohms. The total shunt capacitance is 200 μμ F. Calculate for one stage of this amplifier (i) mid-band gain, (ii] values to coupling condenser to get lower half power frequencies of 3, 10 and 20 c/s. (iii) upper half power frequency and (iv) for coupling condenser of 0·05 μF, voltage gain at frequencies of 5,500 and 50,000 cycles/sec.*

Solution. $\frac{1}{R_{sh}}=\frac{1}{r_p}+\frac{1}{R_L}+\frac{1}{R_g}$

$$=\frac{1}{14\times10^3}+\frac{1}{100\times10^3}+\frac{1}{800\times10^3}$$

$$=0.8268\times10^{-4}\text{ mho.}$$

Hence $\quad R_{sh}=\frac{10^4}{0.8268}$ ohms.

(i) Mid-band gain

$$A_m=-g_m R_{sh}=-\frac{50}{14000}\times\frac{10^4}{.8268}=-43.19$$

(ii) Coupling condenser is given by,

$$C_c=\frac{1}{2\pi f_1 R'_{eq}}$$

where $\quad R'_{eq}=R_g+\frac{r_p\,.\,R_L}{r_p+R_L}$

$$=(800\times10^3)+\frac{14\times10^3\times100\times10^3}{(14+100)\times10^3}$$

$$=812.3\times10^3\text{ ohms.}$$

In order to get lower half power frequency f_1 of 3 c/s.,

$$C_c=\frac{1}{2\pi\times3\times812.3\times10^3}$$

$$=0.0654\times10^{-6}\text{ Farad}$$

In order to get $f_1=10$ c/s,

$$C_c=\frac{1}{2\pi\times10\times812.3\times10^3}$$

$$=0.1962\times10^{-6}\text{ Farad}$$

In order to get $f_1=20$ c/s.,

$$C_c=\frac{1}{2\pi\times20\times812.3\times10^3}$$

$$=0.0981\times10^{-6}\text{ Farad}$$

(iii) Upper half power frequency f_2 is given by,

$$f_2=\frac{1}{2\pi C_s R_{sh}}=\frac{0.8268}{2\pi\times200\times10^{-12}\times10^4}$$

$$=65.8\times10^3\text{ c/s.}$$

(iv) For $C_c=0.05\ \mu F$, lower half power frequency is given by,

$$f_1=\frac{1}{2\pi C_c R'_{eq}}=\frac{1}{2\pi\times0.05\times10^{-6}\times812.3}$$

$$=3.92\text{ cycles/sec.}$$

Frequency of 5 c/s. is close to the lower half power frequency and hence it is required to use the formula for low frequency range. Thus voltage gain at 5 cycles/sec. is given by

$$A=\frac{A_m}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}=\frac{43{\cdot}19}{\sqrt{1+\left(\frac{3{\cdot}92}{5}\right)^2}}$$

$$=33{\cdot}98$$

Frequency of 500 c/s. is greater than $10\,f_1$ and less than $10\,f_2$ and hence it lies in true mid-band. Hence gain at this frequency may be taken as $A_m=43{\cdot}19$.

Frequency of 50 kc/s. lies close to upper half power frequency and hence gain at this frequency is given by

$$A_h=\frac{A_m}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}}=\frac{43{\cdot}19}{\sqrt{1+\left(\frac{50}{65{\cdot}8}\right)^2}}$$

$$=34{\cdot}4$$

Example 4. *A resistance-capacitance coupled triode amplifier has anode load of 25 kilo-ohms., coupling condenser of 0·01 microfarad, dynamic plate resistance of 20 kilo-ohms, amplification factor of tube of 40, grid leak resistance of 500 kilo-ohms and total shunt capacitance of 100 micro-micro farad. Calculate (a) mid-band gain (b) lower and upper high power angular frequencies, (c) stage gain and phase angle at angular frequencies of 100, 5000 and 600×10^3 radians/sec.*

Solution. $g_m=\frac{\mu}{r_p}=\frac{40}{20000}=2\times10^{-3}$ mho.

$$\frac{1}{R_{plg}}=\frac{1}{r_p}+\frac{1}{R_L}+\frac{1}{R_g}$$

$$=\frac{1}{20\times10^3}+\frac{1}{25\times10^3}+\frac{1}{500\times10^3}$$

$$=0{\cdot}92\times10^{-4}\text{ mho}$$

Hence $R_{plg}=\frac{10^4}{0{\cdot}92}$ ohms.

Hence mid-band gain is given by,

$$A_m=-g_m.R_{plg}=-2\times10^{-3}\times\frac{10^4}{0{\cdot}92}$$

$$=-21{\cdot}74.$$

(b) Lower half power angular frequency is given by,

$$\omega_1=\frac{1}{C_c\,R'_{eq}}$$

But $$R'_{eq}=R_g+\frac{r_p \cdot R_L}{r_p+R_L}$$

$$=(500\times 10^3)+\frac{20\times 10^3\times 25\times 10^3}{(20+25)\times 10^3}$$

$$=511{\cdot}1\times 10^3 \text{ ohms.}$$

Hence $$\omega_1=\frac{1}{0{\cdot}01\times 10^{-6}\times 511{\cdot}1\times 10^3}$$

$$=195{\cdot}7 \text{ radians/sec.}$$

Upper half power angular frequency ω_2 is given by

$$\omega_2=\frac{1}{C_s R_{eq}}=\frac{0{\cdot}92}{100\times 10^{-12}\times 10^4}$$

$$=920\times 10^3 \text{ radians/sec.}$$

(c) Voltage gain at angular frequency of 100 radians/s given by

$$A=\frac{A_m}{1-j\dfrac{f_1}{f}}$$

Magnitude $$A=\frac{A_m}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}=\frac{21{\cdot}74}{\sqrt{1+\left(\frac{195{\cdot}7}{100}\right)^2}}$$

$$=9{\cdot}9.$$

Phase angle $$\phi=180°+\tan^{-1}\frac{f_1}{f}$$

$$=180°+\tan^{-1} 1{\cdot}957=243°\ 55'$$

Angular frequency of 5000 radians/sec. lies well within the mid-band. Hence voltage gain is equal to $A_m(=21{\cdot}74)$ and phase angle is 180°.

Angular frequency of 600×10^3 radians/sec. lies close to upper half power frequency. Hence voltage gain at this frequency is given by

$$A=\frac{A_m}{1+j\dfrac{f}{f_2}}$$

Hence magnitude $$A=\frac{A_m}{\sqrt{1+\left(\frac{f}{f_2}\right)^2}}=\frac{21{\cdot}74}{\sqrt{1+\left(\frac{600}{920}\right)^2}}$$

$$=18{\cdot}2$$

Phase angle $\phi = 180° - \tan^{-1} \frac{f}{f_2}$

$$= 180° - \tan^{-1} \frac{600}{920} = 146° \, 54'$$

Example 5. *A resistance-capacitance coupled triode A_1 amplifier stage has plate load of 80 kilo-ohm and coupling capacitance of 0·05 micro-farad. Valve amplification factor is 80 and dynamic plate resistance of 40 kilo-ohm. Grid leak resistance of next stage is 200 kilo-ohms. Calculate (a) mid-band gain, (b) lower half power frequency and (c) magnitude and phase angle of gain at a frequency of $\frac{20}{2\pi}$ cycles/sec. If output capacitance of amplifier valve is 10 μμF, input capacitance of next stage is 150 μμF and stray and wiring capacitance is 70 μμ F, calculate upper half power frequency, gain-bandwidth product and figure of merit of the amplifier tube.*

Solution. $g_m = \frac{\mu}{r_p} = \frac{80}{40 \times 10^3} = 2 \times 10^{-3}$ mho.

$$\frac{1}{R_{plg}} = \frac{1}{r_p} + \frac{1}{R_L} + \frac{1}{R_g}$$

$$= \frac{1}{40 \times 10^3} + \frac{1}{80 \times 10^3} + \frac{1}{200 \times 10^3}$$

$$= \frac{1 \cdot 7}{40 \times 10^3}$$

Hence $R_{plg} = \frac{40 \times 10^3}{1 \cdot 7} = 23 \cdot 53 \times 10^3$ ohms.

(a) Mid-band gain $A_m = -g_m . R_{plg}$

$$= -2 \times 10^{-3} \times 23 \cdot 53 \times 10^3 = -47 \cdot 06$$

Lower half power frequency

$$\omega_1 = \frac{1}{C_c R'_{eq}}$$

$$R'_{eq} = R_g + \frac{r_p \, . \, R_L}{r_p + R_L}$$

$$= (200 \times 10^3) + \frac{40 \times 10^3 \times 80 \times 10^3}{(40 + 80) \times 10^3}$$

$$= 226 \cdot 66 \times 10^3 \text{ ohm.}$$

Hence $\omega_1 = \frac{1}{0 \cdot 05 \times 10^{-6} \times 226 \cdot 66 \times 10^3}$

$$= 88 \cdot 5 \text{ radians/sec.}$$

(c) Voltage gain at angular frequency of 20 radians/sec. is

$$A=\frac{A_m}{1-j\dfrac{f_1}{f}}$$

Magnitude $$A=\frac{A_m}{\sqrt{1+\left(\dfrac{f_1}{f}\right)^2}}=\frac{47\cdot06}{\sqrt{1+\left(\dfrac{88\cdot5}{20}\right)^2}}$$

$$=10\cdot37$$

Phase angle at frequency of 20 radians/sec. is

$$\phi=180^\circ+\tan^{-1}\frac{f_1}{f}$$

$$=180^\circ+\tan^{-1}4\cdot425=257^\circ, 18'$$

Total shunt capacitance $C_s=(10+150+70)=230\ \mu\mu F$.

Hence upper half power frequency f_2 is given by,

$$f_2=\frac{1}{2\pi C_s R_{pl_s}}=\frac{1}{2\pi\times 230\times 10^{-12}\times 23\cdot53\times 10^3}$$

$$=29\cdot39\times 10^3\ \text{c/s.}$$

Gain bandwidth product

$$A_m\times B=\frac{g_m}{2\pi C_s}=\frac{2\times 10^{-3}}{2\pi\times 230\times 10^{-12}}$$

$$=1\cdot38\times 10^6$$

Figure of merit of tube $$=\frac{g_m}{2\pi(C_{pk}+C_g)}$$

$$=\frac{2\times 10^{-3}}{2\pi(10+150)\times 10^{-12}}$$

$$=1\cdot97\times 10^6\ \text{c/s.}$$

Example 6. *A two-stage R. C. coupled A_1 amplifier uses in the first stage a triode having amplification factor of 40 and dynamic plate resistance of 40 kilo-ohms. Load resistance is 160 kilo-ohms and grid leak resistance of the second stage is 2 Meg-ohms. Calculate the value of the coupling condenser such that at a frequency of 50 c/s, the stage again is 4 db below the maximum voltage gain. Also find the voltage gain in db of the first stage at a frequency of 100 cycles/second.*

Solution. Voltage in low frequency range is given by,

$$A_L=\frac{A_m}{1-j\dfrac{X_c}{R'_m}}$$

But at 50 c/s, A_L is 4 db below midband grid A_m so that

$$4=20 \log_{10} \frac{A_m}{A_L}$$

Hence $\frac{A_m}{A_L}=\text{antilog } 0.2=1{\cdot}585$

Hence $1{\cdot}585=\sqrt{1+\left(\frac{X_c}{R'_{eq}}\right)^2}$

or $\left(\frac{X_c}{R'_{eq}}\right)^2=(1{\cdot}585)^2-1=1{\cdot}512$

or $\frac{1}{2\pi f C_c R'_{eq}}=\sqrt{1{\cdot}512}=1{\cdot}23$

But $R'_{eq}=R_g+\frac{r_p \,.\, R_L}{r_p+R_L}=2\times10^6+\frac{40\times10^3\times160\times10^3}{(160+40)\times10^3}$

$$=2{\cdot}032\times10^6$$

Hence $C_c=\frac{1}{2\pi\times50\times2{\cdot}032\times10^6\times1{\cdot}23}$ Farad

$={\cdot}00127$ micro-farad.

$$\frac{1}{R_{plg}}=\frac{1}{r_p}+\frac{1}{r_l}+\frac{1}{R_g}$$

$$=\frac{1}{40\times10^3}+\frac{1}{160\times10^3}+\frac{1}{2000\times10_3}$$

$$=\frac{1{\cdot}27}{40\times10^3} \text{ mho}$$

or $R_{plg}=\frac{40\times10^3}{1{\cdot}27}=31{\cdot}5\times10^3$ ohms

Hence midband gain $A_m=-g_m R_{plg}$

$$=-\frac{40}{40\times10^3}\times31{\cdot}5\times10^3=-31{\cdot}5$$

Voltage gain at a frequency of 100 cycles/sec. is given by

$$A_{100}=\frac{A_m}{\sqrt{1+\left(\frac{X_c}{R'_{eq}}\right)^2}}$$

$$\frac{X_c}{R'_{eq}}=\frac{1}{2\pi f C_c R'_{eq}}=\frac{1}{2\pi\times100\times{\cdot}00127\times10^{-6}\times2{\cdot}032\times10^6}$$

$$=0{\cdot}615$$

$$A_{100}=\frac{31{\cdot}5}{\sqrt{1+({\cdot}615)^2}}=26{\cdot}8$$

ge gain expressed in db is $=20 \log_{10} 26{\cdot}8$

$=28{\cdot}562$ db

Example 7. *A wideband amplifier uses a pentode valve having mutual conductance of 10 mA/V and extremely high dynamic plate resistance. Load resistance is 2500 ohms. Coupling condenser is 0.01 microfarad and grid leak resistance of each stage is 250 kilo-ohms. Find the voltage gain of one stage at a frequency of 30 c/s. Find the value of shunt capacitance that will result in the same voltage gain at a frequency of 3 mega cycles/second.*

Solution.

$$\frac{1}{R_{plg}}=\frac{1}{r_p}+\frac{1}{R_L}+\frac{1}{R_g}$$

$$=\frac{1}{R_L}+\frac{1}{R_g} \text{ since } r_p \text{ is extremely large.}$$

$$=\frac{1}{2500}+\frac{1}{250\times 10^3}$$

$$=\frac{1.01}{2500} \text{ mho.}$$

or $$R_{plg}=\frac{2500}{1.01} \text{ ohms.}$$

Hence midband gain $A_m=-g_m \cdot R_{plg}=-(10\times 10^{-3})\times\dfrac{2500}{1.01}$

$$=-24.75$$

$$R'_{eq}=R_g+\frac{r_p \cdot R_L}{r_p+R_L}$$

$=R_g+R_L$ since r_p is extremely large

$=(250+2.5)\times 10^3=252.5\times 10^3$ ohms.

Voltage gain at 30 c/s is given by

$$A_{30}=\frac{A_m}{\sqrt{1+\left(\dfrac{X_c}{R'_{eq}}\right)^2}}$$

But $\dfrac{X_c}{R'_{eq}}=\dfrac{1}{2\pi f C_c R_{eq}'}=\dfrac{1}{2\pi\times 30\times .01\times 10^{-6}\times 252.5\times 10^3}=2.1$

Hence $A_{30}=\dfrac{24.75}{\sqrt{1+(2.1)^2}}=10.6$

Let C_s be the value of total shunt capacitance in farads

Then $10.6=\dfrac{24.75}{\sqrt{1+(R_{plg}\times 2\pi\times 3\times 10^6\times C_s)^2}}$

Obviously 2.1 must equal $R_{plg}\times 2\pi\times 3\times 10^6\times C_s$

Hence $C_s=\dfrac{2.1}{R_{plg}\times 2\pi\times 3\times 10^6}=\dfrac{2.1\times 1.01}{2500\times 2\pi\times 3\times 10^6}$

$=45\times 10^{-12}$ Farad $=45$ $\mu\mu$F.

Example 8. *A resistance capacitance coupled amplifier has voltage gain 3 db less than midband gain at frequencies of $\frac{100}{2\pi}$ cycles/sec. and $\frac{25}{2\pi}$ mega cycles/sec. The pentode used has mutual conductance of 10 mA/V and extremely high anode slope resistance. The total shunt capacitance in each stage is 16 μμF. Coupling condenser capacity is 0·05 μF. Find (a) the anode load resistance (b) grid leak resistance and (c) midband gain.*

Solution. Lower half power frequency $f_1 = \frac{1}{2\pi\, C_c\, R'_{eq}}$

Hence $R'_{eq} = \frac{1}{2\pi\, C_c f_1} = \frac{1}{2\pi \times 0{\cdot}5 \times 10^{-6} \times \frac{100}{2\pi}} = 200 \times 10^3$ ohms.

$$R'_{eq} = R_g + \frac{r_p \cdot R_L}{r_p + R_L}$$

But $r_p \gg R_L$,

hence $R'_{eq} = R_g + R_L$ so that $R_g + R_L = 200 \times 10^3$ ohms. ... (1)

Upper half power frequency is given by

$$f_2 = \frac{1}{2\pi\, C_s\, R_{plg}}$$

Hence $R_{plg} = \frac{1}{2\pi f_2 C_s} = \frac{1}{2\pi \times \frac{25}{2\pi} \times 10^6 \times 16 \times 10^{-12}} = 2500$ ohms.

$$\frac{1}{R_{plg}} = \frac{1}{r_p} + \frac{1}{R_L} + \frac{1}{R_g}$$

But r_p is extremely large,

hence $\frac{1}{2500} = \frac{1}{R_L} + \frac{1}{R_g} = \frac{R_L + R_g}{R_L \cdot R_g} = \frac{200 \times 10^3}{R_L \cdot R_g}$

Hence $R_L \cdot R_g = 500 \times 10^6$

or $R_L = \frac{500 \times 10^6}{R_g}$ ... (2)

Substituting the value of R_L from equation (2) into equation (1) we get,

$$R_g + \frac{500 \times 10^6}{R_g} = 200 \times 10^3$$

or $R_g^2 - 200 \times 10^3\, R_g + 500 \times 10^6 = 0$

Hence, $R_g = \frac{+200 \times 10^3 \pm \sqrt{(200 \times 10^3)^2 - 4 \times 500 \times 10^6}}{2}$

$= 197{\cdot}45 \times 10^3$ ohms or $2{\cdot}55 \times 10^3$ ohms.

Obviously $R_g = 197{\cdot}45 \times 10^3$ ohms.

Hence $R_L = 2{\cdot}55 \times 10^3$ ohms.

Midband gain $= -gm.\, R_{plg} = -(10 \times 10^{-3}) \times 2500 = -25$

Example 9. *A wideband R. C. coupled pentode amplifier is so designed as to have phase shift not exceeding 30 degrees at frequencies of $\frac{25}{2\pi}$ Mc/s and $\frac{40}{2\pi}$ c/s. The total shunt capacitance is 18 μμF and grid leak resistance is 500 kilo-ohms in each stage. Calculate suitable values of load resistance and coupling condenser. Assume dynamic plate resistance of pentodes to be extremely high.*

Solution. Phase shift in high frequency range is given by,

$$\theta = -\tan^{-1} \frac{R_{sh}}{X_s}$$

Hence $\quad R_{sh} = X_s \cdot \tan\theta = \dfrac{\tan 30°}{2\pi \times \frac{25}{2\pi} \times 10^6 \times 18 \times 10^{-12}}$

$$= 1285 \text{ ohms.}$$

$$\frac{1}{R_{sh}} = \frac{1}{r_p} + \frac{1}{R_L} + \frac{1}{R_g}$$

But r_p is extremely large.

Hence $\dfrac{1}{R_{sh}} = \dfrac{1}{R_L} + \dfrac{1}{R_g}$

or $$\frac{1}{1285} = \frac{1}{R_L} + \frac{1}{500 \times 10^3}$$

or $$\frac{1}{R_L} = \cdot 778 \times 10^{-3} - \cdot 002 \times 10^{-3} = \cdot 776 \times 10^{-3}$$

Hence $\quad R_L = \dfrac{10^3}{0 \cdot 776} = 1289$ ohms.

$$R'_{et} = R_g + \frac{r_p \cdot R_L}{r_p + R_L}. \quad \text{But } r_p \gg R_L.$$

Hence $R'_{et} = R_g + R_L = (500 \times 10^3) + (1 \cdot 289 \times 10^3)$

$= 501 \cdot 3 \times 10^3$ ohms.

In the low frequency range, phase shift is given by

$$\theta = \tan^{-1} \frac{X_c}{R'_{et}}$$

or $\quad \tan\theta = \dfrac{X_c}{R'_{et}} = \dfrac{1}{2\pi f\, C_c\, R'_{et}}$

Hence $\quad C_c = \dfrac{1}{2\pi f \tan\theta\, R'_{et}} = \dfrac{1}{2\pi \times \frac{40}{2\pi} \times 0 \cdot 5774 \times 501 \cdot 3 \times 10^3}$

$= 0 \cdot 865 \times 10^{-6}$ Farad

$= 0 \cdot 865$ μF

Example 8. *A resistance capacitance coupled amplifier has voltage gain 3 db less than midband gain at frequencies of* $\frac{100}{2\pi}$ *cycles/sec. and* $\frac{25}{2\pi}$ *mega cycles/sec. The pentode used has mutual conductance of 10 mA/V and extremely high anode slope resistance. The total shunt capacitance in each stage is 16 μμF. Coupling condenser capacity is 0·05 μF. Find (a) the anode load resistance (b) grid leak resistance and (c) midband gain.*

Solution. Lower half power frequency $f_1 = \frac{1}{2\pi\, C_c\, R'_{eq}}$

Hence $R'_{eq} = \frac{1}{2\pi\, C_c\, f_1} = \frac{1}{2\pi \times 0{\cdot}5 \times 10^{-6} \times \frac{100}{2\pi}} = 200 \times 10^3$ ohms.

$$R'_{eq} = R_g + \frac{r_p \,.\, R_L}{r_p + R_L}$$

But $r_p \gg R_L$,

hence $R'_{eq} = R_g + R_L$ so that $R_g + R_L = 200 \times 10^3$ ohms. ... (1)

Upper half power frequency is given by

$$f_2 = \frac{1}{2\pi\, C_s\, R_{plg}}$$

Hence $R_{plg} = \frac{1}{2\pi\, f_2\, C_s} = \frac{1}{2\pi \times \frac{25}{2\pi} \times 10^6 \times 16 \times 10^{-12}} = 2500$ ohms.

$$\frac{1}{R_{plg}} = \frac{1}{r_p} + \frac{1}{R_L} + \frac{1}{R_g}$$

But r_p is extremely large,

hence $\frac{1}{2500} = \frac{1}{R_L} + \frac{1}{R_g} = \frac{R_L + R_g}{R_L \,.\, R_g} = \frac{200 \times 10^3}{R_L \,.\, R_g}$

Hence $R_L \,.\, R_g = 500 \times 10^6$

or $R_L = \frac{500 \times 10^6}{R_g}$ ... (2)

Substituting the value of R_L from equation (2) into equation (1) we get,

$$R_g + \frac{500 \times 10^6}{R_g} = 200 \times 10^3$$

or $R_g^2 - 200 \times 10^3\, R_g + 500 \times 10^6 = 0$

Hence, $R_g = \frac{+200 \times 10^3 \pm \sqrt{(200 \times 10^3)^2 - 4 \times 500 \times 10^6}}{2}$

$= 197{\cdot}45 \times 10^3$ ohms or $2{\cdot}55 \times 10^3$ ohms.

Obviously $R_g = 197{\cdot}45 \times 10^3$ ohms.

Hence $R_L = 2{\cdot}55 \times 10^3$ ohms.

Midband gain $= -g_m \,.\, R_{plg} = -(10 \times 10^{-3}) \times 2500 = -25$

Example 9. *A wideband R. C. coupled pentode amplifier is so designed as to have phase shift not exceeding 30 degrees at frequencies of $\frac{25}{2\pi}$ Mc/s and $\frac{40}{2\pi}$ c/s. The total shunt capacitance is 18 μμF and grid leak resistance is 500 kilo ohms in each stage. Calculate suitable values of load resistance and coupling condenser. Assume dynamic plate resistance of pentodes to be extremely high.*

Solution. Phase shift in high frequency range is given by,

$$\theta = -\tan^{-1} \frac{R_{plg}}{X_s}$$

Hence $\quad R_{plg} = X_s . \tan\theta = \dfrac{\tan 30^\circ}{2\pi \times \frac{25}{2\pi} \times 10^6 \times 18 \times 10^{-12}}$

$$= 1285 \text{ ohms.}$$

$$\frac{1}{R_{plg}} = \frac{1}{r_p} + \frac{1}{R_L} + \frac{1}{R_g}$$

But r_p is extremely large.

Hence $\dfrac{1}{R_{plg}} = \dfrac{1}{R_L} + \dfrac{1}{R_g}$

or $\qquad \dfrac{1}{1285} = \dfrac{1}{R_L} + \dfrac{1}{500 \times 10^3}$

or $\qquad \dfrac{1}{R_L} = \cdot 778 \times 10^{-3} - \cdot 002 \times 10^{-3} = \cdot 776 \times 10^{-3}$

Hence $\quad R_L = \dfrac{10^3}{0 \cdot 776} = 1289$ ohms.

$$R'_{eq} = R_g + \frac{r_p \cdot R_L}{r_p + R_L}. \quad \text{But } r_p \gg R_L,$$

Hence $R'_{eq} = R_g + R_L = (500 \times 10^3) + (1 \cdot 289 \times 10^3)$

$$= 501 \cdot 3 \times 10^3 \text{ ohms,}$$

In the low frequency range, phase shift is given by

$$\theta = \tan^{-1} \frac{X_c}{R'_{eq}}$$

or $\quad \tan\theta = \dfrac{X_c}{R'_{eq}} = \dfrac{1}{2\pi f\, C_c\, R'_{eq}}$

Hence $\quad C_c = \dfrac{1}{2\pi f \tan\theta\, R'_{eq}} = \dfrac{1}{\text{[illegible]}}$

$$= 0 \cdot 865 \times 10^{-6} \text{ Farad}$$

$$= 0 \cdot 865 \ \mu F$$

Suitable value of coupling condenser may then be one microfarad. This will produce phase shift slightly less than 30 degrees.

Example 10. *Draw a.c. equivalent circuit of one stage of R. C. coupled amplifier. Using Millman theorem, prove that the voltage gain of one stage is given by the expression :*

$$A=\frac{-\mu Y_p Y_c}{(Y_c+Y_g+Y_s)(Y_p+Y_L)-Y_c(Y_g+Y_s)}$$

where $Y_p=\frac{1}{r_p}$, $Y_c=j\omega C_c$, $Y_g=\frac{1}{R_g}$, $Y_s=j\omega C_s$ *and* $Y_L=\frac{1}{R_L}$.

Given that amplification factor μ is equal to 50, dynamic plate resistance=20 kilo-ohms, coupling condenser=·01μF, grid leak resistance R_g=1 meg-ohm, total shunt capacitance=200 μμF and load resistance=20 kilo-ohm, calculate magnitude and phase angle of voltage gain at frequency of $\frac{100}{2\mu}$ cycles/sec.

Solution. $Y_p=\frac{1}{r_p}=\frac{1}{20\times10^3}=\cdot05\times10^{-3}$ mho

$$Y_c=j\omega\ C_c=j\times2\pi\times\frac{100}{2\pi}\times\cdot01\times10^{-6}=j\times10^{-6}\text{ mho}$$

$$Y_g=\frac{1}{R_g}=10^{-6}\text{ mho}$$

$$Y_s=j\omega C_s=j\times100\times200\times10^{-12}=j\times2\times10^{-8}\text{ mho}$$

$$Y_L=\frac{1}{R_L}=\frac{1}{20\times10^3}=\cdot05\times10^{-3}\text{ mho}$$

Hence
$$A=\frac{-50\times\cdot05\times10^{-3}\times j\times10^{-6}}{(j\times10^{-6}+10^{-6}+j2\times10^{-8})(\cdot05\times10^{-3}+\cdot05\times10^{-3})-j\times10^{-6}(10^{-6}+j2\times10^{-8})}$$

$$=\frac{-25j}{[1\cdot0002+1\cdot01\ j]}$$

Hence
$$A=\frac{25}{\sqrt{(1\cdot0002)^2+(1\cdot01)^2}}=17\cdot7$$

Phase angle $=180^\circ+90^\circ-\tan^{-1}\frac{1\cdot01}{1\cdot0002}$

$$=270^\circ+45^\circ\ 18'=224^\circ\ 42'.$$

Example 11. *Voltage gains of 3 stages of a resistance-capacitance coupled amplifier at frequency of 50 c/s. are respectively*

$$20\angle220^\circ,\ 25\angle210^\circ\ \text{and}\ 24\angle230^\circ$$

Find the overall gain and phase angle.

Solution. Overall gain $=20\times 25\times 24=12000$.

Overall phase angle $=(220+210+230)$ degrees

$=660$ degrees $=300$ degrees.

Example 12. *A resistance capacitance coupled amplifier has four identical stages. Each stage uses a pentode having $g_m=10$ mA/V. Load resistance is 10 kilo-ohms, grid leak resistance $R_g=1$ meg-ohm, coupling condenser $C_c=0.01\ \mu F$, and total shunt capacitance $C_s=20\mu\mu F$. Calculate (a) midband voltage gain, (b) half power frequencies, (c) gain bandwidth product of each stage, and (d) gain bandwidth product of overall amplifier. Assume dynamic plate resistance to be extremely large.*

Solution.
$$\frac{1}{R_{sh}}=\frac{1}{r_p}+\frac{1}{R_L}+\frac{1}{R_g}$$
$$=\frac{1}{R_L}+\frac{1}{R_g} \text{ since } r_p \text{ is extremely large.}$$
$$=\frac{1}{10^4}+\frac{1}{10^6}=\frac{1.01}{10^4}$$

Hence
$$R_{sh}=\frac{10^4}{1.01}$$

Midband voltage gain is given by,
$$A_m=-g_m R_{sh}=-(10\times 10^{-3})\times\frac{10^4}{1.01}=-99.01$$

Lower half power frequency $f_1=\dfrac{1}{2\pi C_c R'_{eq}}$

But
$$R'_{eq}=R_g+\frac{r_p\cdot R_L}{r_p+R_L}$$
$$=R_g+R_L \qquad \text{since } r_p>>R_L$$
$$=10^6+10^4=1.01\times 10^6 \text{ ohms.}$$

Hence
$$f_1=\frac{1}{2\pi\times .01\times 10^{-6}\times 1.01\times 10^6}$$
$$=15.75 \text{ cycles/sec.}$$

Upper half power frequency f_2 is given by
$$f_2=\frac{1}{2\pi C_s R_{sh}}=\frac{1.01}{2\pi\times 20\times 10^{-12}\times 10^4}$$
$$=803\times 10^3 \text{ c/s.}$$

Gainband product of one stage is given by
$$A_m\times f_2=99.01\times 803\times 10^3=79.5 \text{ Mc/s.}$$

Overall midband gain $=(99.01)^4=9.601\times 10^7$

Overall upper half power frequency is given by,

$$f_{2n}=f_2\sqrt{(2)^{\frac{1}{n}}-1}$$

$$=0{\cdot}803\times10^6\times\sqrt{(2)^{\frac{1}{4}}-1}$$
$$=0{\cdot}35\times10^6 \text{ c/s.}$$

Overall gain bandwidth product

$$=f_{2n}\times A_{mn}=0{\cdot}35\times10^6\times9{\cdot}601\times10^7$$
$$=3{\cdot}36\times10^{13} \text{ c/s.}$$

Example 13. *A wideband amplifier uses 4 stages of R.C. coupled pentode amplifier. All the stages are identical. Overall bandwidth is required to be 3 Mc/s. The overall shunt capacitance C_s is 15 μμF. Gridelak resistors have values of 1 meg-ohm each. Select suitable value of load resistance. If the lower half power frequency of overall amplifier is 10 cycles/sec., calculate suitable value of coupling condenser. Assume the dynamic plate resistance of each tube to be extremely large. If mutual conductance of each tube is 8 mA/V, calculate overall mid-band gain.*

Solution. Overall bandwidth *i.e.*, overall upper half power frequency is 3 Mc/s.

Hence $$f_2=\frac{3\times10^6}{\sqrt{(2)^{\frac{1}{4}}-1}}=\frac{3\times10^6}{0{\cdot}436}=6{\cdot}87\times10^6 \text{ c/s.}$$

But $$f_2=\frac{1}{2\pi\, C_s\, R_{plg}}$$

Hence $$R_{plg}=\frac{1}{2\pi\, f_2\, C_s}=\frac{1}{2\pi\times6{\cdot}87\times10^6\times15\times10^{-12}}$$
$$=1545 \text{ ohms.}$$

Dynamic plate resistance r_p is extremely large.

Hence $$\frac{1}{1545}=\frac{1}{R_g}+\frac{1}{R_L}=\frac{1}{10^6}+\frac{1}{R_L}$$

Or $$\frac{1}{R_L}=0{\cdot}647\times10^{-3}-10^{-6}=0{\cdot}646\times10^{-3}$$

Or $$R_L=\frac{10^3}{0{\cdot}647}=1548 \text{ ohms}$$

Hence $$R'_{eq}=R_g+\frac{r_p\,.\,R_L}{r_p+R_L}$$
$$=R_g+R_L \qquad \text{since } r_p>>R_L$$
$$=10^6+1{\cdot}5848\times10^3=1001{\cdot}5\times10^3 \text{ ohms.}$$

Lower half power frequency f_1 of each stage is given by,

$$f_1 = f_{1n} \times \sqrt{(2)^{\frac{1}{n}} - 1}$$

n = number of identical stages = 4

$f_{1n} = 10$ c/s.

Hence $f_1 = 10 \times \sqrt{(2)^{\frac{1}{4}} - 1}$

$= 10 \times 0{\cdot}436 = 4{\cdot}36$ c/s.

Hence coupling condenser C_c is given by,

$$C_c = \frac{1}{2\pi f_1 R'_{eq}} = \frac{1}{2\pi \times 4{\cdot}36 \times 1001{\cdot}5 \times 10^3} \text{ Farad}$$

$= 0{\cdot}036\ \mu F.$

Midband gain $A_m = -g_m R_{eq} = -8 \times 10^{-3} \times 1545 = -12{\cdot}36$

Overall gain $= (12{\cdot}36)^4 = 23330.$

Universal voltage gain and phase-shift curves of R. C. coupled amplifier. A study of equations (12·17) and (12·24) for complex voltag gain in the low and high frequency regions reveals a few important characteristics of R C. coupled amplifier These equations are again written below,

$$A_l = \frac{A_m}{1 - j\dfrac{f_1}{f}} \quad \cdots (12{\cdot}17)$$

and

$$A_h = \frac{A_m}{1 + j\dfrac{f}{f_2}} \quad \cdot (12{\cdot}24)$$

Following points are worth noticing —(*i*) at frequencies f_1 and f_2 i.e., at lower and upper half power frequencies, voltage gains are the same and equal $\frac{A_m}{\sqrt{2}}$ *i.e.* 3 *db* below midband gain A_m *Also* phase shift with respect to the phase in midband is $\pm \tan^{-1} 1$ *i.e.* equal to ± 45 degrees, plus sign holding for lower frequency region and minus for high frequency region (*ii*) at frequencies of nf_1 and $\frac{1}{n} f_2$, voltage gains are equal and phase shifts with respect to mid-band condition are equal and opposite Voltage gain for both fre-quencies is $\frac{A_m}{\sqrt{1 + \left(\frac{1}{n}\right)^2}}$ and phase shifts are $\tan^{-1}\left(\frac{1}{n}\right)$ and $-\tan^{-1}\left(\frac{1}{n}\right)$ respectively for frequencies of nf_1

with respect to frequencies f_1 and f_2, voltage gain and phase shift are symmetrically disposed. If relative voltage gain $\frac{A}{A_m}$ in decibels is plotted along vertical axis against relative frequency $\frac{f}{f_1}$ or $\frac{f}{f_2}$ along a log scale, the curve so obtained is universal in nature *i.e.*, it applies to all R.C. coupled amplifier equally well. Fig. 12·13 shows universal voltage gain curve for R.C. coupled amplifier.

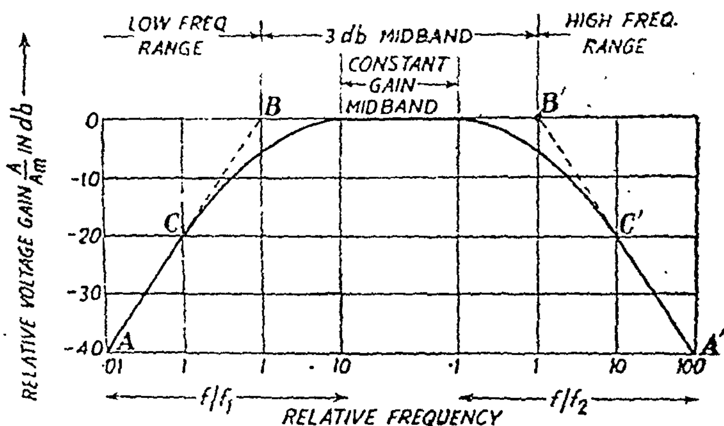


Fig. 12·13. Universal voltage gain curve of R. C. coupled amplifier.

The curve shown in Fig. 12·13 is symmetrical on either side of true middle frequency range *i.e.* range of constant voltage gain. This constant voltage midband extends from frequency 10 f_1 to ·1 f_2 where f_1 and f_2 are lower and upper half power frequencies respectively. Straight lines AB and $A'B'$ have slopes of 6 *db*/octave and -6 *db*/octave respectively. In the region from C to B *i.e.* from frequency ·1 f_1 to f_1 the actual curve departs from linearity. Because of the symmetry of the curve on either side of true midband, only one-half of the entire curve may be drawn to represent universal voltage gain curve. It has, of course, to be interpreted properly in the low and high frequency regions regarding the relative frequency.

Similarly if the numerical value of phase shift with respect to midband phase is plotted against relative frequency f/f_1 or f/f_2 on logarithmic scale, the curve so obtained is shown in Fig. 12·14. This curve again is universal in nature since it applies equally well to all R.-C. coupled amplifiers.

The curve expresses the phase shift at different frequencies in addition to constant 180 degrees phase shift in the true midband. The curve is symmetrical with respect to midband except for the difference in sign of the phase-shift as shown in Fig. 12·14. Only

one-half of the curve in Fig. 12·14 may, therefore, be drawn to represent universal phase characteristic of R. C. coupled amplifier.

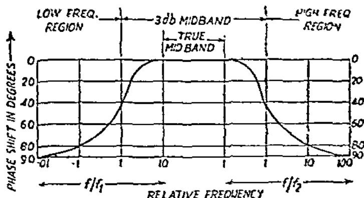


Fig. 12·14. Universal phase characteristic of R. C. coupled amplifier.

Voltage gain characteristic of R. C. Coupled amplifier on complex plane. The response of R. C. coupled amplifier regarding voltage gain and phase angle may be summarized by plotting the complex voltage gain on a complex plane. For each signal frequency, voltage gain vector starting from origin may be drawn such that its length represents the magnitude of the complex voltage gain and its angle with the + X-axis gives the phase angle of the complex voltage gain. Obviously then the projections of this vector on X and Y axes respectively give the real and imaginary parts of the complex voltage gain. Hence these X and Y axes may be referred to as real and imaginary axes. This process may be repeated for a large number of frequencies and tips of these vectors may be joined by a curve. This curve giving the locus of the tip of the voltage gain vector for R. C. coupled amplifier is shown in Fig. 12·15 and may be called the voltage gain characteristic on complex plane. However as a preliminary to the construction of this voltage gain curve we may first draw curves for factor $\dfrac{1}{1-j\dfrac{f_1}{f}}$ for low frequency region and $\dfrac{1}{1+j\dfrac{f}{f_2}}$ for high frequency region. In the frequency range from zero to about $10 f_1$, Eq. (12·17) applies. During this frequency interval, the tip of the vector $\dfrac{1}{1-j\dfrac{f_1}{f}}$ traces out a semicircle in the clockwise direction starting from origin corresponding to $f=0$, and going upto a value $0·99+j0·099$ at point P_1 corresponding to value $f=10 f_1$. In the frequency range from

$10 f_1$ to $0{\cdot}1\, f_2$, the complex voltage gain is constant at the value $1+j0$ indicated by point P. This semicircle is locus of the tip vector A_l/A_m and is shown in upper right hand quadrant in Fig. 12·15.

In the frequency range from $0{\cdot}1\, f_2$ to infinity complex voltage gain is given by Eq. (12·24). During this frequency interval the tip of the vector $\dfrac{1}{1+j\dfrac{f}{f_2}}$ traces out a semicircle in clockwise direction starting from point P_2 $(0{\cdot}99\text{-}j0{\cdot}099)$ corresponding to frequency $f={\cdot}1 f_2$ and going upto origin corresponding to infinitely large frequency. This semicircle is the locus of the tip of vector A_h/A_m and is shown in lower right hand quadrant in Fig. 12·15. Thus the right hand circle in Fig. 12·15 is the locus of the tip of the vector A/A_m over the entire frequency range from zero to infinity. But $A_m=-g_m.R_{plg}$. Hence locus of complex voltage gain A is obtained by multiplying this locus of A/A_m by factor $-g_m.R_{plg}$. The locus corresponding to complex voltage gain A so obtained is then also a circle magnified in magnitude and reversed in phase. This is shown by the left hand circle in Fig. 12·15. For the frequency range 0 to $10 f_1$, locus is represented by the lower left hand semicircle of Fig. 12·15 from origin to point $P_3[-g_m.R_{plg}({\cdot}99+j0{\cdot}099)]$. Locus for middle frequency range from $10 f_1$ to ${\cdot}1 f_2$ is indicated by curve from P_3 to $P_4\,[-g_m.R_{plg}({\cdot}99-j0{\cdot}099)]$. Locus for A for the frequency range ${\cdot}1 f_2$ to infinity is represented by upper left hand circle from point P_4 to origin. Point P_m corresponding to middle frequency $f_0=\sqrt{f_1 f_2}$ has coordinates $[-g_m.R_{plg}, 0]$ corresponding to A_m.

Fig. 12·15. Locus of relative gain A/A_m and complex voltage gain A of R. C. coupled amplifier on complex plane.

In the above construction locus of tip of vector A/A_m has been drawn first following Eqs (12·17) and (12·24) and subsequently locus of complex voltage gain A has been drawn by multiplying the locus of A/A_m by quantity $-g_m.R_{plg}$. This indirect procedure has been followed simply to facilitate understanding. We could

have drawn the locus of the tip vector A directly using the Eqs. (12·17*a*) and (12·24 *a*) for low and high frequency ranges respectively.

Inductance Capacitance Coupled Amplifier

It differs from R.C. coupled amplifier in that an inductor instead of a resistor, is used as the load. Coupling is, however, still done through a coupling condenser. The advantage gained is that no d.c. voltage drop in the load impedance takes place and the corresponding power loss is avoided. The plate circuit efficiency is thus increased. Fig 12·16 shows one stage of inductance-capacitance coupled triode amplifier.

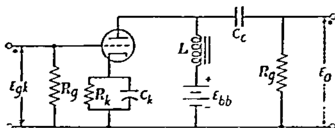


Fig. 12·16. One stage of Inductance-Capacitance coupled triode amplifier.

Another advantage obtainable in inductance-capacitance coupled amplifier is the large voltage gain in the middle frequency range resulting from the fact that the inductive reactance may be made very large. But there exists the disadvantage that in the low frequency range voltage gain falls more rapidly because of rapid reduction in the load impedance with frequency. This results *in higher value of lower half power frequency. There is additional* disadvantage of higher weight and cost of the inductor. Thus no special advantage is obtained by the use of inductance-capacitance coupling. If at all a resistive load is to be avoided, it is preferred to use a transformer coupling in between the stages of a cascade amplifier. Transformer coupling has the advantage over inductance-capacitance coupling of the facility of impedance matching and higher gain due to step up ratio of the transformer. Accordingly I.C. coupled amplifiers are rarely used for interstage coupling. The gain of I. C coupled amplifier is given by Eq. (12·30) with the alterations that Y_t now represents admittance of the inductor.

Inductance-capacitance coupling is, however, often used to couple a load to the tube particularly when the load absorbs con-considerable power. This is known as the "shunt feed" or "parallel feed" of the plate. Fig. 12·17 shows the circuit arrangement of parallel feed. The inductor is in series with the plate supply while load Z_l is connected to the plate through coupling condenser C_c.

In this shunt fed or parallel fed amplifier d c. plate current I_b and a c. plate current I_p flow through parallel paths as shown in Fig. 12·17 No d.c. voltage is then present in the load Z_l because of the blocking condenser C_c. No d c. voltage drop in the load

$10 f_1$ to $0{\cdot}1\, f_2$, the complex voltage gain is constant at the value $1+j0$ indicated by point P. This semicircle is locus of the tip vector A_l/A_m and is shown in upper right hand quadrant in Fig. 12·15.

In the frequency range from $0{\cdot}1\, f_2$ to infinity complex voltage gain is given by Eq. (12·24). During this frequency interval the tip of the vector $\dfrac{1}{1+j\dfrac{f}{f_2}}$ traces out a semicircle in clockwise direction starting from point P_2 $(0{\cdot}99\text{-}j0{\cdot}099)$ corresponding to frequency $f={\cdot}1 f_2$ and going upto origin corresponding to infinitely large frequency. This semicircle is the locus of the tip of vector A_h/A_m and is shown in lower right hand quadrant in Fig. 12·15. Thus the right hand circle in Fig. 12·15 is the locus of the tip of the vector A/A_m over the entire frequency range from zero to infinity. But $A_m=-g_m.R_{plg}$. Hence locus of complex voltage gain A is obtained by multiplying this locus of A/A_m by factor $-g_m.R_{plg}$. The locus corresponding to complex voltage gain A so obtained is then also a circle magnified in magnitude and reversed in phase. This is shown by the left hand circle in Fig. 12·15. For the frequency range 0 to $10 f_1$, locus is represented by the lower left hand semicircle of Fig. 12·15 from origin to point $P_3[-g_m.R_{plg}({\cdot}99+j0{\cdot}099)]$. Locus for middle frequency range from $10 f_1$ to ${\cdot}1\, f_2$ is indicated by curve from P_3 to P_4 $[-g_m.R_{plg}({\cdot}99-j0{\cdot}099)]$. Locus for A for the frequency range ${\cdot}1\, f_2$ to infinity is represented by upper left hand circle from point P_4 to origin. Point P_m corresponding to middle frequency $f_0=\sqrt{f_1 f_2}$ has coordinates $[-g_m.R_{plg}, 0]$ corresponding to A_m.

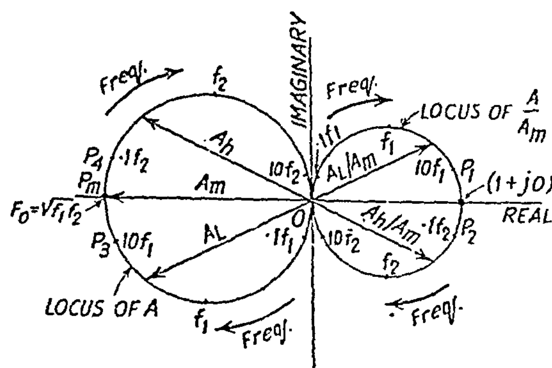


Fig. 12·15. Locus of relative gain A/A_m and complex voltage gain A of R. C. coupled amplifier on complex plane.

In the above construction locus of tip of vector A/A_m has been drawn first following Eqs (12·17) and (12·24) and subsequently locus of complex voltage gain A has been drawn by multiplying the locus of A/A_m by quantity $-g_m.R_{plg}$. This indirect procedure has been followed simply to facilitate understanding. We could

have drawn the locus of the tip vector A directly using the Eqs. (12·17a) and (12·24 a) for low and high frequency ranges respectively.

Inductance Capacitance Coupled Amplifier

It differs from R.C. coupled amplifier in that an inductor instead of a resistor, is used as the load. Coupling is, however, still done through a coupling condenser. The advantage gained is that no d c. voltage drop in the load impedance takes place and the corresponding power loss is avoided. The plate circuit efficiency is thus increased. Fig 12·16 shows one stage of inductance-capacitance coupled triode amplifier.

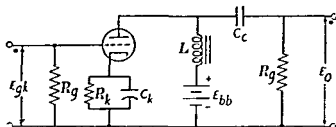


Fig. 12·16. One stage of Inductance-Capacitance coupled triode amplifier.

Another advantage obtainable in inductance-capacitance coupled amplifier is the large voltage gain in the middle frequency range resulting from the fact that the inductive reactance may be made very large. But there exists the disadvantage that in the low frequency range voltage gain falls more rapidly because of rapid reduction in the load impedance with frequency. This results in higher value of lower half power frequency. There is additional disadvantage of higher weight and cost of the inductor. Thus no special advantage is obtained by the use of inductance-capacitance coupling. If at all a resistive load is to be avoided, it is preferred to use a transformer coupling in between the stages of a cascade amplifier. Transformer coupling has the advantage over inductance-capacitance coupling of the facility of impedance matching and higher gain due to step up ratio of the transformer. Accordingly I.C. coupled amplifiers are rarely used for interstage coupling. The gain of I. C coupled amplifier is given by Eq. (12 30) with the alterations that Y_l now represents admittance of the inductor.

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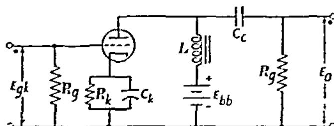


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impedance takes place therefore. This has an additional advantage that one may touch the load without any danger of electrical shock. Further d.c. magnetic saturation is avoided whenever load happens

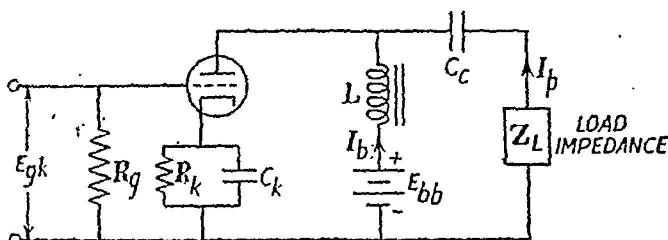


Fig. 12·17. Shunt fed amplifier

to be such a device. Reactance of inductance L is kept large and that of coupling condenser C_c is made small so that almost entire a. c. plate current flows through the load impedance.

UNTUNED TRANSFORMER COUPLED AMPLIFIER

Untuned transformer coupling is used at audio frequencies and uses iron-cored transformer to increase the permeability and hence the inductance. Such transformer coupling may be used fo one of the following three uses :—

(*i*) to couple the output of one stage of a cascade amplifier to the input of the next stage.

(*ii*) to couple the output power of a power amplifier to the load.

and (*iii*) to couple the output of a microphone to the input of the audio amplifier.

Accordingly these transformers are termed respectively as : (*i*) interstage transformer (*ii*) output transformer and (*iii*) input transformer.

Interstage Transformer Coupled Amplifier.—Fig. 12·18 shows basic circuit of a transformer-coupled amplifier using interstage transformer.

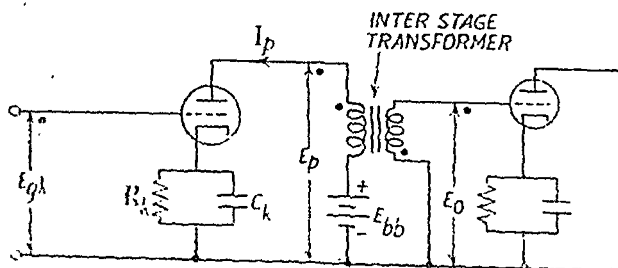


Fig. 12·18. Circuit of Transformer coupled amplifier.

The interstage transformer serves three purposes : (*i*) it isolates the d.c. components of voltages and currents in the adjacent stages (*ii*) it couples the a c. output of one stage to the input of the next and (*iii*) it provides simple means for either stepping up or stepping down voltages, currents and impedances.

Such iron cored transformer may be used only at audio frequencies since eddy current and hysteresis losses in the core are excessive at higher frequencies. At audio frequencies, a high permeability material (like stalloy) is used for core and the coupling between primary and secondary is very tight The co-efficient of [illegible] rs of primary [illegible] and on these [illegible] by a number of layers of secondary winding The ratio of secondary a c. voltage to primary a c. voltage is equal to secondary to primary turns ratio.

[illegible] Input [illegible] s stage. [illegible] er with A_1 operation is extremely large so that stray and distributed capacitances in transformer are not negligible and govern the frequency characteristics of the amplifier.

Transformer coupled amplifier has the following advantages over R. C. coupled amplifier :—(*i*) because of step up ratio of transformer it allows voltage gain greater than amplification factor of the tube (*ii*) there is very small d.c. voltage drop in the primary of the transformer so that for a given plate supply voltage transformer coupled amplifier provides larger amplitude of a.c. voltage across the primary as compared with the R C. coupled amplifier. (*iii*) centre tapped secondary allows it to provide pushpull input to the next stage if necessary. With R C. coupling, an additional stage called "phase-invertor" is necessary if the next stage is a pushpull amplifier.

The main disadvantages are : (*i*) higher cost of transformer coupled amplifier and (*ii*) stray magnetic field linkage.

Voltage gain of inter stage transformer coupled untuned amplifier. A. C. equivalent circuit is shown in Fig. 12 19.

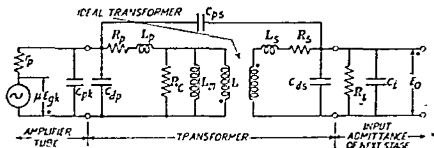


Fig. 12·19. A. C. equivalent circuit of inter-stage transformer coupled untuned amplifier.

In the above a.c. equivalent circuit, the various symbols are,

R_p, R_s—effective resistance of primary and secondary windings respectively.

L_p, L_s—primary and secondary leakage inductances respectively.

R_c—Shunt resistance corresponding to core loss.

L_m—magnetising inductance of primary.

C_{dp}, C_{ds}—primary and secondary distributed capacitance respectively.

C_{ps}—interwinding capacitance.

N_s/N_p—secondary to primary turns ratio.

R_i, C_i—input resistance and capacitance respectively of next stage.

Solution of a.c. equivalent circuit of Fig. 12·19 is quite complex and is usually not justified. Hence approximate analysis is made making the following simplifying assumptions :—

(1) Reactance of C_{dp} and C_{pk} are generally large compared with the plate resistance of the tube and hence neglected.

(2) Coefficient of coupling is large and hence primary leakage inductance L_p is much smaller than the primary incremental magnetising inductance L_m. L_p may, therefore, be shifted to the right of the parallel combination of L_m and R_c without causing appreciable error.

(3) Interwinding capacitance C_{ps} may be replaced by an equivalent shunt capacitance in either the primary or secondary circuit. When transferred to the secondary side it becomes $C_{ps}\left(\frac{a\mp 1}{a}\right)$ where a is the secondary to primary turns ratio N_s/N_p. The psoitive sign applies for the winding direction shown in Fig. 12·19 while negative sign applies for the reverse winding direction. It may be seen that such a condenser connected across the secondary results in approximately the same current at the secondary terminals as occurs with C_{ps} itself.

(4) In a well designed transformer R_c is usually large compared with effective resistance R_p of the transformer primary or dynamic plate resistance r_p of the triode and hence R_c may be omitted without causing much error.

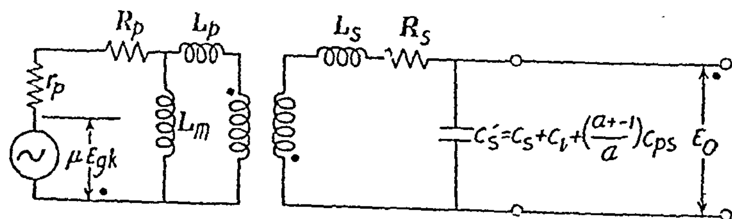


Fig. 12·20. Simplified a.c. equivalent circuit of transformer coupled amplifier.

(5) R_i in shunt with the secondary is usually large and hence omitted.

With the above simplifying assumptions, the a.c. equivalent circuit of Fig. 12·19 reduces to one shown in Fig. 12·20.

To simplify the equivalent circuit further, all the secondary circuit elements are referred to the primary. R_s, L_s and C'_s when transferred from secondary to primary become respectively $R_s\left(\frac{N_p}{N_s}\right)^2$, $L_s\left(\frac{N_p}{N_s}\right)$ and $C'_s\left(\frac{N_s}{N_p}\right)^2$. These may then be added to the ... present on the primary resulting ... of Fig. 12·21. Let these total ...

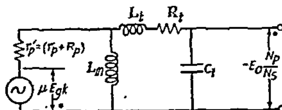


Fig. 12·21. Final a.c. equivalent circuit of transformer coupled amplifier

Then
$$R_t = R_s\left(\frac{N_p}{N_s}\right)^2 \quad .. \quad (12·25)$$

$$L_t = L_p + L_s\left(\frac{N_p}{N_s}\right)^2 \quad .. \quad (12·26)$$

and
$$C_t = C'_s\left(\frac{N_s}{N_p}\right)^2 \quad .. \quad (12·27)$$

Further output voltage E_0 when transferred to the primary becomes $-E_0\frac{N_p}{N_s}$.

Typical values of parameters in the equivalent circuit for transformer coupled amplifier are :—

R_p=1000 ohms ; R_s=10,000 ohms, L_m=20 H,

L_t=0·2H and C_t=600 $\mu\mu F$. It may be noted that magnetising inductance L_m is much greater than total leakage inductance L_t referred to the primary. Ratio of L_m to L_t, in accordance with the above values, is 100 but it may be much larger for high quality better designed transformers.

Since there are two inductances namely L_t and L_m, in the circuit, there are two resonant frequencies (*i*) parallel resonant frequency of L_m and C_t. This is a very low frequency and at this resonant frequency reactance of L_t may be neglected (*ii*) series resonant frequency of L_t and C_t. This frequency is so high that the reactance of L_m at this frequency is exceedingly high amounting to almost an open circuit.

For the purpose of analysis, the entire audio frequency range is divided into 3 ranges namely low, middle and high frequency ranges as in the case of R.C. coupled amplifier.

(I) **Low Frequency Range.**—This frequency range lies much below the parallel resonant frequency of L_m and C_t. At such low frequencies, the reactance of C_t is so high that it virtually constitutes an open circuit and hence may be neglected. Hence negligible current flows through L_t and R_t. Thus the voltage drop in L_t and R_t may be neglected. Fig. 12·21 shows the approximate a.c. equivalent circuit for low frequency range.

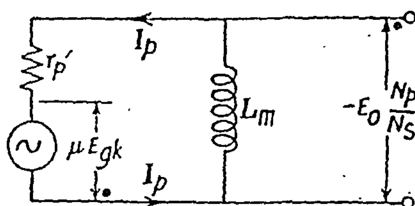


Fig. 12·21. Approximate a.c. equivalent circuit of transformer coupled amplifier in low frequency range.

Then plate current $\mathbf{I}_p = \dfrac{\mu\, \mathbf{E}_{gk}}{(r_p+R_p)+j\omega L_m}$

Voltage across $L_m = -j\omega L_m\, \mathbf{I}_p = -\dfrac{\mu\, \mathbf{E}_{gk}\, j\omega L_m}{(r_p+R_p)+j\omega L_m}$

This voltage must be equal to $-\mathbf{E}_0 \dfrac{N_p}{N_s}$

Then
$$-\mathbf{E}_0 \frac{N_p}{N_s} = \frac{-\mu\, \mathbf{E}_{gk}\, j\omega L_m}{r_p'+j\omega L_m} \quad \text{... (12·28)}$$

Hence complex voltage gain in the low frequency range is given by,

$$\mathbf{A}_l = \frac{\mathbf{E}_0}{\mathbf{E}_{gk}} = \mu \frac{N_s}{N_p} \cdot \frac{j\,\omega L_m}{r'_p+j\omega L_m} \quad \text{... (12·29)}$$

or
$$\mathbf{A}_l = \mu \frac{N_s}{N_p} \cdot \frac{1}{1-j\dfrac{r'_p}{\omega L_m}} \quad \text{... (12·30)}$$

Magnitude
$$A_l = \mu \frac{N_s}{N_p} \frac{1}{\sqrt{1+\left(\dfrac{r'_p}{\omega L_m}\right)^2}} \quad \text{... (12·31)}$$

and phase angle
$$\phi = \tan^{-1} \frac{r'_p}{\omega L_m} \quad \text{... (12·32)}$$

Let us define a frequency f_1 such that

$$2\pi f_1 L_m = r'_p \quad \text{... (12·33)}$$

Then $$A_l = \frac{\mu \dfrac{N_s}{N_p}}{1 - j f_1/f} \quad \text{... (12·34)}$$

Magnitude $$A_l = \frac{\mu \dfrac{N_s}{N_p}}{\sqrt{1 + \left(\dfrac{f_1}{f}\right)^2}} \quad \text{... (12·35)}$$

f_1 is obviously the lower half power frequency *i.e.* frequency at which voltage gain falls to ·707 or $3db$ below the midband gain of $\mu \dfrac{N_s}{N_p}$.

(II) **Middle Frequency Range.**—From Eq. (12·35) for voltage gain in the low frequency range it is seen that at lower half power frequency f_1 (*i.e.* when $\omega L_m = r'_p$) the voltage gain is $\dfrac{\mu N_s/N_p}{\sqrt{2}}$ *i.e.* $3\,db$ below the value $\mu \dfrac{N_s}{N_p}$. Middle frequency range begins from frequency f_1 upwards. At any frequency greater than f_1 (*i.e.*, $\omega L_m > 10\, r'_p$) the voltage gain from Eq. (12·35) may be seen to be constant at the value $\mu \dfrac{N_s}{N_p}$. Thus true middle frequency range begins at a frequency $10 f_1$ whereas $3db$ middle frequency range begins from frequency f_1. Thus we observe that in the true middle frequency range voltage gain is constant at value $\mu \dfrac{N_s}{N_p}$.

The same result may be obtained from the a.c. equivalent circuit for the middle frequency range. In the middle frequency range, frequency is large enough to make reactance ωL_m quite large so that L_m may be omitted in the equivalent circuit. At the same time the frequency is small enough to make the reactance of C_t very large so that C_t may also be omitted in the equivalent circuit. The a.c. equivalent circuit for middle frequency range is then as shown in Fig. 12·22.

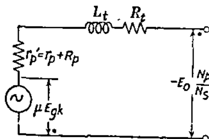


Fig. 12·22. A.C. equivalent circuit of transformer coupled amplifier in the middle frequency range.

From a.c. equivalent circuit of Fig. 12·22,

$$-\mathrm{E}_0 \frac{N_p}{N_s} = -\mu\, \mathrm{E}_{gk} \qquad \ldots (12·36)$$

Hence voltage gain in the middle frequency range is given by,

$$\mathrm{A}_m = \frac{\mathrm{E}_0}{\mathrm{E}_{gk}} = \mu\, \frac{N_s}{N_p} \qquad \ldots (12·37)$$

At the lower end of middle frequency range, the capacitance C_t resonates with inductance L_m so that the inpedance of parallel combination of L_m and C_t becomes very large. But this is of no great importance because already the reactances of L_m and C_t have been neglected.

As the frequency is increased, the shunt condenser C_t can no longer be ignored. It offers progressively reducing impedance and hence allows progressively increasing current to flow through it. Voltage drop, therefore, takes place in the series impedance elements r_p', L_t and R_t. As the frequency is increased the voltage gain, therefore, progressively falls in a manner similar to that in R.C. coupled amplifier in the high frequency range. Situation is, however, further complicated by series resonance between C_t and L_t. This reduces the impedance of combination L_t and C_t so that the driving current and output voltage increases tremendously. The effect is, however, felt in a narrow band of frequencies surrounding the resonant frequency. The band of frequencies effected in this manner depends upon the circuit Q or magnification factor. As a rough approximation we may say that at a frequency less than $\frac{f_2}{10}$, this effect is not felt. This frequency $\frac{f_2}{10}$ may then be taken as the upper limit of true middle frequency range.

(III) **High Frequency Range.**—In the high frequency range, reactance of L_m is very large and may be omitted in the a.c. equivalent circuit. The reactance of C_t is, however, small and hence C_t cannot be ignored in the high frequency range. Fig. 12·23 shows the a.c. equivalent circuit of transformer coupled amplifier in the high frequency range.

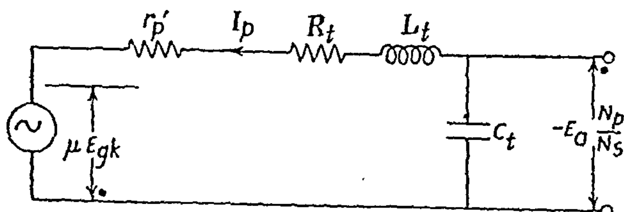


Fig. 12·23. A.C. equivalent circuit of transformer coupled amplifier in the high frequency range.

From the a. c. equivalent circuit of Fig. 12·23, plate current I_p is given by,

$$I_p = \frac{\mu E_{gk}}{(r'_p + R_t) + j\left(\omega L_t - \frac{1}{\omega C_t}\right)} \quad \ldots \quad \ldots \quad (12\cdot38)$$

This current flowing through C_t produces the voltage $-E_o \frac{N_p}{N_s}$.

Hence,

$$-E_o \frac{N_p}{S_s} = -\frac{\mu \, E_{gk} \frac{1}{j\omega C_t}}{(r'_p + R_t) + j\left(\omega L_t - \frac{1}{\omega C_t}\right)} \quad \ldots (12\cdot39)$$

Hence complex voltage gain in the high frequency range is given by,

$$A_h = \frac{E_0}{E_{gk}} = \mu \, \frac{N_s}{N_p} \cdot \frac{\frac{1}{j\omega C_t}}{(r'_p + R_t) + j\left(\omega L_t - \frac{1}{\omega C_t}\right)} \quad \ldots (12\cdot40)$$

From Eq. (12·40) we see that as the frequency is increased, voltage gain reduces because of reduction of reactance $\frac{1}{\omega C_t}$ and increase of reactance ωL_t. But, as has been said earlier, situation gets complicated by the occurrence of resonance between L_t and C_t. The frequency of this resonance is given by,

$$f_2 = \frac{1}{2\pi \sqrt{L_t \cdot C_t}} \quad \ldots (12\cdot41)$$

The value of voltage gain A_h at resonance as well as the frequency range in which resonance effect is prominent, depend upon the magnification factor Q_0 of resonant circuit. Q_0 is given by the relation,

$$Q_0 = \frac{\omega_2 \, L_t}{(r'_p + R_t)} \quad \ldots (12\cdot42)$$

where $\omega_2 = 2\pi f_2$.

The high frequency range is taken as starting from frequency $f_2/10$ upwards. The frequency response of interstage transformer coupled audio amplifier is given in Fig. 12·24. On the Y axis, relative voltage gain A/A_m in decibels is plotted. Relative frequency is plotted along the X-axis. For simplicity, frequency is expressed in terms of f_1 (lower half power frequency) in the low frequency range and in terms of f_2 (the resonant frequency of L_t and C_t) in the high frequency range.

It may be noted that true middle frequency range is given by the relation,

$$\frac{10 \, r'_p}{L_m} < \omega < \frac{1}{10\sqrt{L_t \cdot C_t}} \quad \ldots (12\cdot43)$$

In the high frequency range the circuit Q has to be carefully chosen to obtain almost constant voltage gain over as large frequency range in the high frequency region as possible. Excessively large resonance peak should be avoided. Typical value of Q_0 is 0·7 to 0·8.

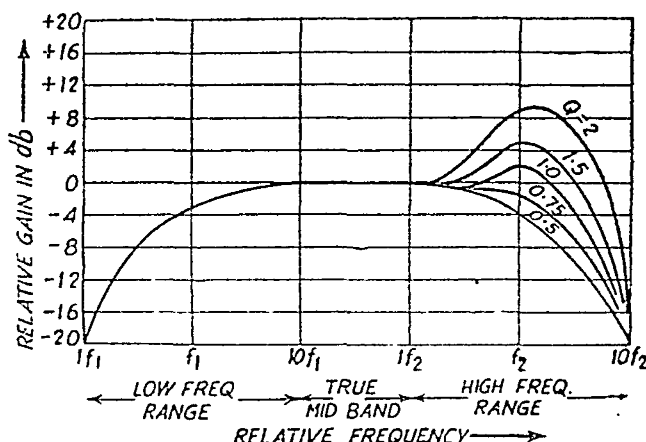


Fig. 12·24. Frequency response of transformer coupled amplifier.

Desirable features of the transformer :

(*i*) To extend the middle frequency range on the lower side, the incremental magnetising inductance L_m of the primary winding should be made larger.

(*ii*) The primary winding resistance R_p should be made smaller and so also plate resistance r_p of the tube.

(*iii*) To extend the middle frequency range on the upper side, f_2 may be made larger by decrease in the leakage inductance L_l, by decrease in the distributed capacitance of the winding and by decrease in the interwinding distributed capacitance.

But it may readily be seen that these requirements are contradictory. If L_m is increased by increase in the number of primary turns, then the number of secondary turns must also be increased in order to maintain the turns ratio constant. This increase in the number of turns results in an increase in the value of distributed capacitance and leakage inductance. If attempt is made to increase the turns ratio in order to increase the voltage gain of the amplifier, it results in decrease of L_m and increase of secondary leakage inductance and distributed capacitance C_s. Hence the middle frequency range decreases with the increase of turns ratio and this consideration, therefore, limits the maximum turns ratio. Typical value of turns ratio $\frac{N_s}{N_p}$ is three. Further in order to avoid an excessively large peak in the high frequency range, Q_o should not in general exceed

about 0·8. In accordance with Eq. (12·42) Q_o may be reduced by decreasing the leakage inductance L_l or by increasing the dynamic plate resistance r_p and the winding resistances R_p and R_s. But any increase in r_p and primary winding resistance R_p deteriorates the low frequency response. Hence the secondary winding resistance alone may be increased.

Because of high value of their r_p, pentodes are inferior to triodes in low frequency response. If pentode has to be used, a damping resistance should be put across the primary winding to reduce the effective value of plate resistance r_p.

Input-Transformer Coupling :

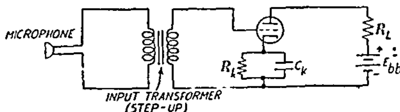


Fig. 12·25. Input Transformer Coupling

Fig. 12·25 shows the arrangement of feeding the output of a microphone to the input of an A_1 voltage amplifier through an input transformer which is a step up transformer. The transformer serves three purposes : (*i*) to couple the output of microphone to the input of the amplifier (*ii*) to provide d.c isolation between microphone circuit and amplifier input circuit and (*iii*) to effect impedance matching between low microphone impedance and comparatively high input impedance of A_1 amplifier used.

Example 14. *A transformer coupled cascade amplifier uses triode having amplification factor of 20 and dynamic plate resistance of 8000 ohms. Following particulars relate to the transformer : effective primary resistance $R_p=1000$ ohms ; effective secondary resistance $R_s=9900$ ohms ; incremental magnetising inductance of primary $=20$ henries ; leakage inductances referred to the primary$=0·2$ henry ; effective total capacitance across the primary$=600$ μμF and turns ratio $\frac{N_s}{N_p}=3$.*

Calculate (i) Midband gain, (ii) Lower half power frequency, (iii) Q_o of series resonance in the high frequency range, (iv) constant gain midband and (v) gain at frequency of 40 c/s.

Solution. Midband gain$=\mu \cdot \frac{N_s}{N_p}=20\times 3=60$

$$r'_p=R_p+r_p=8000+1000=9000 \text{ ohms.}$$

Let f_1 be the lower half-power frequency in c./s.

Then $\quad 2\pi f_1 L_m=r'_p$

so that $$f_1=\frac{r'_p}{2\pi L_m}=\frac{9000}{2\pi\times 20}$$
$$=71{\cdot}6 \text{ cycles/sec.}$$

Frequency of resonance of L_t and C_t is given by

$$f_2=\frac{1}{2\pi\sqrt{L_t.C_t}}=\frac{1}{2\pi\sqrt{0{\cdot}2\times 600\times 10^{-12}}}$$
$$=14{\cdot}5\times 10^3 \text{ c/s.}$$
$$Q_o=\frac{2\pi f_2 L_t}{(r'_p+R_t)}=\frac{2\pi\times 14{\cdot}5\times 10^3\times 0{\cdot}2}{\left[9000+\frac{9900}{(3)^2}\right]}$$
$$=1{\cdot}805$$

True midband $=(0{\cdot}1 f_2-10f_1)=(0{\cdot}1\times 14{\cdot}5\times 10^3-716)=734$ c/s.

Gain at a frequency of 40 c/s. is given by,

$$A_l=\frac{A_m}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}=\frac{60}{\sqrt{1+\left(\frac{71{\cdot}6}{40}\right)^2}}$$
$$=29{\cdot}3.$$

EXERCISES

1. An R.C. coupled amplifier uses pentode having amplification factor $\mu=100$ and dynamic plate resistance $r_p=500$ kilo-ohms. Load resistance$=100$ kilo-ohms, grid leak resistance of each stage $=500$ kilo-ohms, coupling condenser $C_c=0\ 1\ \mu F$, plate to cathode capacitance $C_{pk}=3\ \mu\mu F$, input capacitance$=5\ \mu\mu F$, stray and wiring capacitance $C_w=2\ \mu\mu F$. Calculate lower and upper half power frequencies, midband gain and gain-bandwidth product.

2. An R.C. coupled amplifier uses triode with amplification factor$=25$, anode slope resistance $r_p=10000$ ohms, load resistance $R_l=10000$ ohms, grid leak resistanee $R_g=100$ kilo-ohms, coupling condenser $C_c=0\ 05\ \mu F$ and total shunt capacitance $C_s=100\ \mu\mu F$. Calculate lower and upper half-power frequencies, midband gain and magnitudes and phase angles of complex voltage gain at angular frequencies of 10, 100, 10,000 and 100,000 radians/sec.

3. A resistance-capacitance coupled amplifier uses triodes having amplification factor of 20 and anode slope resistance of 8000 ohms. Each stage has load resistance of 12000 ohms, grid leak, resistance of 200 kilo-ohms and total shunt capacitance of $80\ \mu\mu F$. Calculate midband voltage amplification and upper cross-over frequency. Also calculate the values of coupling condenser to get lower cross-over frequencies of 10, 50 and 100 radians/sec. For coupling condenser of $0{\cdot}01\ \mu F$, calculate voltage gain in magnitude and phase angle at angular frequencies of 20, 200 and 200,000 radians/sec.

4. In a resistance-capacitance coupled amplifier pentodes used (ha)ve amplification factor=150 and dynamic plate resistance=150 (kil)o-ohms. Each stage uses load resistance of 20 kilo-ohms, grid (lea)k resistance of 250 kilo-ohms and coupling condenser of value (...)1 μF. Each stage has plate to cathode capacitance of 3 μμF, input (c)apacitance of next stage of 2 μμF and stray and wiring capacitance (o)f 5 μμF. Calculate for each stage voltage gain in the middle fre(q)uency range, lower and upper half power frequencies, gain band-width product and figure of merit of the tube used.

5. In a two-stage R.C coupled amplifier, the first stage uses a triode having amplification factor=20 and incremental plate resistance=20,000 ohms. First stage has load resistance=60 kilo-ohms and grid leak resistance of second stage=1 Meg ohm. Calculate the capacitance of the coupling condenser in order to have at a frequency of 20 c/s., voltage gain of first stage 2 db below the midband gain. With this value of coupling condenser find the magnitude and phase angle of voltage gain of first stage at a frequency of 10 cycles/sec.

6. A resistance capacitance coupled amplifier uses pentode having mutual conductance g_m = 10mA/V and very high anode slope resistance. Each stage has load resistance of 4000 ohms, grid leak resistance of 500 kilo-ohms and coupling condenser of capacity 0.01 micro-farad. Find the midband voltage gain, lower half power frequency and voltage gain at 10 c/s. Find the value of total shunt capacity which will result in voltage gain at 1 Mc/s equal to that at 10 c/s. Calculate also the gain bandwidth product with this value of total shunt capacitance.

7. A single stage of a resistance-capacitance coupled amplifier has half power frequencies of 3 c/s and 3 Mc/s. The pentode used has mutual conductance of 12 milli-mhos. Total shunt capacitance C_s is 18 μμF. Coupling capacitance is 0.02 μF. Calculate (i) plate load resistance (ii) grid leak resistance and (iii) midband gain.

8. A resistance-capacitance coupled pentode amplifier is to be used as a wideband amplifier. The total shunt capacitance in each stage is 12 μμF and grid leak resistance is 2 Meg-ohms. The phase shift at frequencies of $\frac{40}{2\pi}$ c/s and $\frac{20}{2\pi}$ meg cycles/sec is not to exceed 20 degrees. Calculate suitable values of load resistance and coupling condenser. The dynamic plate resistance of pentode is 1 Meg. ohm.

9. Draw the circuit diagram of a two-stage R.C couple(d) amplifier using triodes. Draw the voltage generator form of a.(c.) equivalent circuit for one stage of this amplifier. Using Millma(n's) theorem derive the following expression for the voltage gain of on(e) stage

where $\quad Y_p=\frac{1}{r_p}$, $Y_c=j\omega\ C_c$, $Y_g=\frac{1}{R_g}$, $Y_s=j\omega C_s$ and $X_l=\frac{1}{R_l}$

10. In a 3-stage R.C. coupled amplifier each stage is identical and uses a triode having amplification factor $\mu=20$, anode slope resistance $r_p=20$ kilo-ohms, load resistance $R_l=20$ kilo-ohms, grid leak resistance=500 kilo-ohms, coupling condenser=0·05 μF and total shunt capacitance $C_s=120$ $\mu\mu F$. Calculate (*i*) midband voltage gain, (*ii*) lower and upper half power frequencies, (*iii*) gain bandwidth product of each stage, (*iv*) overall half power frequencies and (*v*) overall gain bandwidth product. If stray and wiring capacitance equal the input and output capacitances of tubes, calculate the figure of merit of the tube.

11. A transformer coupled cascade amplifier uses triodes having amplification factor $\mu=15$ and plate resistance $r_p=10{,}000$ ohms. The parameters of the transformer are : effective primary resistance=900 ohms ; effective secondary resistance=9,000 ohms ; incremental magnetising inductance of primary=18 henries ; leakage inductances referred to the primary=0·2 henry ; effective total capacitances referred to the primary=640 $\mu\mu F$. Turns ratio $\frac{N_s}{N_p}=3$. Calculate (*i*) voltage gain in the middle frequency range, (*ii*) lower half power frequency, (*iii*) series resonant frequency, (*iv*) Q_0 at series resonance (*v*) constant gain middle frequency range and *vi*) voltage at a frequency of 30 cycles/second.

CHAPTER XIII

FEEDBACK AMPLIFIERS

In a feedback amplifier, a voltage proportional to the amplifier's [output] voltage or current, or a combination of the two, is fedback [an]d superimposed on the input signal voltage.

This feedback amplifier may be of one of the following types :—

(i) *Positive or regenerative feedback amplifier* :—Here the feedback voltage adds to the input signal voltage and hence increases the amplification of the amplifier.

(ii) *Negative, inverse or degenerative feedback* .—Here the feedback voltage subtracts from the input signal voltage and hence decreases the amplification of the amplifier.

General Theory of vacuum tube feedback amplifiers. All methods of feedback in vacuum tube amplifiers involve feedback of a "voltage" to the input terminals of some stage of amplifier. If this voltage fedback is proportional to the output voltage of the same or a succeeding stage by a factor which is independent of the magnitude of the load impedance, the feedback is called a "voltage feedback". On the othe hand, if the voltage feedback is proportional to the output current by a factor which is independent of the load impedance, the feedback is called a "current feedback" These two methods produce different overall behaviour of the amplifier in many respects

Fig. 13 1. shows schematic circuit of a voltage feedback amplifier. The feedback network returns a portion E_{fb} of the output voltage E_0 to the input This voltage E_{fb} gets added to the input signal voltage E_i to produce grid-to-cathode voltage E_{gk}

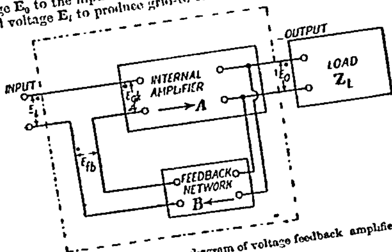


Fig. 13 1. Block schematic diagram of voltage feedback amplifier.

Complex voltage gain of the internal amplifier is given by

$$\mathbf{A}=\frac{\mathbf{E}_0}{\mathbf{E}_{gk}} \quad \ldots (13{\cdot}1)$$

Here $\mathbf{E}_{gk}$ is the input voltage to the internal amplifier. Transfer voltage ratio of the feedback network

$$=\beta=\frac{\mathbf{E}_{fb}}{\mathbf{E}_0} \quad \ldots (13{\cdot}2)$$

Hence $\quad \mathbf{E}_{fb}=\mathbf{A}\,\beta\,\mathbf{E}_{gk} \quad \ldots (13{\cdot}3)$

The quantity $\mathbf{A}\beta$ is called the "loop transmission", "loop gain", "feedback factor" or "return ratio". It is the factor by which the input voltage $\mathbf{E}_{gk}$ to the internal amplifier is multiplied in traversing through the amplifier and back around the loop through the feedback network to the input.

But $\quad \mathbf{E}_{gk}=\mathbf{E}_i+\mathbf{E}_{fb} \quad \ldots (13{\cdot}4)$

$$=\mathbf{E}_i+\mathbf{A}\beta\,\mathbf{E}_{gk}$$

so that $\quad \mathbf{E}_{gk}=\dfrac{\mathbf{E}_i}{1-\mathbf{A}\beta} \quad \ldots (13{\cdot}5)$

Hence voltage gain of the feedback amplifier is given by,

$$\mathbf{A}_{fb}=\frac{\mathbf{E}_0}{\mathbf{E}_i}=\frac{\mathbf{A}\,\mathbf{E}_{gk}}{\mathbf{E}_i}=\frac{\mathbf{A}}{1-\mathbf{A}\beta} \quad \ldots (13{\cdot}6)$$

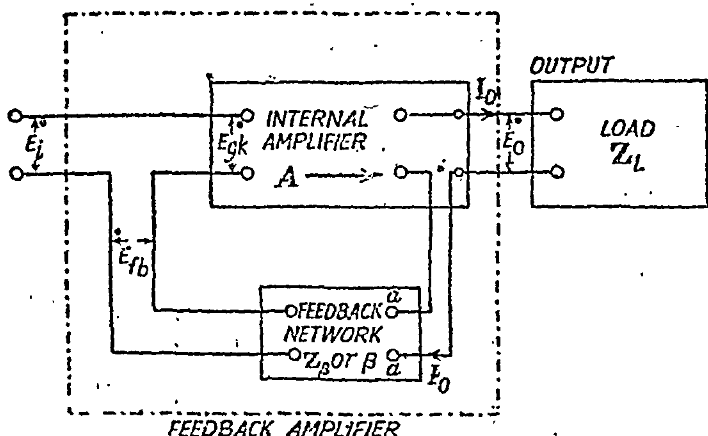


Fig. 13·2. Block schematic diagram of current feedback amplifier.

Fig. 13·2 shows the block schematic diagram of current feedback amplifier. In this case the output load current $\mathbf{I}_0$ flows through the feedback network and develops the feedback voltage $\mathbf{E}_{fb}$. The feedback network may then be represented by a complex transfer impedance $\mathbf{Z}_\beta$ defined by the relation,

$$\mathbf{Z}_\beta=\frac{\mathbf{E}_{fb}}{\mathbf{I}_0} \quad \ldots (13{\cdot}7)$$

Hence $$\beta=\frac{E_{fb}}{E_0}=\frac{E_{fb}}{I_0}\times\frac{I_0}{E_0}=\frac{Z_\beta}{Z_l} \qquad \text{... (13·8)}$$

where Z_l is the impedance of the load. For current feedback, ratio β is thus a function of the load impedance whereas for voltage feedback it is not. This dependence of β on Z_l distinguishes current ... a practical test ... r amplifier is to ... a short circuit ... ge feedback if E_{fb} ... is ... is

Significance of feedback equation. A number of significant inferences may be drawn from the feedback equation (13·6) given below

$$A_{fb}=\frac{A}{1-A\beta} \qquad \text{... (13·6)}$$

(i) If numerical value $|1-A\beta|>1$, A_{fb} is less than A and hence feedback is degenerative or inverse or negative.

(ii) If numerical value $|1-A\beta|<1$, A_{fb} is greater than A, and hence feedback is positive or regenerative.

(iii) If $A\beta=+1$, $1-A\beta=0$. Hence A_{fb} becomes infinite. ... the condition of the ... h zero input. The ... as an oscillator, *i.e.*, an electronic device for generating alternating voltage.

(iv) If $|A\beta|\gg 1$, $1-A\beta\approx -A\beta$

Hence $$A_{fb}\approx\frac{A}{-A\beta}=-\frac{1}{\beta} \qquad \text{... (13·9)}$$

Following inferences may be drawn from Eqn. (13·9) :

(*a*) The gain is inversely proportional to β, which means that the frequency response characteristic of such a feedback amplifier with high feedback factor ($A\beta\gg 1$) is inverse of the response characteristic of the feedback network. By proper design of the feedback network the amplifier may be made to have any desired frequency response. Further it is now possible, to correct for the non-uniform response o... by using ... feedback ... much simpler than the design of a network having inverse characteristics. Further, if feedback is obtained through ...

(*b*) If $|A\beta|\gg 1$, amplification is independent of the load impedance in the output of the amplifier provided that the load does

not form a part of the feedback network. However when β is made dependent upon load impedance, the amplification can be made to vary in a desired manner with load impedance.

(c) If $|A\beta| \gg 1$, the amplification is independent of A and hence the tube factors. The amplification is unaffected by variations of battery voltage or aging of tubes. Hence the gain stability of an amplifier is improved by feedback if the feedback factor is large.

If $|A\beta| \gg 1$, then $|1-A\beta| = |A\beta|$ ... (13·10)

If Aβ is put in form $A\beta = a + jb$ ... (13·11)

Then from Eqn. (13·10),

$$\sqrt{(1-a)^2+b^2} = \sqrt{a^2+b^2}$$

or $$a = \tfrac{1}{2}$$ ...(13·12)

Thus the amplification with feedback may also be made to depend only on β by making the real part of the complex feedback factor equal to 0·5.

Amount of feedback in decibels. Amount of feedback is sometimes expressed in decibels in accordance with the relation,

$$db \text{ of feedback} = 20 \log_{10} \frac{1}{|1-A\beta|}$$ ...(13·13)

FEEDBACK AMPLIFIER CIRCUITS

There are two types of feedback namely voltage feedback and current feedback. In either case, the feedback may be either positive or negative depending upon whether overall voltage gain has been increased or decreased by feedback. But in many cases the feedback circuit is so complex, that it is difficult to specify whether the feedback is voltage feedback or current feedback. The following two rules are, however, of considerable help in distinguishing the type of feedback :—

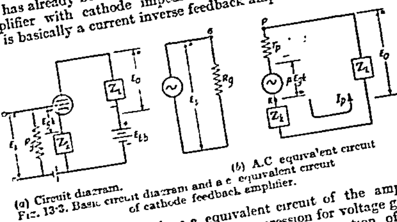
(i) In current feedback the ratio of feedback voltage to the load current is independent of the load impedance but the feedback ratio β is dependent on the load impedance and (ii) in voltage feedback, the feedback ratio β is independent of load impedance.

Hereunder we shall consider some of the common current feedback and voltage feedback circuits.

Current feedback circuits. Following two current feedback circuits are analysed here: (a) cathode feedback amplifier and (b) transformer coupled amplifier with current inverse feedback.

(a) **Cathode feedback amplifier.** Fig. 13·3 (a) shows the circuit diagram of cathode feedback amplifier. The a.c. plate current flowing through the cathode impedance Z_k produces an a.c. voltage drop which is superimposed on the input voltage E_i to constitute the voltage E_{gk}. Polarity of this voltage is such as to subtract from the signal voltage E_i and hence overall voltage gain is reduced. This circuit, therefore, provides current inverse feedback. This current

has already been studied in the last chapter treating it as plifier with cathode impedance. It may now be observed is basically a current inverse feedback amplifier.



(a) Circuit diagram. (b) A.C equivalent circuit

Fig. 13·3. Basic circuit diagram and a c equivalent circuit of cathode feedback amplifier.

Fig. 13 3 (b) shows the a c equivalent circuit of the amplifier. The circuit may be analysed to get expression for voltage gain by one of the following two methods (i) by direct application of feedback equation and (ii) by application of circuit theory.

First Method. From the a c equivalent circuit, a c. plate current I_p is given by,

$$I_p = \frac{\mu E_{gk}}{r_p + Z_l + Z_k} \quad \ldots(13{\cdot}14)$$

Hence output voltage E_0 is given by,

$$E_0 = -I_p . Z_l = \frac{-\mu E_{gk} . Z_l}{r_p + Z_l + Z_k} \quad \ldots(13\ 15)$$

Hence voltage gain of internal amplifier is given by,

$$A = \frac{E_0}{E_{gk}} = \frac{-\mu Z_l}{r_p + Z_l + Z_k} \quad \ldots(13{\cdot}16)$$

Again feedback voltage E_{fb} is given by,

$$E_{fb} = \frac{Z_k}{Z_l} \cdot E_0 \quad \ldots(13{\cdot}17)$$

Hence feedback ratio β is given by,

$$\beta = \frac{E_{fb}}{E_0} = \frac{Z_k}{Z_l} \quad \ldots(13{\cdot}18)$$

Hence overall voltage gain of the feedback amplifier is given by,

$$A_{fb} = \frac{A}{1 - A\beta} = \frac{\dfrac{-\mu Z_l}{r_p + Z_l + Z_k}}{1 - \dfrac{-\mu Z_l}{r_p + Z_l + Z_k} \times \dfrac{Z_k}{Z_l}}$$

or

$$A_{fb} = \frac{-\mu Z_l}{r_p + Z_l + (\mu + 1) Z_k} \quad \ldots(13{\cdot}19)$$

not form a part of the feedback network. However when β is made dependent upon load impedance, the amplification can be made to vary in a desired manner with load impedance.

(c) If $|A\beta| \gg 1$, the amplification is independent of **A** and hence the tube factors. The amplification is unaffected by variations of battery voltage or aging of tubes. Hence the gain stability of an amplifier is improved by feedback if the feedback factor is large.

If $|A\beta| \gg 1$, then $|1-A\beta| = |A\beta|$ ... (13·10)

If $A\beta$ is put in form $A\beta = a+jb$ ... (13·11)

Then from Eqn. (13·10),

$$\sqrt{(1-a)^2+b^2} = \sqrt{a^2+b^2}$$

or $$a=\tfrac{1}{2} \qquad \text{...(13·12)}$$

Thus the amplification with feedback may also be made to depend only on β by making the real part of the complex feedback factor equal to 0·5.

Amount of feedback in decibels. Amount of feedback is sometimes expressed in decibels in accordance with the relation,

$$db \text{ of feedback} = 20 \log_{10} \frac{1}{|1-A\beta|} \qquad \text{...(13·13)}$$

FEEDBACK AMPLIFIER CIRCUITS

There are two types of feedback namely voltage feedback and current feedback. In either case, the feedback may be either positive or negative depending upon whether overall voltage gain has been increased or decreased by feedback. But in many cases the feedback circuit is so complex, that it is difficult to specify whether the feedback is voltage feedback or current feedback. The following two rules are, however, of considerable help in distinguishing the type of feedback :—

(*i*) In current feedback the ratio of feedback voltage to the load current is independent of the load impedance but the feedback ratio β is dependent on the load impedance and (*ii*) in voltage feedback, the feedback ratio β is independent of load impedance.

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Second Method. Study of grid circuit reveals that,

$$\mathbf{E}_{gk}=\mathbf{E}_i-\mathbf{I}_p\,.\,\mathbf{Z}_k \quad ...(13\cdot20)$$

Substituting the value of $\mathbf{E}_{gk}$ from Eqn. (13·20) into Eqn. (13·14) we get,

$$\mathbf{I}_p=\frac{\mu\,[\mathbf{E}_i-\mathbf{I}_p\,.\,\mathbf{Z}_k]}{r_p+\mathbf{Z}_k+\mathbf{Z}_l} \quad ...(13\cdot21)$$

Rearranging Eqn. (13·21) we get,

$$\mathbf{I}_p=\frac{\mu\,\mathbf{E}_i}{r_p+\mathbf{Z}_l+(\mu+1)\mathbf{Z}_k} \quad ...(13\cdot22)$$

Hence overall voltage with feedback in given by,

$$\mathbf{A}_{fb}=\frac{\mathbf{E}_0}{\mathbf{E}_i}=\frac{-\mathbf{I}_p.\,\mathbf{Z}_l}{\mathbf{E}_i}$$

or

$$\mathbf{A}_{fb}=\frac{-\mu\,\mathbf{Z}_l}{r_p+\mathbf{Z}_l+(\mu+1)\mathbf{Z}_k} \quad ...(13\cdot23)$$

This Eqn. (13·23) for $\mathbf{A}_{fb}$ is the same as Eqn. (13·19) obtained according to the first method.

In may be observed that in most of the amplifiers self bias is used. This is achieved by using a parallel combination of a resistance R_k and condenser C_k in the cathode lead. The value of bypass condenser C_k is chosen to be so large that even at the lowest frequency contained in the input signal, the reactance of C_k is very small with the result that for a.c. operation impedance of this total cathode impedance $\mathbf{Z}_k$ may be neglected. No feedback is achieved in such a case. If, however, inverse current feedback is required, C_k is omitted from the circuit, leaving resistor R_k alone in the cathode lead. The feedback ratio β is equal to $\frac{R_k}{\mathbf{Z}_l}$ and the overall gain $\mathbf{A}_{fb}$ is given by expression (13·23) if $\mathbf{Z}_k$ is replaced by R_k.

From Eqn. (13·22) it is possible to draw a.c. equivalent circuit of Fig. 13·4 which an alternative form for the a.c. equivalent circuit of Fig. (13·3*b*).

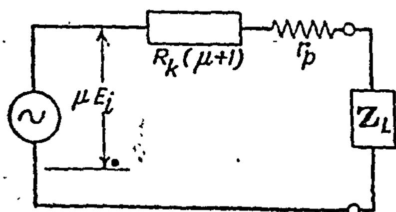


Fig. 13·4. Alternative form of a.c. equivalent circuit for cathode feedback amplifier.

It is obvious from Fig. 13·4 that the circuit consists of a voltage source $\mathbf{E}_{tf}=\mu\mathbf{E}_i$ with an internal impedance of $\mathbf{Z}_{tf}$ given by,

$$\mathbf{Z}_{tf}=r_p+(\mu+1)R_k \quad ...(13\cdot24)$$

Without feedback, the internal impedance $\mathbf{Z}_t$ is simply (r_p+R_k). Thus negative current feedback has increased the internal impedance by an amount μR_k. The ratio of internal impedances with and without feedback is given by,

$$\frac{\mathbf{Z}_{tf}}{\mathbf{Z}_t}=\frac{r_p+(\mu+1)R_k}{r_p+R_k}=1+\mu\frac{R_k}{r_p+R_k} \quad ...(13\cdot25)$$

(b) Transformer coupled amplifier with current inverse [illegible]

Fig. 13·5.

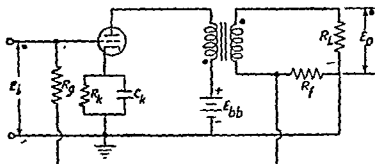


Fig. 13·5. Transformer coupled voltage amplifier with current inverse feedback

Voltage Feedback Circuit. Two circuits are described : (*i*) R.C. coupled amplifier with voltage inverse feedback and (*ii*) Transformer coupled amplifier with voltage inverse feedback.

(i) R.C. coupled amplifier with voltage inverse feedback.

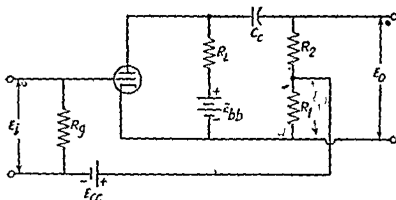


Fig. 13·6. R.C. coupled amplifier with voltage inverse feedback.

Fig. 13·6 shows an R.C. coupled amplifier with voltage inv feedback. Voltage developed across load resistance R_L is coupled coupling condenser C_c to shunt resistors R_1 and R_2. Voltage de loped across R_1 is fed to the input in such a way as to subtract fr the input voltage E_i. Feedback ratio β is given by the relation,

$$\beta=\frac{R_1}{R_1+R_2}$$

Let $\mathbf{Z}_l$ indicate the total load impedance *i.e.*, the impedance of load resistances R_l along with (R_1+R_2) and shunt capacitances. Reactance of coupling condenser C_c is assumed to be negligible.

Then the voltage gain of the internal amplifier is given by,

$$A=\frac{-\mu \mathbf{Z}_l}{r_p+\mathbf{Z}_l} \quad \text{... (13·27)}$$

Then overall voltage gain with feedback is given by,

$$A_{fb}=\frac{A}{1-A\beta}=\frac{-\mu\mathbf{Z}_l/(r_p+\mathbf{Z}_l)}{1+\dfrac{\mu\mathbf{Z}_l.\,\beta}{r_p+\mathbf{Z}_l}}$$

or

$$A_{fb}=\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l+\beta\mu\mathbf{Z}_l}$$

or

$$A_{fb}=\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l\ (1+\beta\mu)} \quad \text{...(13·28)}$$

Eqn. (13·28) may be put in the alternative form.

$$A_{fb}=\frac{\dfrac{-\mu}{1+\mu\beta}\mathbf{Z}_l}{\dfrac{r_p}{1+\mu\beta}+\mathbf{Z}_l}=\frac{-\mu'\mathbf{Z}_l}{r'_p+\mathbf{Z}_l} \quad \text{...(13·29)}$$

where $\mu'=\dfrac{\mu}{1+\mu\beta}$ and $r_p'=\dfrac{r_p}{1+\mu\beta}$ ...(13·30)

Equation (13·30) suggests that the amplifier may be represented by the a.c. equivalent circuit shown in Fig. 13·7. Evidently the circuit of Fig. 13·7 resembles a voltage source of open circuit voltage $\mathbf{E}_{lf}$ and internal impedance $\mathbf{Z}_{lf}$ where $\mathbf{E}_{lf}$ and $\mathbf{Z}_{lf}$ are given by,

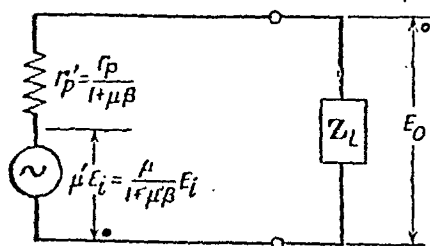


Fig. 13·7. Alternative form of a.c. equivalent for R.C. coupled amplifier with voltage inverse feedback.

$$\mathbf{E}_{lf}=\frac{\mu}{1+\mu\beta}\mathbf{E}_i \quad \text{...(13·31)}$$

and $$\mathbf{Z}_{lf}=\frac{r_p}{1+\mu\beta} \quad \text{...(13·32)}$$

But the internal impedance of the amplifier without feedback is given by,

$$\mathbf{Z}_i=r_p \quad \text{...(13·33)}$$

Thus the voltage inverse feedback reduces the internal impedance. The ratio of internal impedances with and without feedback is given by,

$$\frac{\mathbf{Z}_{lf}}{\mathbf{Z}_i}=\frac{1}{1+\mu\beta} \quad \text{...(13·34)}$$

The tube behaves as if it has an effective amplification factor $\mu' = \frac{\mu}{1+\mu\beta}$ and dynamic plate resistance $r'_p = \frac{r_p}{1+\mu\beta}$. Thus the effective amplification factor and dynamic plate resistance have been reduced by the factor $\frac{1}{1+\mu\beta}$.

In the R C. coupled amplifier circuit discussed above, reactance of coupling condenser C_c has been neglected and $\mathbf{Z}_l$ is taken as the total load impedance consisting of R_l, (R_1+R_2) and C_s in parallel. In general, resistance (R_1+R_2) is much larger than R_l and hence may be omitted. Again all shunt capacitances as may arise from interelectrode capacities, stray and wiring capacitance etc. may be neglected except at high frequencies. Hence in the normal frequency range not much error is caused by replacing impedance $\mathbf{Z}_l$ by load resistance R_l. At very low or at high frequencies, however, effects of coupling condenser C_c and shunt capacitance C_s become prominent and have to be considered in deriving the gain of internal amplifier. In that case no direct expression like Eqn. (13 28) may be used for calculating $A_{f\!b}$. General procedure then will be to calculate the gain of the internal amplifier i.e. the value of **A** for the relevant frequency using the usual formulae for R C coupled amplifier and then to use the feedback equation (13 6) to determine the voltage gain with feedback $A_{f\!b}$.

(ii) Transformer coupled amplifier with voltage inverse feedback. Fig 13·8 shows basic circuit of transformer coupled

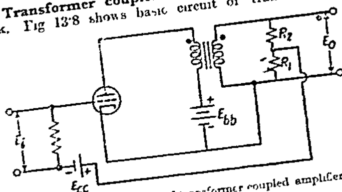


Fig. 13 8. Basic circuit of transformer coupled amplifier with voltage inverse feedback

voltage amplifier with voltage inverse feedback Fraction $\frac{R_1}{R_1+R_2}$ of the a.c. output voltage $\mathbf{E}_0$ is fed back to the input. The feedback is positive or negative depending upon the winding directions of the transformers. For the winding direction shown in Fig. 13·8, feedback is negative.

Compound Feedback. In certain feedback amplifier circuits both current and voltage feedbacks are used. Fig. 13 9 shows one stage of R C. coupled amplifier having compound feedback i.e. both current feedback and voltage feedback. A.C. plate current $\mathbf{I}_p$

Let $\mathbf{Z}_l$ indicate the total load impedance *i.e.*, the impedance of load resistances R_L along with (R_1+R_2) and shunt capacitances. Reactance of coupling condenser C_c is assumed to be negligible.

Then the voltage gain of the internal amplifier is given by,

$$A = \frac{-\mu \mathbf{Z}_l}{r_p + \mathbf{Z}_l} \quad \ldots (13{\cdot}27)$$

Then overall voltage gain with feedback is given by,

$$A_{fb} = \frac{A}{1 - A\beta} = \frac{-\mu \mathbf{Z}_l/(r_p+\mathbf{Z}_l)}{1 + \dfrac{\mu \mathbf{Z}_l . \beta}{r_p + \mathbf{Z}_l}}$$

or

$$A_{fb} = \frac{-\mu \mathbf{Z}_l}{r_p + \mathbf{Z}_l + \beta\mu \mathbf{Z}_l}$$

or

$$A_{fb} = \frac{-\mu \mathbf{Z}_l}{r_p + \mathbf{Z}_l\ (1+\beta\mu)} \quad \ldots (13{\cdot}28)$$

Eqn. (13·28) may be put in the alternative form.

$$A_{fb} = \frac{\dfrac{-\mu}{1+\mu\beta} \mathbf{Z}_l}{\dfrac{r_p}{1+\mu\beta} + \mathbf{Z}_l} = \frac{-\mu' \mathbf{Z}_l}{r'_p + \mathbf{Z}_l} \quad \ldots (13{\cdot}29)$$

where

$$\mu' = \frac{\mu}{1+\mu\beta} \text{ and } r_p' = \frac{r_p}{1+\mu\beta} \quad \ldots (13{\cdot}30)$$

Equation (13·30) suggests that the amplifier may be represented by the a.c. equivalent circuit shown in Fig. 13·7. Evidently the circuit of Fig. 13·7 resembles a voltage source of open circuit voltage $\mathbf{E}_{lf}$ and internal impedance $\mathbf{Z}_{lf}$ where $\mathbf{E}_{lf}$ and $\mathbf{Z}_{lf}$ are given by,

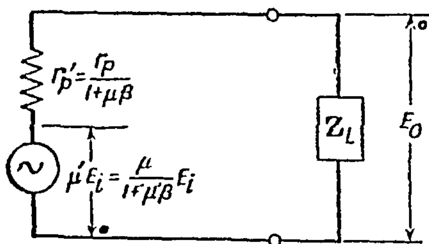


Fig. 13·7. Alternative form of a.c. equivalent for R.C. coupled amplifier with voltage inverse feedback.

$$\mathbf{E}_{lf} = \frac{\mu}{1+\mu\beta} \mathbf{E}_i \quad \ldots (13{\cdot}31)$$

$$\text{and } \mathbf{Z}_{lf} = \frac{r_p}{1+\mu\beta} \quad \ldots (13{\cdot}32)$$

But the internal impedance of the amplifier without feedback is given by,

$$\mathbf{Z}_l = r_p \quad \ldots (13{\cdot}33)$$

Thus the voltage inverse feedback reduces the internal impedance. The ratio of internal impedances with and without feedback is given by,

$$\frac{\mathbf{Z}_{lf}}{\mathbf{Z}_l} = \frac{1}{1+\mu\beta} \quad \ldots (13{\cdot}34)$$

The tube behaves as if it has an effective amplification factor of $\mu' = \frac{\mu}{1+\mu\beta}$ and dynamic plate resistance $r'_p = \frac{r_p}{1+\mu\beta}$. Thus both the effective amplification factor and dynamic plate resistance have been reduced by the factor $\frac{1}{1+\mu\beta}$.

In the R.C. coupled amplifier circuit discussed above, reactance of coupling condenser C_c has beer
the total load impedance consisting of
In general, resistance (R_1+R_2) is mu
may be omitted. Again all shunt capacitances as may arise from interelectrode capacities, stray and wiring capacitance etc. may be neglected except at high frequencies. Hence in the normal frequency
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be used for
calculating A_{f_b}. General procedure then will be to calculate the gain of the internal amplifier i.e. the value of A for the relevant frequency using the usual formulae for R.C. coupled amplifier and then to use the feedback equation (13.6) to determine the voltage gain with feedback A_{fb}.

***(ii)* Transformer coupled amplifier with voltage inverse feedback.** Fig. 13.8 shows basic circuit of transformer coupled

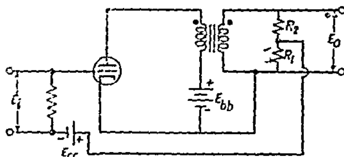


Fig. 13.8. Basic circuit of transformer coupled amplifier with voltage inverse feedback.

voltage amplifier with voltage inverse feedback. Fraction $\frac{R_1}{R_1+R_2}$ of the a.c. output voltage E_0 is fed back to the input. The feedback is positive or negative depending upon the winding directions of the transformers. For the winding direction shown in Fig. 13.8, feedback is negative.

Compound Feedback. In certain feedback amplifier circuits both current and voltage feedbacks are used. Fig. 13.9 shows one stage of R.C. coupled amplifier having compound feedback *i.e.* both current feedback and voltage feedback. A.C. plate current I_p

Let $\mathbf{Z}_l$ indicate the total load impedance *i.e.*, the impedance of load resistances R_l along with (R_1+R_2) and shunt capacitances. Reactance of coupling condenser C_c is assumed to be negligible.

Then the voltage gain of the internal amplifier is given by, ... (13·27)

$$\mathbf{A}=\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l}$$

Then overall voltage gain with feedback is given by,

$$\mathbf{A}_{fb}=\frac{\mathbf{A}}{1-\mathbf{A}\beta}=\frac{-\mu\mathbf{Z}_l/(r_p+\mathbf{Z}_l)}{1+\dfrac{\mu\mathbf{Z}_l.\beta}{r_p+\mathbf{Z}_l}}$$

or

$$\mathbf{A}_{fb}=\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l+\beta\mu\mathbf{Z}_l}$$

or

$$\mathbf{A}_{fb}=\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l\ (1+\beta\mu)} \quad ...(13·28)$$

Eqn. (13·28) may be put in the alternative form.

$$\mathbf{A}_{fb}=\frac{\dfrac{-\mu}{1+\mu\beta}\,\mathbf{Z}_l}{\dfrac{r_p}{1+\mu\beta}+\mathbf{Z}_l}=\frac{-\mu'\mathbf{Z}_l}{r'_p+\mathbf{Z}_l} \quad ...(13·29)$$

where

$$\mu'=\frac{\mu}{1+\mu\beta}\text{ and } r_p'=\frac{r_p}{1+\mu\beta} \quad ...(13·30)$$

Equation (13·30) suggests that the amplifier may be represented by the a.c. equivalent circuit shown in Fig. 13·7. Evidently the circuit of Fig. 13·7 resembles a voltage source of open circuit voltage $\mathbf{E}_{tf}$ and internal impedance $\mathbf{Z}_{tf}$ where $\mathbf{E}_{tf}$ and $\mathbf{Z}_{tf}$ are given by,

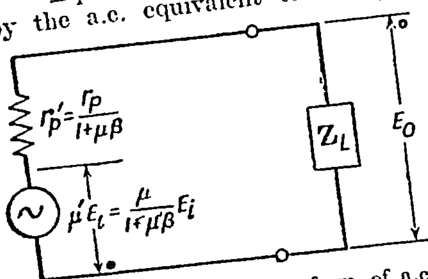


Fig. 13·7. Alternative form of a.c. equivalent for R.C. coupled amplifier with voltage inverse feedback.

$$\mathbf{E}_{tf}=\frac{\mu}{1+\mu\beta}\mathbf{E}_i \quad ...(13·31$$

$$\text{and } \mathbf{Z}_{tf}=\frac{r_p}{1+\mu\beta} \quad ...(13·3$$

But the internal impedance of the amplifier without feedback is given by,

$$\mathbf{Z}_t=r_p \quad ...(13$$

Thus the voltage inverse feedback reduces the internal impedance. The ratio of internal impedances with and without feedback is given by, ...(1

$$\frac{\mathbf{Z}_{tf}}{\mathbf{Z}_t}=\frac{1}{1+\mu\beta}$$

The tube behaves as if it has an effective amplification factor of $\mu' = \frac{\mu}{1+\mu\beta}$ and dynamic plate resistance $r'_p = \frac{r_p}{1+\mu\beta}$. Thus both the effective amplification factor and dynamic plate resistance have been reduced by the factor $\frac{1}{1+\mu\beta}$.

In the R C. coupled amplifier circuit discussed above, reactance of coupling condenser C_c has been neglected and Z_l is taken as the total load impedance consisting of R_l, (R_1+R_2) and C_s in parallel. In general, resistance (R_1+R_2) is much larger than R_l and hence may be omitted. Again all shunt capacitances as may arise from interelectrode capacities, stray and wiring capacitance etc. may be neglected except at high frequencies. Hence in the normal frequency [illegible] Z_l by load [illegible] ver, effects [illegible] prominent [illegible] amplifier. [illegible] be used for calculating A_{fb}. General procedure then will be to calculate the gain of the internal amplifier i e. the value of A for the relevant frequency using the usual formulae for R.C. coupled amplifier and then to use the feedback equation (13 6) to determine the voltage gain with feedback A_{fb}.

***(ii)* Transformer coupled amplifier with voltage inverse feedback.** Fig. 13·8 shows basic circuit of transformer coupled

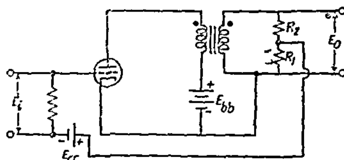


Fig. 13 8. Basic circuit of transformer coupled amplifier with voltage inverse feedback.

voltage amplifier with voltage inverse feedback Fraction $\frac{R_1}{R_1+R_2}$ of the a.c. output voltage E_0 is fed back to the input The feedback is positive or negative depending upon the winding directions of the transformers. For the winding direction shown in Fig. 13 8, feedback is negative.

Compound Feedback. In certain feedback amplifier circuits both current and voltage feedbacks are used. Fig 13 9 shows one stage of R C. coupled amplifier having compound feedback i.e both current feedback and voltage feedback. A.C. plate current I_p

flowing through cathode resistance R_k produces a voltage drop which gets added to the input voltage $\mathbf{E}_i$ in such a polarity as to reduce

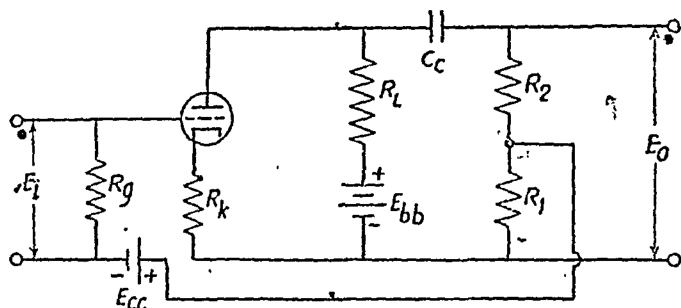


Fig. 13·9. R.C. coupled amplifier with compound feedback

it. This, therefore, provides current inverse feedback. Fraction $\frac{R_1}{R_1+R_2}$ of the a.c. output voltage $\mathbf{E}_0$ is fed back to the input to provide voltage inverse feedback.

The feedback ratio β is given by,

$$\beta=\beta_1+\beta_2=\frac{R_1}{R_1+R_2}+\frac{R_k}{\mathbf{Z}_l} \quad \text{...(13·35)}$$

where $\mathbf{Z}_l$ is the complete load impedance including R_l, (R_1+R_2) etc.

Resistance (R_1+R_2) is much greater than R_l so that in computation of voltage gain without voltage feedback, (R_1+R_2) may be neglected in shunt with load resistance R_l. Similarly reactance of coupling condenser C_c is small compared with (R_1+R_2) and hence may be neglected over normal frequency range. Then R_l may be used instead of $\mathbf{Z}_l$ in Eqn. (13·35).

Voltage gain without voltage feedback but with current feedback is given by,

$$\mathbf{A}_c=\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l+(\mu+1)R_k} \quad \text{...(13·36)}$$

Then voltage amplification with current feedback as well as voltage feedback is given by,

$$\mathbf{A}_{fb}=\frac{\mathbf{A}_c}{1-\mathbf{A}_c\beta_1}=\frac{\dfrac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l+(\mu+1)R_k}}{1-\beta\left[\dfrac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l+(\mu+1)R_k}\right]} \quad \text{...(13·37)}$$

or

$$\mathbf{A}_{fb}=\frac{-\mu\mathbf{Z}_l}{r_p+(\mu+1)R_k+(1+\mu\beta_1)\mathbf{Z}_l} \quad \text{...(13·38)}$$

Eqn. (13·38) may be put in the alternative form

$$A_{fb}=\frac{-\frac{\mu}{1+\mu\beta_1}Z_l}{\frac{r_p+(\mu+1)R_k}{1+\mu\beta_1}+Z_l} \quad ...(13\cdot39)$$

$$=\frac{-\mu'' Z_l}{r_p''+Z_l} \quad ...(13\cdot40)$$

where $$\mu''=\frac{\mu}{1+\mu\beta_1} \quad ...(13\cdot41)$$

and $$r''_p=\frac{r_p+(\mu+1)R_k}{1+\mu\beta_1} \quad ...(13\cdot42)$$

Eqn. (13·40) suggests that the circuit of Fig. 13·9 may be represented by the a.c. equivalent circuit of Fig. 13·10.

Obviously the circuit of Fig. 13 10 resembles a voltage source of open circuit voltage E_{if} and internal impedance Z_{if} given by

$$E_{if}=\frac{\mu}{1+\mu\beta_1}E_i \quad ...(13\cdot43)$$

and $$Z_{if}=\frac{r_p+(\mu+1)R_k}{1+\mu\beta_1} \quad ...(13\cdot44)$$

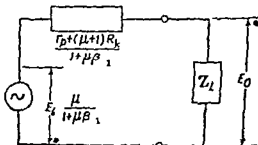


Fig. 13·10. A C. equivalent circuit of amplifier of Fig 13 9.

Current inverse feedback increases the value of internal impedance whereas *voltage* feedback reduces it. Thus Z_{if} may be less than or greater than Z_l depending upon relative effects of two types of feedback.

Feedback over several stages In several cases feedback covers ... tion. Let feedback cover n ... shown in line diagram of ... then the overall voltage gain A_{fb} is given by,

$$A_{fb}=\frac{A_1 A_2 ....A_n}{1-A_1 A_2 ...... A_n\beta} \quad ...(13\cdot45)$$

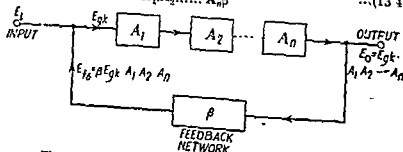


Fig. 13·11. Line diagram of multi-stage feedback amplifier.

For feedback to be negative, quantity $A_1.A_2......A_n\beta$ must be negative. Multi-stage feedback amplifiers may assume a large number of circuit configurations. A few simple cases are considered here.

(*i*) **Multi-stage current feedback amplifier.** Fig. 13·12 shows the circuit diagram of a two-stage amplifier in which case a voltage proportional to the output current is fed back to the input of

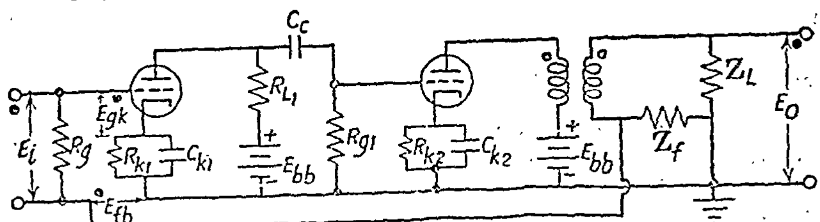


Fig. 13·12. Two-stage current inverse feedback amplifier.

the first stage in such a way as to subtract from the input causing inverse feedback.

Feedback ratio $\beta = \frac{-Z_f}{Z_l}$. The overall voltage gain with feed back is given by,

$$A_{fb} = \frac{A_1.A_2}{1 - A_1A_2\,\beta} \quad ...(13·46)$$

where A_1 and A_2 are the voltage amplifications of the two stages.

(*ii*) **Multi-stage voltage feedback amplifier** : Fig. 13·13 shows the circuit diagram of a three-stage R. C. coupled amplifier with voltage inverse feedback from the output of third stage to the input of first stage.

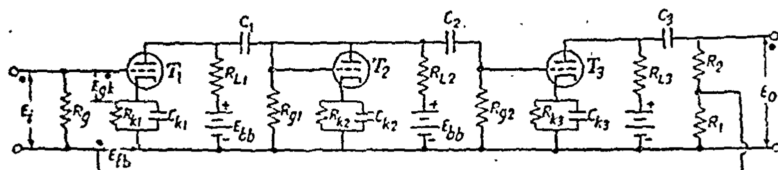


Fig. 13·13. Three-stage R.C. coupled amplifier with voltage inverse feedback.

Let the voltage gain of the three stages be A_1, A_2 and A_3. Each of these stages produces a phase-shift of 180 degrees in the middle frequency range and overall feedback ratio β is positive. Thus negative feedback is obtained. The overall voltage gain with feedback is given by,

$$A_{fb} = \frac{A_1.\,A_2.\,A_3}{1 - A_1.\,A_2.\,A_3\,\beta} \quad ...(13·47)$$

where $\quad \beta = \frac{R_1}{R_1 + R_2}$

Let $\mathbf{Z}_l$ be the total load impedance. This includes actual load resistance R_{l3}, shunt resistance (R_1+R_2) and interelectrode and other shunt capacitances. Then the voltage gain of the last stage is given by,

$$\mathbf{A}_2=\frac{-\mu\,\mathbf{Z}_l}{r_p+\mathbf{Z}_l} \quad \text{...(13·48)}$$

where μ and r_p are respectively the amplification factor and incremental plate resistance of the last stage.

Further let the voltage gain of all amplifier stages covered by feedback except the last one be $\mathbf{A}$ so that in the circuit of Fig. 13·13, $\mathbf{A}=\mathbf{A}_1\ \mathbf{A}_2$. Thus overall voltage gain with feedback is,

$$\mathbf{A}_{fb}=\frac{\mathbf{A}\times\left(\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l}\right)}{1-\mathbf{A}\left(\frac{-\mu\,\mathbf{Z}_l}{r_p+\mathbf{Z}_l}\right)\beta}$$

$$=\frac{-\mu\,\mathbf{Z}_l}{r_p+\mathbf{Z}_l\,(1+\mathbf{A}\mu\beta)} \quad \text{...(13·49)}$$

This Eqn. (13·49) applies to any multi-stage voltage feedback amplifier provided that $\mathbf{A}$ is the voltage gain of all the previous stages included in the feedback loop, β is the feedback ratio and $\mathbf{Z}_l$ is the load impedance of the last stage.

Eqn. (13·49) may be put in the alternative form,

$$\mathbf{A}_{fb}=\frac{\frac{-\mu}{1+\mathbf{A}\mu\beta}\mathbf{Z}_l}{\frac{r_p}{1+\mathbf{A}\mu\beta}+\mathbf{Z}_l} \quad \text{...(14·50)}$$

Eqn. (13·50) suggests that this feedback amplifier may be represented by a.c. equivalent circuit of Fig. 13·14.

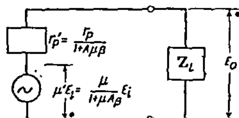


Fig. 13·14. A.C. equivalent circuit of multi-stage voltage feedback amplifier.

Evidently the circuit is of a voltage source $\mathbf{E}_{tf}$ and internal impedance $\mathbf{Z}_{tf}$ given by the relations,

$$\mathbf{E}_{tf}=\frac{\mu}{1+\mathbf{A}\mu\beta} \quad \text{...(13·51)}$$

and $\mathbf{Z}_{tf}=r'_p=\frac{r_p}{1+\mathbf{A}\mu\beta}$

Fig. 13·15 shows two- stage voltage inverse feedback amplifier in which the last stage is a transformer coupled amplifier,

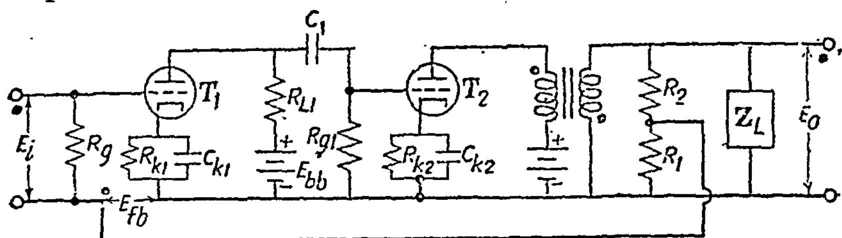


Fig. 13·15. Two-stage voltage inverse feedback amplifier.

The transformer introduces a phase shift of 180 degrees for the winding directions shown. Eqn. (13·45) gives the overall voltage gain.

Multi-stage compound feedback amplifier. A few typical multi-stage compound feedback amplifiers will be discussed here. Fig. 13·16 shows the circuit diagram of a three-stage R.C. coupled amplifier with current inverse feedback obtained by using a resistor R_k in the last stage and with voltage inverse feedback from the output of third stage to the input of the first stage.

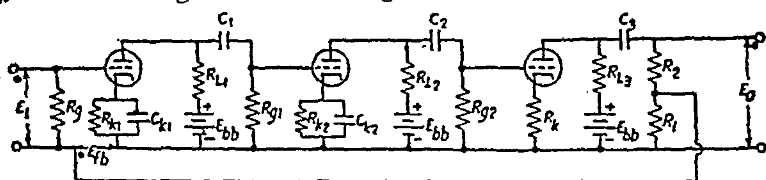


Fig. 13·16. Three-stage R.C. coupled amplifier with compound feedback.

Obviously this circuit is the same as that of Fig. 13·13 with bypass condenser C_k in the last stage removed to produce current inverse feedback.

Obviously overall voltage gain with feedback is given by,

$$A_{fb}=\frac{A_1.\ A_2.\ A_3}{1-A_1.\ A_2.\ A_3\beta} \qquad ...(13·53)$$

where $$\beta=\frac{R_1}{R_1+R_2} \qquad ...(13·54)$$

A_1 and A_2 are voltage gains of first and second stages and

$$A_3=\frac{-\mu Z_l}{r_p+(\mu+1)R_k+Z_l} \qquad ...(13·55)$$

where μ and r_p are amplification factor and dynamic plate resistance respectively of the last stage and Z_l is the total load impedance including R_{l3}, (R_1+R_2) and shunt capacitances.

Let $A_1\times A_2=A$ ...(13·56)

Then A_{fb} may be put as,

$$A_{fb}=\frac{A\left[\dfrac{-\mu\, Z_l}{r_p+(\mu+1)R_k+Z_l}\right]}{1-A\beta\left[\dfrac{-\mu\, Z_l}{r_p+(\mu+1)R_k+Z_l}\right]} \qquad ...(13·57)$$

or $$A_{f}=\frac{-A\mu Z_l}{r_p+(\mu+1)R_k+Z_l(1+A\beta\mu)} \quad ...(13\cdot58)$$

Eqn. (13·58) may be put in the alternative form below,

$$A_{f}=\frac{-A\dfrac{\mu}{1+A\beta\mu}Z_l}{\dfrac{r_p+(\mu+1)R_k}{1+A\beta\mu}+Z_l} \quad ...(13\cdot59)$$

From Eqn. (13·59), it is obvious that the effective internal impedance of this amplifier is given by,

$$Z_{if}=\frac{r_p+(\mu+1)R_k}{1+A\beta\mu} \quad ...(13\cdot60)$$

Fig. 13·17 shows the circuit diagram of a two stage R.C. coupled [illegible] ance R_1 provides current [illegible] togethers provide voltage [illegible]

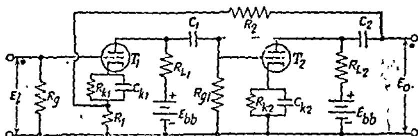


Fig. 13·17. Two-stage R.C. coupled amplifier with compound feedback.

Let A_1 and A_2 be the voltage gains of the first and second stages respectively and let β be the feedback ratio of voltage inverse feedback from the output last stage to the input of the first stage. Then overall voltage gain of the amplifier with compound feedback is given by,

$$A_{f}=\frac{A_1 A_2}{1-A_1 A_2\beta} \quad ...(13\cdot61)$$

where $$\beta=\frac{-R_1}{R_1+R_2} \quad ...(11\cdot62)$$

A_1=voltage gain of first stage with current feedback

$$=\frac{-\mu_1 Z_{l1}}{r_{p1}+(\mu_1+1)R_1+Z_{l1}} \quad ...(13\cdot63)$$

where μ_1 and r_{p1} are respectively amplification factor and dynamic plate resistance of first stage and Z_{l1} is the total load impedance of the first stage (assuming reactance of coupling condenser to be negligible)

and $$A_2=\frac{-\mu \,.\, Z_l}{r_p+Z_l} \quad ...(13\cdot64)$$

where μ and r_p refer to the second tube and Z_l is the total load impedance.

Fig. 13·15 shows two- stage voltage inverse feedback amplifier in which the last stage is a transformer coupled amplifier,

Fig. 13·15. Two-stage voltage inverse feedback amplifier.

The transformer introduces a phase shift of 180 degrees for the winding directions shown. Eqn. (13·45) gives the overall voltage gain.

Multi-stage compound feedback amplifier. A few typical multi-stage compound feedback amplifiers will be discussed here. Fig. 13·16 shows the circuit diagram of a three-stage R.C. coupled amplifier with current inverse feedback obtained by using a resistor R_k in the last stage and with voltage inverse feedback from the output of third stage to the input of the first stage. Obviously this circuit is the same as that of Fig. 13·13 with bypass condenser C_k in the last stage removed to produce current inverse feedback.

Fig. 13·16. Three-stage R.C. coupled amplifier with compound feedback.

Obviously overall voltage gain with feedback is given by,

$$A_{fb}=\frac{A_1.\ A_2.\ A_3}{1-A_1.\ A_2.\ A_3\beta} \quad ...(13·53)$$

where $$\beta=\frac{R_1}{R_1+R_2} \quad ...(13·54)$$

A_1 and A_2 are voltage gains of first and second stages and

$$A_3=\frac{-\mu Z_l}{r_p+(\mu+1)R_k+Z_l} \quad ...(13·55)$$

where μ and r_p are amplification factor and dynamic plate resistance respectively of the last stage and Z_l is the total load impedance including R_{l3}, (R_1+R_2) and shunt capacitances.

Let $A_1\times A_2=A$ ...(13·56)

Then A_{fb} may be put as,

$$A_{fb}=\frac{A\left[\dfrac{-\mu\ Z_l}{r_p+(\mu+1)R_k+Z_l}\right]}{1-A\beta\left[\dfrac{-\mu\ Z_l}{r_p+(\mu+1)R_k+Z_l}\right]} \quad ...(13·57)$$

or $$A_{fb} = \frac{-A\mu\, Z_l}{r_p + (\mu+1)R_k + Z_l(1+A\beta\mu)} \qquad ...(13\cdot58)$$

Eqn. (13·58) may be put in the alternative form below,

$$A_{fb} = \frac{-A\dfrac{\mu}{1+A\beta\mu}Z_l}{\dfrac{r_p+(\mu+1)R_k}{1+A\beta\mu} + Z_l} \qquad ...(13\cdot59)$$

From Eqn. (13·59), it is obvious that the effective internal impedance of this amplifier is given by,

$$Z_{if} = \frac{r_p + (\mu+1)R_k}{1+A\beta\mu} \qquad ...(13\cdot60)$$

Fig. 13·17 shows the circuit diagram of a two stage R C. coupled [illegible] ance R_1 provides current [illegible] togethers provide voltage [illegible]

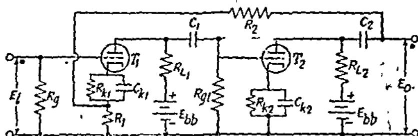


Fig. 13·17. Two-stage R.C. coupled amplifier with compound feedback.

Let A_1 and A_2 be the voltage gains of the first and second stages respectively and let β be the feedback ratio of voltage inverse feedback from the output last stage to the input of the first stage. Then overall voltage gain of the amplifier with compound feedback is given by,

$$A_{fb} = \frac{A_1 A_2}{1 - A_1 A_2 \beta} \qquad ...(13\cdot61)$$

where $$\beta = \frac{-R_1}{R_1 + R_2} \qquad ...(11\cdot62)$$

A_1 = voltage gain of first stage with current feedback

$$= \frac{-\mu_1 Z_{l1}}{r_{p1} + (\mu_1+1)R_1 + Z_{l1}} \qquad ...(13\cdot63)$$

where μ_1 and r_{p1} are respectively amplification factor and dynamic plate resistance of first stage and Z_{l1} is the total load impedance of the first stage (assuming reactance of coupling condenser to be negligible)

and $$A_2 = \frac{-\mu \,.\, Z_l}{r_p + Z_l} \qquad ...(13\,64)$$

where μ and r_p refer to the second tube and Z_l is the total load impedance.

Fig. 13·15 shows two- stage voltage inverse feedback amplifier in which the last stage is a transformer coupled amplifier,

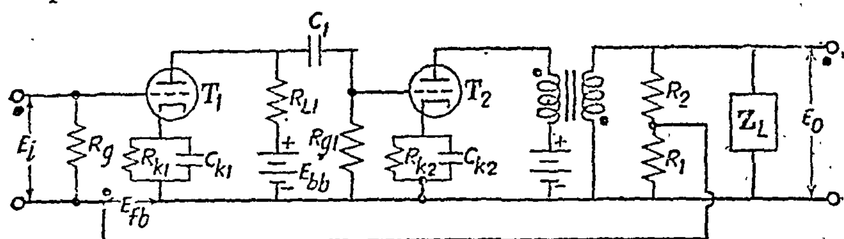


Fig. 13·15. Two-stage voltage inverse feedback amplifier.

The transformer introduces a phase shift of 180 degrees for the winding directions shown. Eqn. (13·45) gives the overall voltage gain.

Multi-stage compound feedback amplifier. A few typical multi-stage compound feedback amplifiers will be discussed here. Fig. 13·16 shows the circuit diagram of a three-stage R.C. coupled amplifier with current inverse feedback obtained by using a resistor R_k in the last stage and with voltage inverse feedback from the out-

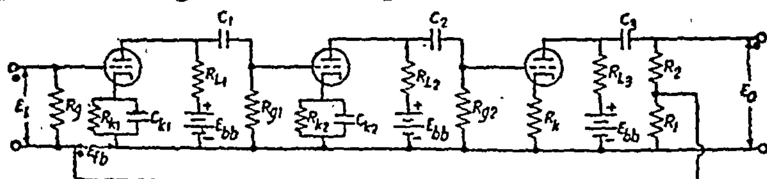


Fig. 13·16. Three-stage R.C. coupled amplifier with compound feedback.

put of third stage to the input of the first stage. Obviously this circuit is the same as that of Fig. 13·13 with bypass condenser C_k in the last stage removed to produce current inverse feedback.

Obviously overall voltage gain with feedback is given by,

$$\mathbf{A}_{fb}=\frac{\mathbf{A}_1.\,\mathbf{A}_2.\,\mathbf{A}_3}{1-\mathbf{A}_1.\,\mathbf{A}_2.\,\mathbf{A}_3\beta} \qquad \text{...(13·53)}$$

where
$$\beta=\frac{R_1}{R_1+R_2} \qquad \text{...(13·54)}$$

$\mathbf{A}_1$ and $\mathbf{A}_2$ are voltage gains of first and second stages and

$$\mathbf{A}_3=\frac{-\mu\mathbf{Z}_l}{r_p+(\mu+1)R_k+\mathbf{Z}_l} \qquad \text{...(13·55)}$$

where μ and r_p are amplification factor and dynamic plate resistance respectively of the last stage and $\mathbf{Z}_l$ is the total load impedance including R_{l3}, (R_1+R_2) and shunt capacitances.

Let $\mathbf{A}_1\times\mathbf{A}_2=\mathbf{A}$ ...(13·56)

Then $\mathbf{A}_{fb}$ may be put as,

$$\mathbf{A}_{fb}=\frac{\mathbf{A}\left[\dfrac{-\mu\,\mathbf{Z}_l}{r_p+(\mu+1)R_k+\mathbf{Z}_l}\right]}{1-\mathbf{A}\beta\left[\dfrac{-\mu\,\mathbf{Z}_l}{r_p+(\mu+1)R_k+\mathbf{Z}_l}\right]} \qquad \text{...(13·57)}$$

or $$A_{f} = \frac{-A\mu\, Z_l}{r_p+(\mu+1)R_k+Z_l(1+A\beta\mu)} \qquad \ldots(13{\cdot}58)$$

Eqn. (13·58) may be put in the alternative form below,

$$A_{f} = \frac{-A\dfrac{\mu}{1+A\beta\mu}Z_l}{\dfrac{r_p+(\mu+1)R_k}{1+A\beta\mu}+Z_l} \qquad \ldots(13{\cdot}59)$$

From Eqn. (13·59), it is obvious that the effective internal impedance of this amplifier is given by,

$$Z_{if} = \frac{r_p+(\mu+1)R_k}{1+A\beta\mu} \qquad \ldots(13{\cdot}60)$$

Fig. 13·17 shows the circuit diagram of a two stage R.C. coupled amplifier w... ance R_1 provides current inverse fee... togethers provide voltage inverse fee...

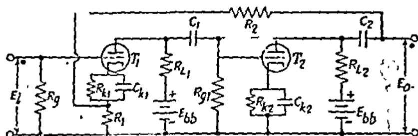


Fig. 13·17. Two-stage R.C. coupled amplifier with compound feedback.

Let A_1 and A_2 be the voltage gains of the first and second stages respectively and let β be the feedback ratio of voltage inverse feedback from the output last stage to the input of the first stage. Then overall voltage gain of the amplifier with compound feedback is given by,

$$A_{f} = \frac{A_1 A_2}{1-A_1 A_2 \beta} \qquad \ldots(13{\cdot}61)$$

where $$\beta = \frac{-R_1}{R_1+R_2} \qquad \ldots(11{\cdot}62)$$

A_1 = voltage gain of first stage with current feedback

$$= \frac{-\mu_1 Z_{l1}}{r_{p1}+(\mu_1+1)R_1+Z_{l1}} \qquad \ldots(13{\cdot}63)$$

where μ_1 and r_{p1} are respectively amplification factor and dynamic plate resistance of first stage and Z_{l1} is the total load impedance of the first stage (assuming reactance of coupling condenser to be negligible)

and $$A_2 = \frac{-\mu\, .\, Z_l}{r_p+Z_l} \qquad \ldots(13{\cdot}64)$$

where μ and r_p refer to the second tube and Z_l is load impedance.

Hence $$1000=\frac{8000}{1-8000\beta}$$

or $$1-8000\beta=8$$

Hence $$\beta=\frac{-7}{8000}=\cdot 000875.$$

Example 5. *An amplifier with negative feedback has voltage gain of 40. To produce a specified output the input voltage required without feedback is 0·1 volt whereas the input must be increased to 2·4 volts to produce the same output when feedback has been provided. Find the value of feedback ratio β and voltage gain without feedback.*

Solution. Let **A** be the voltage gain without feedback.

Then $$A=\frac{40\times 2\cdot 4}{0\cdot 1}=960$$

But $$A_{fb}=\frac{A}{1-A\beta}$$

Hence $$40=\frac{960}{1-960\beta}$$

or $$1-960\beta=\frac{960}{40}=24$$

Hence $$\beta=\frac{-23}{960}=-0\cdot 02395.$$

Example 6. *If the overall gain of a negative feedback amplifier is 66·02 db and the attenuation of the feedback path is 80 db, calculate the voltage gain of the amplifier without feedback.*

Solution. $$-80=20\log_{10}\beta$$

Hence $$\beta=\frac{1}{10^4}=0\cdot 0001$$

If feedback is to be negative, feedback factor **Aβ** must be negative. If voltage gain without feedback is taken as positive, β must be negative. Again voltage gain with feedback must also be positive.

Let A_{fb} indicate the voltage gain with feedback.

Hence $$66\cdot 02=20\log_{10}A_{fb}$$

Hence $$A_{fb}=2000.$$

Applying the feedback equation,

$$A_{fb}=\frac{A}{1-A\beta}$$

or $$2000=\frac{A}{1+0\cdot 0001\,A}$$

or $$A=2000+0\cdot 2A$$

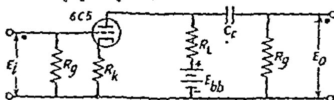
or $$0\cdot 8A=2000$$

Hence $$A=\frac{2000}{0\cdot 8}=2500.$$

Example 7. *Following values relate to the single stage R.C. coupled amplifier with current inverse feedback :*

$R_l = 20$ *kilo-ohms,* $R_g = 1$ *Meg-ohm.* $E_{bb} = 250$ *volts,* $\mu = 20$, $g_m = 2000\ \mu$ *mhos*

Reactance of coupling condenser C_c *may be neglected. Shunt capacitances may also be neglected. Calculate the value of cathode resistor* R_k *to produce voltage gain of* -6.



Solution. Let $\mathbf{Z}_l$ be the impedance of the parallel combination of R_g and R_l. Hence

$$\mathbf{Z}_l = \frac{R_g \cdot R_l}{R_g + R_l} = \frac{(1000 \times 10^3)(20 \times 10^3)}{(1000 + 20) \times 10^3}$$

$$= 19.6 \times 10^3 \text{ ohms.}$$

$$r_p = \frac{\mu}{g_m} = \frac{20}{2000 \times 10^{-6}} = 10^4 \text{ ohms.}$$

Voltage gain of this amplifier is given by,

$$\mathbf{A}_{fb} = \frac{-\mu\, Z_l}{r_p + (\mu + 1)R_k + Z_l}$$

Hence $$-6 = \frac{-20 \times 19.6 \times 10^3}{10^4 + (20 + 1)R_k + (19.6 \times 10^3)}$$

or $$21\ R_k + (29.6 \times 10^3) = \frac{20 \times 19.6 \times 10^3}{6} = 65.33 \times 10^3$$

Hence $$R_k = \frac{35.73 \times 10^3}{21} = 1.71 \times 10^3 \text{ ohms.}$$

Example 8. *For the amplifier shown, calculate the voltage gain for the following frequencies of the signal : 1, 10, 100, 1,000 and 10,000 c/s. What will be the voltage of this amplifier with fixed bias ?*

Solution. Voltage gain of the amplifier shown is given by,

$$\mathbf{A}_{fb} = \frac{-\mu\, \mathbf{Z}_l}{r_p + (\mu + 1)\mathbf{Z}_k + \mathbf{Z}_l}$$

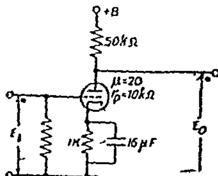
where $\mathbf{Z}_k$ is the impedance in the cathode circuit and $\mathbf{Z}_l$ is the load impedance.

For the given circuit,

$\mu = 20$,

$r_p = 10^4$ ohms

$\mathbf{Z}_l = 50 \times 10^3$ ohms



Hence $$1000=\frac{8000}{1-8000\beta}$$

or $$1-8000\beta=8$$

Hence $$\beta=\frac{-7}{8000}=\cdot000875.$$

Example 5. *An amplifier with negative feedback has voltage gain of 40. To produce a specified output the input voltage required without feedback is 0·1 volt whereas the input must be increased to 2·4 volts to produce the same output when feedback has been provided. Find the value of feedback ratio β and voltage gain without feedback.*

Solution. Let **A** be the voltage gain without feedback.

Then $$\mathbf{A}=\frac{40\times2\cdot4}{0\cdot1}=960$$

But $$\mathbf{A}_{fb}=\frac{\mathbf{A}}{1-\mathbf{A}\beta}$$

Hence $$40=\frac{960}{1-960\beta}$$

or $$1-960\beta=\frac{960}{40}=24$$

Hence $$\beta=\frac{-23}{960}=-0\cdot02395.$$

Example 6. *If the overall gain of a negative feedback amplifier is 66·02 db and the attenuation of the feedback path is 80 db, calculate the voltage gain of the amplifier without feedback.*

Solution. $$-80=20\log_{10}\beta$$

Hence $$\beta=\frac{1}{10^4}=0\cdot0001$$

If feedback is to be negative, feedback factor **A**β must be negative. If voltage gain without feedback is taken as positive, β must be negative. Again voltage gain with feedback must also be positive.

Let A_{fb} indicate the voltage gain with feedback.

Hence $$66\cdot02=20\log_{10}A_{fb}$$
Hence $$A_{fb}=2000.$$
Applying the feedback equation,

$$\mathbf{A}_{fb}=\frac{\mathbf{A}}{1-\mathbf{A}\beta}$$

or $$2000=\frac{\mathbf{A}}{1+0\cdot0001\,\mathbf{A}}$$

or $$\mathbf{A}=2000+0\cdot2A$$

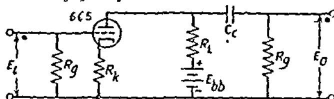
or $$0\cdot8A=2000$$

Hence $$A=\frac{2000}{0\cdot8}=2500.$$

Example 7. *Following values relate to the single stage R.C. coupled amplifier with current inverse feedback :*

R_l=20 kilo-ohms, R_g=1 Meg-ohm. E_{bb}=250 volts, μ=20, g_m=2000 μ mhos

Reactance of coupling condenser C_c may be neglected. Shunt capacitances may also be neglected. Calculate the value of cathode resistor R_k to produce voltage gain of −6.



Solution. Let Z_l be the impedance of the parallel combination of R_g and R_l. Hence

$$Z_l=\frac{R_g.\,R_l}{R_g+R_l}=\frac{(1000\times 10^3)(20\times 10^3)}{(1000+20)\times 10^3}$$

$$=19.6\times 10^3 \text{ ohms}$$

$$r_p=\frac{\mu}{g_m}=\frac{20}{2000\times 10^{-6}}=10^4 \text{ ohms.}$$

Voltage gain of this amplifier is given by,

$$A_{fb}=\frac{-\mu\, Z_l}{r_p+(\mu+1)R_k+Z_l}$$

Hence $\quad -6=\dfrac{-20\times 19.6\times 10^3}{10^4+(20+1)R_k+(19.6\times 10^3)}$

or $21\,R_k+(29.6\times 10^3)=\dfrac{20\times 19.6\times 10^3}{6}=65.33\times 10^3$

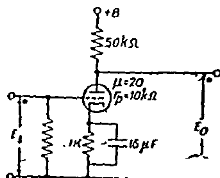
Hence $\quad R_k=\dfrac{35.73\times 10^3}{21}=1.71\times 10^3$ ohms.

Example 8. *For the amplifier shown, calculate the voltage gain for the following frequencies of the signal : 1, 10. 100, 1,000 and 10,000 c/s. What will be the voltage of this amplifier with fixed bias ?*

Solution. Voltage gain of the amplifier shown is given by,

$$A_{fb}=\frac{-\mu\, Z_l}{r_p+(\mu+1)Z_k+Z_l}$$

where Z_k is the impedance in the cathode circuit and Z_l is the load impedance.



For the given circuit,

$\mu=20$,

$r_p=10^4$ ohms

$Z_l=50\times 10^3$ ohms

$$\text{and } \mathbf{Z}_k=\frac{R_k\cdot\frac{1}{j\omega C_k}}{R_k+\frac{1}{j\omega C_k}}$$

$$=\frac{R_k}{1+j\omega C_kR_k}$$

$R_k=1000$ ohms and $C_k=16\ \mu F$.

At frequency of 1 c/s :

$$\mathbf{Z}_k=\frac{1000}{1+j\,2\pi\times1\times16\times10^{-6}\times1000}$$

$$=10^3[1-j\,0{\cdot}01]$$

$$\text{Hence}\qquad \mathbf{A}_{fb}=\frac{-20\times50\times10^3}{10^4+(50\times10^3)+(20+1)(1-j\cdot0{\cdot}01)\times10^3}$$

$$=\frac{-1000}{81-j\,0{\cdot}21}$$

$0{\cdot}21j$ may be neglected when compared with 81.

$$\text{Hence}\qquad \mathbf{A}_{fb}=\frac{-1000}{81}=-12{\cdot}35$$

At frequency of 10 c/s :

$$\mathbf{Z}_k=\frac{1000}{1+j\,0{\cdot}1}=990{\cdot}2[1-j\,0{\cdot}1]$$

$$\text{Hence}\qquad \mathbf{A}_{fb}=-\frac{-1000}{60+21\times{\cdot}9902[1-j\,0{\cdot}1]}=\frac{-1000}{80{\cdot}79-j\,2{\cdot}091}$$

Again $j\,2{\cdot}091$ may be neglected as compared with $80{\cdot}79$.

$$\text{Hence}\qquad \mathbf{A}_{fb}=-\frac{1000}{80{\cdot}79}=-12{\cdot}38$$

At frequency of 100 c/s :

$$\mathbf{Z}_k=\frac{1000}{1+j\,1{\cdot}006}=498{\cdot}4[1-j\,1{\cdot}006$$

$$\text{Hence}\qquad \mathbf{A}_{fb}=\frac{-1000}{60+21\times0{\cdot}4984[1-j\,1{\cdot}006]}$$

$$=\frac{-1000}{70{\cdot}43-j\,10{\cdot}50}$$

Numerical value

$$\mathbf{A}_{fb}=\frac{1000}{\sqrt{(70{\cdot}43)^2+(10{\cdot}50)^2}}=14{\cdot}05$$

At frequency of 1,000 c/s :

$$\mathbf{Z}_k=\frac{1000}{1+j\,10{\cdot}06}=9{\cdot}88[1-j\,10{\cdot}06]$$

Hence $\quad A_{fb}=\dfrac{-1000}{60+21\times 0{\cdot}88[1-j\ 10{\cdot}06]\times 10^{-3}}$

$$=\frac{-1000}{60{\cdot}2-j\ 2{\cdot}09}$$

or $\quad A_{fb}=\dfrac{1000}{\sqrt{(60{\cdot}2)^2+(2{\cdot}09)^2}}=16{\cdot}6.$

At frequency of 10,000 c/s .

$$Z_L=\frac{1000}{1+j\ 100{\cdot}6}=0{\cdot}988\ (1-j\ 100{\cdot}6)$$

Hence $\quad A_{fb}=\dfrac{-1000}{60+21\times 0{\cdot}0988\ (1-j\ 100{\cdot}6)\times 10^{-3}}$

$$=\frac{-1000}{60-j\ 0{\cdot}209}$$

or $\quad A_{fb}=\dfrac{1000}{\sqrt{(60)^2+(0{\cdot}209)^2}}=16{\cdot}67$

With fixed bias voltage gain is given by,

$$A=\frac{-\mu R_l}{r_p+R_l}=\frac{-20\times 50\times 10^3}{(10+50)\times 10^3}=-16{\cdot}67.$$

Example 9. *A valve has anode slope resistance of 7,500 ohms and amplification factor 20. It is used as an amplifier with anode load of 10,000 ohms and cathode bias resistor of 1,000 ohms. A signal of 2 mV r.m.s. is applied between grid and earth. No cathode bypass capacitor is used. Calculate the a.c. output voltage.*

Solution. Voltage gain with feedback through cathode resistor is given by,

$$A_{fb}=\frac{-\mu R_l}{r_p+(\mu+1)\ R_k+R_l}$$

$$=\frac{-20\times 10^4}{(7{\cdot}5\times 10^3)+(20+1)\times 10^3+10^4}$$

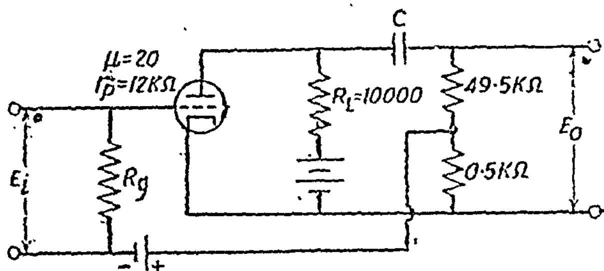
$$=\frac{-200}{38{\cdot}5}=-5{\cdot}194$$

Input voltage $=2\ mV$ r.m.s.

Hence a.c. output

$$=2\times 5{\cdot}194=10{\cdot}388\ mV \text{ r.m.s.}$$

Example 10. *In the circuit shown, calculate the voltage amplification of the amplifier with feedback and also the effective internal impedance of coupling*



Solution. Voltage gain with feedback is given by

$$A_{fb}=\frac{A}{1-A\beta}$$

where A=voltage gain without feedback

$$=\frac{-\mu Z_l}{r_p+Z_l}$$

where Z_l is impedance of R_l and 50 kΩ in parallel.

and β=feedback ratio

$$=\frac{0\cdot5}{0\cdot5+49\cdot5}=0\cdot01$$

Hence
$$A_{fb}=\frac{\dfrac{-\mu Z_l}{r_p+Z_l}}{1-\dfrac{-\mu Z_l}{r_p+Z_l}\beta}=\frac{-\mu Z_l}{r_p+Z_l(1+\mu\beta)}$$

$$Z_l=\frac{(10\times10^3)\times(50\times10^3)}{60\times10^3}=8\cdot333\times10^3 \text{ ohms}$$

$$A_{fb}=\frac{-20\times8\cdot333\times10^3}{(12\times10^3)+8\cdot333\times10^3\ (1+20\times\cdot01)}$$
$$=-7\cdot575.$$

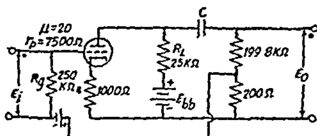
Expression for voltage gain with feedback may be put in the form,

$$A_{fb}=\frac{-\dfrac{\mu}{1+\mu\beta}Z_l}{\dfrac{r_p}{1+\mu\beta}+Z_l}$$

Hence effective internal impedance of the amplifier is given by,

$$Z_{if}=\frac{r_p}{1+\mu\beta}=\frac{12000}{1+20\times0\cdot01}=\frac{12000}{1\cdot2}$$
$$=10000 \text{ ohms.}$$

Example 11. *For the amplifier shown, calculate the voltage gain and effective internal impedance. Reactance of coupling condenser may be neglected.*



Solution. Feedback ratio

$$\beta=\frac{200}{200+(199\ 8\times 10^3)}=0{\cdot}001$$

Total load impedance $\mathbf{Z}_l$=parallel combination of 25 kΩ and 200 kΩ

$$=\frac{(25\times 10^3)\times(200\times 10^3)}{(200+25)\times 10^3}$$

$$=22\ 22\times 10^3 \text{ ohms.}$$

For the compound feedback shown overall voltage gain is given by

$$\mathbf{A}_{fb}=\frac{\mathbf{A}}{1-\mathbf{A}\beta}$$

where **A** *is the voltage of amplifier without voltage feedback.*

$$\mathbf{A}=\frac{-\mu\ \mathbf{Z}_l}{r_p+(\mu+1)\ R_k+\mathbf{Z}_L}$$

Hence
$$\mathbf{A}_{fb}=\frac{-\mu\ \mathbf{Z}_l}{r_p+(\mu+1)\ R_k+(1+\mu\beta)\ \mathbf{Z}_l} \quad ...(1)$$

$$=\frac{-20\times 22\ 22\times 10^3}{(7{\cdot}5\times 10^3)+(20+1)10^3+(1+20\times 0\ 001)22\ 2\times 10^3}$$

$$=-8{\cdot}68$$

Equation (1) may be put in the form

$$\mathbf{A}_{fb}=\frac{-\dfrac{\mu}{1+\mu\beta}\ \mathbf{Z}_l}{\dfrac{r_p+(\mu+1)\ R_k}{(1+\mu\beta)}+\mathbf{Z}_l} \quad ...(2)$$

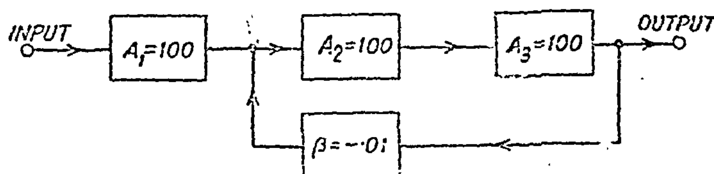
Equation (2) suggests that effective internal impedance of the amplifier is given by,

$$\mathbf{Z}_{if}=\frac{r_p+(\mu+1)\ R_k}{1+\mu\beta}=\frac{7500+(20+1)\ 1000}{1+20\times 0{\cdot}001}$$

$$=28000 \text{ ohms.}$$

Example 12. *An amplifier has three stages each with a gain of 100. Negative feedback is now applied by feeding $\frac{1}{100}$ of the output voltage to the input of the second stage. Find the overall voltage gain of the amplifier with feedback. Again if the negative feedback is applied to the input of the first stage, calculate the overall voltage gain with feedback.*

Solution.



A=voltage gain of last two stages without feedback

$=100\times100=10^4.$

For feedback to be negative feedback factor $A\beta$ must be negative.

Since $A=+10^4$, β must be negative. Obviously then

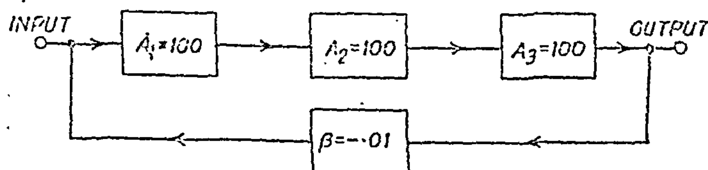
$\beta=-0{\cdot}01.$

Hence voltage of last two stages with feedback

$$=\frac{A}{1-A\beta}=\frac{10^4}{1+10^4\times0{\cdot}01}=99$$

Then overall gain of all three stages

$=99\times100=9900.$



Next feedback is made to cover all the three stages. Again feedback ratio is required to be negative to make $A\beta$ negative.

Hence $\beta=-0{\cdot}01.$

A=voltage gain of all the three stages without feedback $=100\times100\times100=10^6.$

Hence overall voltage of all the three stages with negative feedback

$$=\frac{A}{1-A\beta}=\frac{10^6}{1+10^6\times0{\cdot}01}=\frac{10^6}{10001}$$

$=99{\cdot}99.$

Example 13. *In a resistance capacitance coupled amplifier, the load resistance R_l is 100 kilo-ohms, grid leak resistance is 1 Meg-ohm, coupling condenser C_c is 0·01 micro-farad and total shunt capacitance is 500 μμF. The tube has amplification factor of 20 and dynamic plate resistance of 10 kilo-ohms. Calculate the voltage gain of the amplifier at frequencies of 10, 100 and 10,000 c/s.*

If negative feedback is provided by feeding $\frac{1}{20}$ of the output voltage back to the input, calculate the voltage gain at the above frequencies.

Solution. Voltage amplification of R.C. coupled amplifier in the low frequency range is given by the expression,

$$A_l = \frac{A_m}{1 - j\frac{f_1}{f}}$$

where

$$A_m = \text{midband gain} = -g_m R_{plg}$$

R_{plg} = parallel combination of r_p, R_l and R_g

f_1 = lower half power frequency

$$= \frac{1}{2\pi R'_{eq} C_c}$$

$$R'_{eq} = R_g + \frac{r_p R_l}{r_p + R_l}$$

$$= 10^6 + \frac{10^4 \times 10^6}{10^4 + 10^5} = 1009 \times 10^3 \text{ ohms.}$$

Hence

$$f_1 = \frac{1}{2\pi \times 1009 \times 10^3 \times 0{\cdot}01 \times 10^{-6}}$$

$$= 15{\cdot}78 \text{ c/s.}$$

$$R_{plg} = \frac{1}{r_p} + \frac{1}{R_l} + \frac{1}{R_g}$$

$$= \frac{1}{10^4} + \frac{1}{10^5} + \frac{1}{10^6} = \frac{1{\cdot}11}{10^4}$$

Hence

$$R_{plg} = \frac{10^4}{1{\cdot}11} = 9010 \text{ ohms}$$

$$g_m = \frac{\mu}{r_p} = \frac{20}{10^4} = 20 \times 10^{-4} \text{ mho}$$

$$A_m = -g_m \cdot R_{plg} = -20 \times 10^{-4} \times 9010 = -18{\cdot}02.$$

At frequency of 10 cycles/sec

Without feedback,

$$A_l = \frac{A_m}{1 - j\frac{f_1}{f}} = \frac{-18{\cdot}02}{1 - j\frac{15{\cdot}78}{10}} = \frac{-18{\cdot}02}{1 - j\,1{\cdot}578}$$

Numerical value

$$A_l = \frac{18{\cdot}02}{\sqrt{1 + (1{\cdot}578)^2}} = 9{\cdot}654.$$

With negative feedback, voltage gain is given by

$$A_{fb}=\frac{A_l}{1-A_l\beta}=\frac{\dfrac{-18\cdot02}{1-j\,1\cdot578}}{1+\dfrac{18\cdot02}{1-j\,1\cdot578}\times0\cdot05}$$

$$=\frac{-18\cdot02}{1\cdot901-j\,1\cdot578}$$

Numerical value

$$A_{fb}=\frac{18\cdot02}{\sqrt{(1\cdot901)^2+(1\cdot578)^2}}=7\cdot302.$$

At frequency of 100 cycles/sec :

$$A=\frac{-18\cdot02}{1-j\,\dfrac{15\cdot78}{100}}=\frac{-18\cdot02}{1-j\,0\cdot1578}$$

Numerical value

$$A=\frac{18\cdot02}{\sqrt{1+(0\cdot1578)^2}}=17\cdot8.$$

With negative feedback, voltage gain at 100 c/s is given by

$$A_{fb}=\frac{A}{1-A\beta}=\frac{\dfrac{-18\cdot02}{1-j\,0\cdot1578}}{1+\dfrac{18\cdot02}{1-j\,0\cdot1578}\times0\cdot05}$$

$$=\frac{-18\cdot02}{1\cdot901-j\,0\cdot1578}$$

Numerical value

$$A_{fb}=\frac{18\cdot02}{\sqrt{(1\cdot901)^2+(0\cdot1578)^2}}=9\cdot45.$$

Upper half power frequency

$$f_2=\frac{1}{2\pi\,C_sR_{plo}}=\frac{1}{2\pi\times500\times10^{-12}\times9010}$$

$$=35\cdot3\times10^3\text{ c/s.}$$

At frequency of 10 kc/s :

Voltage gain

$$A_h=\frac{A_m}{1+j\,\dfrac{f}{f_2}}=\frac{-18\cdot02}{1+j\,\dfrac{10\times10^3}{35\cdot3\times10^3}}$$

$$=\frac{-18\cdot02}{1+j\,0\cdot283}$$

Numerical value

$$A_h = \frac{18.02}{\sqrt{1+(0.283)^2}} = 17.3.$$

Voltage gain at 10 kc/s with voltage inverse feedback is given by,

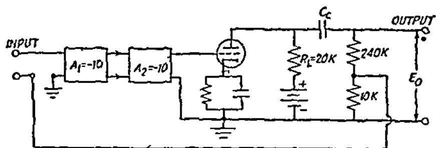
$$A_{fh} = \frac{A_h}{1-A_h\beta} = \frac{\dfrac{-18.02}{1+j\,0.283}}{1-\dfrac{-18.02}{1+j\,0.283}\times 0.05}$$

$$= \frac{-18.02}{1.901+j\,0.283}$$

Numerical value

$$A_{fh} = \frac{18.02}{\sqrt{(1.901)^2+(0.283)^2}} = 9.4.$$

Example 14. *In a three-stage R. C. coupled negative feedback amplifier, first two stages have a gain of -10 each at the operating frequency and the last stage uses a load resistance R_l of 20 kilo-ohms, potential divider of 250 kΩ as shown. Reactance of coupling condenser C_c may be neglected at the frequency. A fraction $\frac{1}{25}$ of the output voltage of last stage is feedback to the input of the first stage as shown in the circuit diagram.*



Tube has amplification factor of 20 and dynamic plate resistance of 10,000 ohms. Calculate the voltage amplification of the amplifier with and without feedback. If the voltage gain of the first two stages gets reduced to -9 each, find the overall gain of all three stages. Calculate also the effective internal impedance of the amplifier.

Solution. Let A_{12} represent the voltage gain of first two stages and A_3 be the gain of last stage.

Then $\qquad A_3 = \dfrac{-\mu Z_l}{r_p + Z_l}$

where $\qquad Z_l$ = the total impedance of last stage

and overall gain of amplifier with feedback is given by,

$$A_{fb}=\frac{A_{12}\,A_1}{1-A_{12}\,A_3\,\beta}=\frac{A_{12}\cdot\dfrac{-\mu Z_l}{r_p+Z_l}}{1+A_{12}\dfrac{\mu Z_l}{r_p+Z_l}\cdot\beta}$$

$$=\frac{-\mu Z_l\,A_{12}}{r_p+Z_l(1+\mu\beta A_{12})}$$

$$A_{12}=(-10)(-10)=+100$$

$$Z_l=\frac{(20\times10^3)(250\times10^3)}{(250+20)\,10^3}=18{\cdot}5\times10^3 \text{ ohms.}$$

$$\beta=0{\cdot}04$$

Hence
$$A_{fb}=\frac{-20\times18{\cdot}5\times10^3\times100}{10^4+18{\cdot}5\times10^3\,(1+20\times0{\cdot}04\times100)}$$

$$=-24{\cdot}66.$$

Also voltage gain of all the three stages without feedback is given by,

$$A=A_{12}\cdot\left[\frac{-\mu Z_l}{r_p+Z_l}\right]=\frac{-100\times20\times18{\cdot}5\times10^3}{10^4+(18{\cdot}5\times10^3)}$$

$$=-1298.$$

If gain of first two stages reduced to -9 each, then

$$A_{12}=(-9)(-9)=81$$

Hence voltage gain of all the three stages without feedback is given by $A=A_{12}\times A_3=81\times(-12{\cdot}98)=-1052$ and overall voltage gain with feedback is given by

$$A_{fb}=\frac{-\mu A_{12}\,.\,Z_l}{r_p+Z_l(1+\mu\beta A_{12})}$$

$$=\frac{-20\times81\times18{\cdot}5\times10^3}{10^4+18{\cdot}5\times10^3(1+20\times0{\cdot}04\times81)}$$

$$=-24{\cdot}4$$

Effective internal impedance of this feedback amplifier is given by

$$Z_{tf}=\frac{r_p}{1+\mu\beta A_{12}}$$

when $A_{12}=100$,

$$Z_{tf}=\frac{10^4}{1+20\times0{\cdot}04\times100}=\frac{10^4}{81}=12{\cdot}35 \text{ ohms.}$$

when $A_{12}=81$,

$$Z_{tf}=\frac{10^4}{1+20\times0{\cdot}04\times81}=\frac{10^4}{65{\cdot}8}=15{\cdot}2 \text{ ohms.}$$

Example 15. *A three-stage R.C. coupled amplifier has all the stages exactly identical. Each stage has load resistance of 20,000 ohms, coupling condenser of 0·01 micro-farad, and grid leak resistance of*

200 kilo-ohms. Tubes used have amplification factor of 20 and dynamic plate resistance of 8,000 ohms. A fraction $\frac{1}{100}$ of the output of the last stage is fed to the input of the first to provide negative feedback. Calculate the overall gain of the amplifier with and without feedback at frequencies of 10 and 100 cycles/second.

Solution. Gain of each stage in or close to low frequency range is given by,

$$A=\frac{-g_m R_{eq}}{1-j\frac{X_c}{R'_{eq}}}$$

$$g_m=\frac{\mu}{r_p}=\frac{20}{8000}=2.5\times10^{-3}\text{ mho.}$$

$$\frac{1}{R_{eq}}=\frac{1}{r_p}+\frac{1}{R_l}+\frac{1}{R_g}$$

$$=\frac{1}{8\times10^3}+\frac{1}{20\times10^3}+\frac{1}{200\times10^3}$$

$$=\frac{36}{200\times10^3}$$

Hence $\qquad R_{eq}=\frac{200\times10^3}{36}=5555$ ohms

$$R'_{eq}=R_g+\frac{r_p\,.\,R_l}{r_p+R_l}$$

$$=(200\times10^3)+\frac{(8\times10^3)(20\times10^3)}{(8+20)10^3}$$

$$=205.7\times10^3\text{ ohms.}$$

At frequency of 10 c/s :

$$X_c=\frac{1}{2\pi\times10\times0.01\times10^{-6}}=1.59\times10^6\text{ ohms.}$$

Hence voltage gain of each stage at 10 c/s is given by,

$$A=\frac{-(2.5\times10^{-3})\times5555}{1-j\frac{1.59\times10^6}{205.7\times10^3}}=\frac{-13.88}{1-j\,7.74}$$

Hence voltage gain of all the three stages without feedback

$$A_t=A\times A\times A=\frac{(-13.88)^3}{(1-j\,7.74)^3}=\frac{-26.74}{-1.79+j\,4.41}$$

$$\text{Numerical } A_t=\frac{26.74}{\sqrt{(1.79)^2+(4.41)^2}}=5.61$$

Since there are three stages in R.C. coupled amplifier and feedback is required to be negative, hence $\beta=+.01$. Voltage gain of

complete amplifier with feedback is given by,

$$A_{fb}=\frac{A_t}{1-A_t\beta}=\frac{\dfrac{-26\cdot74}{-1\cdot79+j\,4\cdot41}}{1-\dfrac{-26\cdot74}{-1\cdot79+j\,4\cdot41}\times0\cdot01}$$

$$=\frac{-26\cdot74}{-1\cdot523+j\,4\cdot41}$$

Numerical value

$$A_{fb}=\frac{26\cdot74}{\sqrt{(1\cdot523)^2+(4\cdot41)^2}}=5\cdot73$$

Note : We observe that the circuit is intended to provide negative feedback and it actually does provide negative feedback in the middle frequency range but at low frequencies such as the one under consideration, the feedback may become positive and increase the output voltage. This results because of excessive phase shift at frequencies far below lower half power frequency. At 10 c/s, the gain has actually increased from 5·61 to 5·73 due to feedback.

At frequency of 100 c/s:

Voltage gain of each stage is given by,

$$A=\frac{-13\cdot88}{1-j\,0\cdot774}$$

Hence overall gain without feedback is given by,

$$A_t=(A)^3=\frac{(-13\cdot88)^3}{(1-j\,0\cdot774)^3}=\frac{+2674}{0\cdot8+j\,1\cdot78}$$

Numerical value

$$=\frac{2674}{\sqrt{(0\cdot8)^2+(1\cdot78)^2}}=1400$$

Voltage gain of overall amplifier with feedback is given by,

$$A_{fb}=\frac{A_t}{1-A_t\beta}=\frac{\dfrac{2674}{0\cdot8+j\,1\cdot78}}{1-\dfrac{2674}{0\cdot8+j\,1\cdot78}\times0\cdot01}$$

$$=\frac{2674}{-25\cdot94+j\,1\cdot78}$$

Numerical value

$$A_{fb}=\frac{2674}{\sqrt{(25\cdot94)^2+(1\cdot78)^2}}=103$$

Note. We observe that with the same feedback arrangement, feedback is negative *i.e.* voltage is reduced at a frequency of 100 c/s.

Example 16. *In the two stages R. C. coupled amplifier circuit of Fig. 13·17, let $R_{11}=25\ k\Omega$, $R_{01}=250\ k\Omega$, $C_1=0\cdot1\ \mu F$, $R_{12}=50\ k\Omega$, $C_2=0\cdot1\ \mu F$, $R_2=249\ k\Omega$ and $R_1=1\ k\Omega$. Let amplification factor for*

the first and second tube be 20 and 50 respectively and dynamic plate resistance be 10 kΩ and 20 kΩ respectively. Assume R_k and C_k combinations to have negligible a.c. impedance. Calculate the overall voltage gain with feedback and also the effective internal impedance. Assume coupling condensers C_1 and C_2 to have negligible reactance at the operating frequency.

Solution. For the given circuit, voltage gain with feedback is given by,

$$A_{fb}=\frac{-\mu_2 A_1 Z_l}{r_{p2}+Z_l(1+A_1\mu_2\beta)}$$

where μ_2, r_2 refer to the second tube. Z_l is the load impedance of the second stage, β is the feedback ratio and A_1 is the voltage gain of the first stage with current feedback through R_1.

$$\text{Here } \beta=-\frac{R_1}{R_1+R_2}=-\frac{1}{249+1}=-\frac{1}{250}=-0{\cdot}004$$

$$\mu_2=50,\ r_{p2}=20\times10^3 \text{ ohms.}$$

$$Z_l=\frac{(50\times10^3)(250\times10^3)}{(50+250)\times10^3}=41{\cdot}66\times10^3 \text{ ohms.}$$

$$A_1=\frac{-\mu_1 Z_{l1}}{r_{p1}+(\mu_1+1)R_1+Z_{l1}}$$

Z_{l1}=parallel combination of R_{l1} and R_{g1}

$$=\frac{R_{l1}\times R_{g1}}{R_{l1}+R_{g1}}=\frac{(20\times10^3)(250\times10^3)}{(20+250)\times10^3}$$

$$=18{\cdot}5\times10^3 \text{ ohms}$$

Hence
$$A_1=\frac{-20\times18{\cdot}5\times10^3}{(10\times10^3)+(20+1)\times10^3+(18{\cdot}5\times10^3)}$$
$$=-7{\cdot}47$$

Hence
$$A_{fb}=\frac{-50\times(-7{\cdot}47)\times41{\cdot}66\times10^3}{(20\times10^3)+41{\cdot}66\times10^3[1+(-7{\cdot}47)(-0{\cdot}004)(50)]}$$
$$=125{\cdot}2$$

Effective internal impedance is given by,

$$Z_{of}=\frac{r_{p2}}{1+A_1\beta\mu_2}=\frac{20\times10^3}{1+(-7{\cdot}47)(-0{\cdot}004)\times50}$$
$$=8020 \text{ ohms.}$$

FREEBACK AMPLIFIERS CHARACTERISTICS

It was the positive feedback which was first thought of and

(4) *Reduction in noise* :—Let the internal amplifier be divided into two parts having voltage gains of $\mathbf{A}_1$ and $\mathbf{A}_2$ so that overall voltage gain without feedback is $\mathbf{A}=\mathbf{A}_1.\mathbf{A}_2$

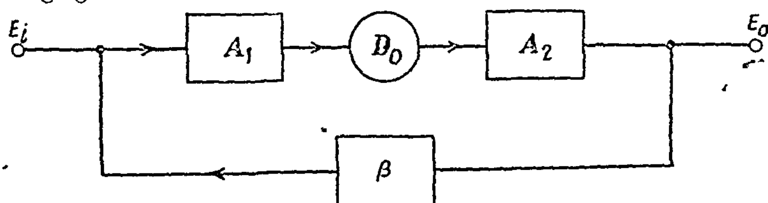


Fig. 13·18. Block schematic diagram showing reduction of noise by negative feedback.

Let $\mathbf{D}_o$ be the noise voltage generated within the internal amplifier. In the absence of feedback $\mathbf{D}_o$ gets amplified to $\mathbf{N}_o=\mathbf{A}_2.\mathbf{D}_o$ in the output. With negative feedback, let $\mathbf{N}'_o$ be the noise output. This noise output $\mathbf{N}'_o$ after passing through feedback network becomes $\beta\mathbf{N}'_o$ and again after amplification through $\mathbf{A}_1$ amplifier stage becomes $\beta\mathbf{N}'_o\,\mathbf{A}_1$

Hence $\qquad \mathbf{A}_2\times[\beta\mathbf{N}'_o\mathbf{A}_1+\mathbf{D}_o]=\mathbf{N}'_o$

or $\qquad \mathbf{A}_2 D_o+A\beta N'_o=N'_o$

or
$$N'_o=\frac{\mathbf{A}_2 D_o}{1-\mathbf{A}\beta}=\frac{\mathbf{N}_o}{1-\mathbf{A}\beta} \qquad \ldots(13\cdot72)$$

Thus we conclude that by negative feedback the noise output voltage due to noise generated within the amplifier, is reduced in the same proportion as the voltage gain *i.e.* by a factor $\dfrac{1}{1-\mathbf{A}\beta}$.

This seems to represent a real reduction in noise. However to obtain a specified output voltage the voltage gain of feedback amplifier has to be increased either by adjustment of circuit parameters or by the addition of amplifier stages. In the latter case, noise will also be amplified in the same proportion as signal. Additional amplifier stages will introduce additional noise. Then the overall noise of the amplifier with negative feedback may be higher than that in the amplifier without feedback. If, however, required additional gain is obtained by the readjustment of the circuit parameters, reduction in noise may take place.

(5) *Modification of input and output impedances* :—Effect of negative feedback on input and output impedances is discussed in the following articles.

Effective internal impedance with feedback

It has been seen earlier that current inverse feedback increases the effective internal impedance and voltage feedback decreases the effective internal impedance. A generalized treatment to the matter will be given now.

Following notations will be used :

β—Feedback ratio.

$\mathbf{A}$—Voltage gain without feedback, with load connected.

$\mathbf{A}_{fb}$—Voltage gain with feedback, with load connected.

A_t—Voltage gain without feedback, with load open circuited.
E_t—Effective internal potential source without feedback.
E_{tf}—Effective internal potential source with feedback.
Z_t—Effective internal impedance without feedback *i e.* the Thevenin's impedance of the equivalent network. It is the impedance found on looking back from the output terminals into the source with the load open circuited.
Z_{tf}—Effective internal impedance with feedback.
Z_o—Output terminal impedance without feedback
Z_{of}—Output terminal impedance with feedback
E_o—Output voltage
E_i—Input voltage to the amplifier.
E_{fb}—Feedback voltage.

Hereunder we will derive general expressions for Z_{tf} and E_{tf} for the two cases of voltage feedback and current feedback.

Effective internal impedance with voltage feedback. Fig. 13·19 shows block schematic diagram of voltage feedback amplifier. In

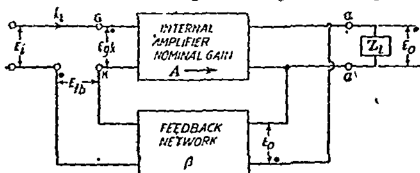


Fig. 13·19. Block schematic diagram of voltage feedback amplifier.

Fig. 13 19, it may be seen that feedback may be removed by removing the lead running from point *P* to feedback network and connecting it from *P* to the cathode *K*. Then the Thevenin's potential source equivalent of this circuit is given by the circuit of Fig. 13·20. In this equivalent circuit Z_t is the effective internal impedance without feedback and A_t is the gain without feedback on open circuit.

With no feedback, E_{gk} is the same as E_i. But in any amplifier without feedback, with load disconnected, magnitude of the voltage gain of the circuit is simply μ of the tube and hence $A_t = -\mu$, negative sign appearing because of the assigned positive direction of $A_t E_t$ in Fig. 13·20. Also the internal impedance without feedback is r_p so that $Z_t = r_p$.

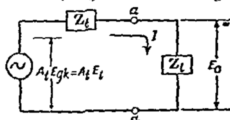


Fig. 13·20. A.C. equivalent circuit of amplifier of Fig. 13·19, with feedback removed.

Next we consider the situation with voltage feedback applied. With feedback, $E_{gk} = E_i + \beta E_o$. Hence the equivalent

circuit of amplifier of Fig. 13·19 with voltage feedback is shown in

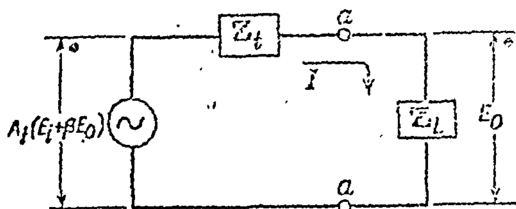


Fig. 13·21. A.C. equivalent circuit of amplifier of Fig. 13·19 with voltage feedback.

Fig. 13·21. It is interesting to note that though the circuit of Fig. 13·21 is the equivalent of Fig. 13·19 with feedback, it is not a Thevenin's voltage source equivalent representation because here both $\mathbf{Z}_t$ and $\mathbf{A}_t(\mathbf{E}_i+\beta\mathbf{E}_o)$ are functions of the load.

Then $$\mathbf{I}=\frac{\mathbf{A}_t(\mathbf{E}_i+\beta\mathbf{E}_o)}{\mathbf{Z}_t+\mathbf{Z}_l} \qquad ...(13·73)$$

Hence $$\mathbf{E}_l=\mathbf{I}.\,\mathbf{Z}_l=\frac{\mathbf{A}_t(\mathbf{E}_i+\beta\mathbf{E}_o)\mathbf{Z}_l}{\mathbf{Z}_t+\mathbf{Z}_l} \qquad ...(13·74)$$

Rearranging Eqn. (13·74) we get,

$$\mathbf{E}_0=\frac{\mathbf{A}_t\,\mathbf{E}_i\,\mathbf{Z}_l}{\mathbf{Z}_t+\mathbf{Z}_l(1-\mathbf{A}_t\beta)} \qquad ...(13·75)$$

$$\text{Hence } \mathbf{I}=\frac{\mathbf{E}_o}{\mathbf{Z}_l}=\frac{\dfrac{\mathbf{A}_t}{1-\mathbf{A}_t\beta}\mathbf{E}_i}{\dfrac{\mathbf{Z}_t}{1-\mathbf{A}_t\beta}+\mathbf{Z}_l} \qquad ...(13·76)$$

From Eqn. (13·76), Thevenin's equivalent network for the circuit of Fig. 13·19 may be drawn as shown in Fig. 13·22.

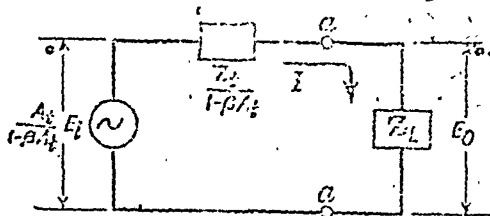


Fig. 13·22. Thevenin's equivalent network of amplifier of Fig. 13·19.

Hence $$\mathbf{E}_{tf}=\frac{\mathbf{A}_t}{1-\beta\mathbf{A}_t}\cdot\mathbf{E}_i \qquad (13·77)$$

and $$\mathbf{Z}_{tf}=\frac{\mathbf{Z}_t}{1-\beta\mathbf{A}_t} \qquad ...(13·78)$$

For negative feedback $(1-\beta A_t)$ is greater than unity so that effective internal impedance with voltage feedback is less than the

internal impedance without feedback. Equations (13·77) and (13·78) agree with equation (13·30) since $A_t = -\mu$ and $Z_t = r_p$.

Effective internal impedance with current feedback. Fig. 13·23 shows block diagram of a current feedback amplifier.

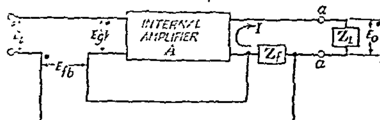


Fig. 13·23 Block diagram of current feedback amplifier.

Z_f is a part of the feedback circuit and is not a part of the load impedance Z_l. The a c equivalent circuit is shown in Fig. 13 24.

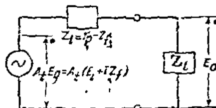


Fig. 13·24. A C equivalent circuit of current feedback amplifier of Fig. 13·23

Here Z_t is the total internal impedance looking back from the load and hence consists of r_p and Z_f.

From equivalent circuit of Fig 13·24,

$$A_t(E_t + IZ_f) = I(Z_t + Z_l)$$

so that
$$I = \frac{A_t E_t}{Z_t + Z_l - Z_f A_t} \quad ...(13·79)$$

Equation (13·79) suggests the circuit given in Fig. 13·25 as the Thevenin's equivalent circuit for current feedback amplifier of Fig. 13·23.

From Fig. 13·25, effective internal impedance of the current feedback amplifier is given by,

$$Z_{tf} = Z_t - A_t Z_f \quad ...(13·80)$$

and open circuit potential with current feedback is given by,

$$E_{tf} = A_t E_t \quad ...(13·81)$$

Equations (13·80) and (13·81) agree with Eqn. (13·24) since $A_t = -\mu$ and $Z_t = r_p + Z_f$ in the case of current feedback.

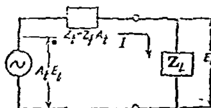


Fig. 13·25. Thevenin's equivalent current feedback amplifier Fig. 13·23.

Output terminal impedance with feedback

The output terminal impedance of a circuit is the impedance seen while looking back from the output terminals into the energy source under the condition that load impedance is connected in circuit and the input potential or input current is reduced to zero (*i.e.* the input voltage source is replaced by a short circuit or input current source is replaced by an open circuit). Obviously then the output terminal impedance of an amplifier is nothing but a parallel combination of load impedance $\mathbf{Z}_l$ and effective internal impedance $\mathbf{Z}_{tf}$. The latter quantity depends upon the feedback condition.

Following notations are used :

$\mathbf{Z}_o$—Output terminal impedance without feedback.

$\mathbf{Z}_{of}$—Output terminal impedance with feedback.

The output terminal impedance $\mathbf{Z}_o$ may be determined by applying a potential $\mathbf{E}_o$ across the output terminals and noting the current $\mathbf{I}_o$ that flows into the output terminals. The ratio of $\mathbf{E}_o$ to $\mathbf{I}_o$ then gives the output terminal impedance $\mathbf{Z}_o$ as shown in Fig. 13·26. Same procedure apply for Z_{of}.

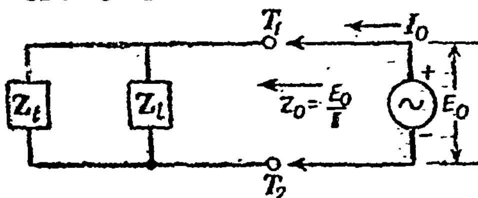


Fig. 13·26. Determination of output terminal impedance of an amplifier.

Considerable confusion exists regarding the meaning of the word output impedance. Often it is used to indicate effective internal impedance $\mathbf{Z}_t$. It is, therefore, convenient and desirable to use the words "internal impedance" and "output terminal impedance" as defined above and symbolized by $\mathbf{Z}_t$ and $\mathbf{Z}_o$ respectively.

Obviously then,

$$\mathbf{Z}_o = \frac{\mathbf{Z}_t \cdot \mathbf{Z}_l}{\mathbf{Z}_t + \mathbf{Z}_l} \qquad \ldots(13·82)$$

and

$$\mathbf{Z}_{of} = \frac{\mathbf{Z}_{tf} \cdot \mathbf{Z}_l}{\mathbf{Z}_{tf} + \mathbf{Z}_l} \qquad \ldots(13·83)$$

where

$$\mathbf{Z}_{tf} = \frac{\mathbf{Z}_t}{1 - \beta \mathbf{A}_t} \text{ for voltage feedback} \qquad \ldots(13·84)$$

and

$$\mathbf{Z}_{tf} = \mathbf{Z}_t - \mathbf{A}_t \cdot \mathbf{Z}_f \text{ for current feedback} \qquad \ldots(13·85)$$

Expressions for $\mathbf{Z}_{tf}$ in terms of $\mathbf{Z}_o$

(*a*) **Voltage feedback :**—In order to find the output terminal impedance with voltage feedback, the internal source voltage in the amplifier is reduced to zero and the internal amplifier circuit is represented by an impedance $\mathbf{Z}_o$. A potential source of $\mathbf{E}_o$ volts is

applied to the output terminals T_1-T_2 as shown in Fig. 13·27. Then, because of voltage feedback, an internal voltage source of $A\beta E_o$ volts appears in series with Z_o as shown in Fig. 13·27.

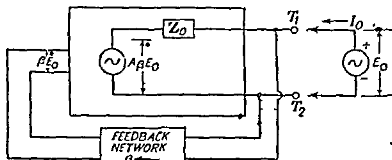


Fig. 13·27. Determination of output terminal impedance Z_{of} in voltage feedback amplifier.

Current I_o sent by the external voltage source E_o is given by,

$$I_o = \frac{E_o - A\beta E_o}{Z_o} \qquad \text{... (13·86)}$$

Hence effective output terminal impedance with voltage feedback is given by,

$$Z_{of} = \frac{E_o}{I_o} = \frac{Z_o}{1 - A\beta} \qquad \text{... (13·87)}$$

Eqn. (13·87) shows that with voltage feedback, effective output terminal impedance is reduced by the same factor as the voltage gain.

(*b*) **Current feedback.** To determine Z_{of}, in this case also input signal voltage E_i is reduced to zero and a voltage source of voltage E_o is applied to the output terminals. Fig. 13·28 shows the circuit arrangement.

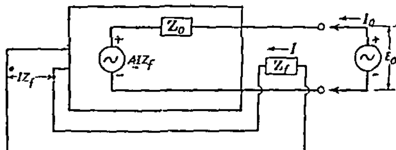


Fig. 13·28. Determination of output terminal impedance Z_{of} in current feedback amplifier

Let I be the current that flows through the feedback impedance Z_f. Then the feedback voltage is IZ_f. This feedback appears as AIZ_f on the plate side as shown in Fig. 13·28.

Fig. 13·31 shows the polar diagram or Nyquist diagram obtained for an arbitrary amplifier. P is any point on the diagram. Then length $OP = A\beta$ and $\angle AOP = \phi$. A closed curve is obtained if diagram is drawn for all frequencies from zero to infinity.

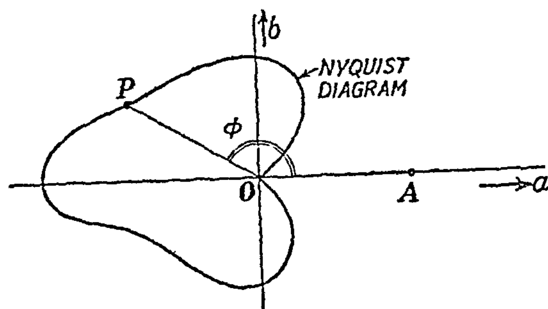


Fig. 13·31. Nyquist diagram of an arbitrary amplifier.

It may, however, be noted that equal arc lengths or angles of polar diagram do not necessarily represent equal frequency intervals.

By means of the relation $\sqrt{(1-a)^2+(b)^2}=1$, it may be shown that the locus of all values of Aβ for which $(1-A\beta)=1$ is a circle of unit radius having its centre at the point (1, 0) as shown in Fig. 13·32. It follows, therefore, that for all frequencies for which vector Aβ terminates within this circle, $(1-A\beta)<1$, and hence amplifier is regenerative and for all values of frequency for which Aβ terminates outside this circle, the amplifier is degenerative.

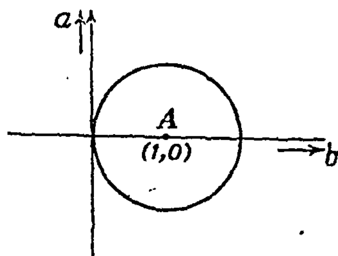


Fig. 13·32. Nyquist diagram for the relation $|1-A\beta|=1$.

The beneficial effects of feedback are attained only when the feedback is degenerative *i.e.* when $(1-A\beta)>1$ and the vector Aβ terminates outside the above mentioned circle. For all values of frequency for which Aβ terminates within the circle, the feedback is positive.

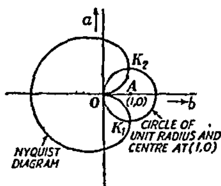
Often there exists a misconception that a negative feedback amplifier has necessarily negative feedback at all frequencies *i.e.* its gain is reduced by feedback at all operating frequencies. This is not true. Feedback amplifiers are designed so that the feedback voltage is opposite in phase with input voltage *i.e.* Aβ is negative, in the middle frequency range where the response is maximum and almost constant. But in general, the polar diagram extends towards the positive side at extremely low and high frequencies and a certain portion of Nyquist diagram may lie within the positive feedback circle.

It is, of course, not necessary to keep Aβ negative at the maximum response frequency. To obtain negative feedback all that is necessary is to keep $|1-A\beta|>1$. If $A\beta$ is real, this may be achieved when (*i*) $A\beta<0$ *i.e.* negative or (*ii*) $A\beta>+2$. It is, however, preferred to keep Aβ negative *i.e.* to keep the feedback voltage in phase opposition with the input voltage in the middle frequency range. In that case the amplifier is unconditionally stable whereas

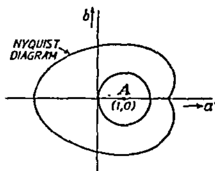
when $A\beta > +2$, amplifier is conditionally stable. To find the condition under which feedback amplifier with $A\beta > +2$, will be stable it is required to establish the condition of oscillation. For oscillations to take place, $|1-A\beta|$ should be equal to zero or $A\beta$ should be equal to one. This means that the vector $A\beta$ should terminate at point $A(1, 0)$. Thus if the polar diagram passes through the point A, oscillations get started at the corresponding frequency and the amplifier gets converted into an oscillator.

Nyquist had shown that for oscillations to be set up, it is not necessary for the curve to pass through the point (1, 0) and even if it encloses the point A oscillations will be sustained. This requirement may be met with ease in several common types of amplifiers. To avoid oscillations, therefore, practical negative feedback amplifiers are designed so that feedback voltage is opposite in phase with the input voltage in the middle of the frequency band where the response is maximum and at no point does the curve include the point (1, 0).

To study the phenomenon of conditional stability of amplification, three specific cases of Nyquist diagram are studied.

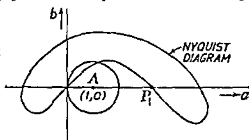


(a) Unconditionally stable Amplifier.



(b) Unconditionally oscillating amplifier.

In the Nyquist diagram of Fig 13 33 (a), for all frequencies for which $A\beta$ terminates at any point from K_1 to O or from O to K_2, quantity $|1-A\beta| < 1$, and hence feedback is positive. For all other frequencies, $|1-A\beta| > 1$ and the feedback is negative. At no frequency, however, does the vector $A\beta$ either terminate at point $A(1, 0)$ or enclose it and hence there is no possibility of oscillation taking place. The amplifier is thus unconditionally stable.



(c) Conditionally stable amplifier.

Fig. 13 33. Nyquist diagram of three arbitrary feedback amplifiers.

The Nyquist diagram

Fig. 13·31 shows the polar diagram or Nyquist diagram obtained for an arbitrary amplifier. P is any point on the diagram. Then length $OP = A\beta$ and $\angle AOP = \phi$. A closed curve is obtained if diagram is drawn for all frequencies from zero to infinity.

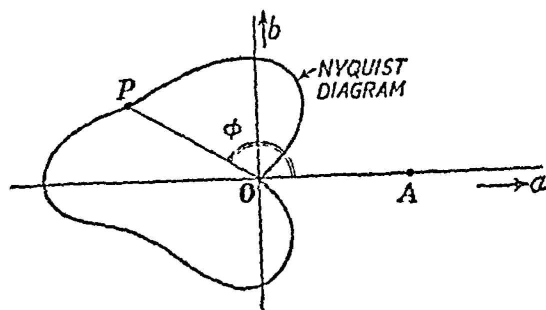


Fig. 13·31. Nyquist diagram of an arbitrary amplifier.

It may, however, be noted that equal arc lengths or angles of polar diagram do not necessarily represent equal frequency intervals.

By means of the relation $\sqrt{(1-a)^2+(b)^2}=1$, it may be shown that the locus of all values of **Aβ** for which $(1-A\beta)=1$ is a circle of unit radius having its centre at the point (1, 0) as shown in Fig. 13·32. It follows, therefore, that for all frequencies for which vector **Aβ** terminates within this circle, $(1-A\beta)<1$, and hence amplifier is regenerative and for all values of frequency for which **Aβ** terminates outside this circle, the amplifier is degenerative.

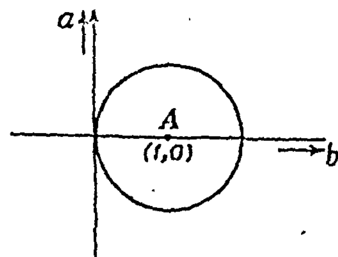


Fig. 13·32. Nyquist diagram for the relation $|1-A\beta|=1$.

The beneficial effects of feedback are attained only when the feedback is degenerative *i.e.* when $(1-A\beta)>1$ and the vector **Aβ** terminates outside the above mentioned circle. For all values of frequency for which **Aβ** terminates within the circle, the feedback is positive.

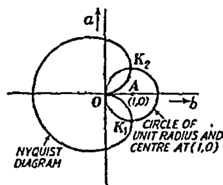
Often there exists a misconception that a negative feedback amplifier has necessarily negative feedback at all frequencies *i.e.* its gain is reduced by feedback at all operating frequencies. This is not true. Feedback amplifiers are designed so that the feedback voltage is opposite in phase with input voltage *i.e.* **Aβ** is negative, in the middle frequency range where the response is maximum and almost constant. But in general, the polar diagram extends towards the positive side at extremely low and high frequencies and a certain portion of Nyquist diagram may lie within the positive feedback circle.

It is, of course, not necessary to keep **Aβ** negative at the maximum response frequency. To obtain negative feedback all that is necessary is to keep $|1-A\beta|>1$. If $A\beta$ is real, this may be achieved when (*i*) $A\beta<0$ *i.e.* negative or (*ii*) $A\beta>+2$. It is, however, preferred to keep **Aβ** negative *i.e.* to keep the feedback voltage in phase opposition with the input voltage in the middle frequency range. In that case the amplifier is unconditionally stable whereas

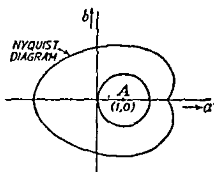
when $A\beta > +2$, amplifier is conditionally stable. To find the condition under which feedback amplifier with $A\beta > +2$, will be stable it is required to establish the condition of oscillation. For oscillations to take place, $|1-A\beta|$ should be equal to zero or $A\beta$ should be equal to one. This means that the vector $A\beta$ should terminate at point $A(1, 0)$. Thus if the polar diagram passes through the point A, oscillations get started at the corresponding frequency and the amplifier gets converted into an oscillator.

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To study the phenomenon of conditional stability of amplification, three specific cases of Nyquist diagram are studied.

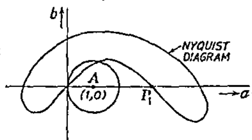


(a) Unconditionally stable Amplifier.



(b) Unconditionally oscillating amplifier.

In the Nyquist diagram of Fig. 13·33 (a), for all frequencies for which $A\beta$ terminates at any point from K_1 to O or from O to K_2, quantity $|1-A\beta| < 1$, and hence feedback is positive. For all other frequencies, $|1-A\beta| > 1$ and the feedback is negative. At no frequency, however, does the vector $A\beta$ either terminate at point $A(1, 0)$ or enclose it and hence there is no possibility of oscillation taking place. The amplifier is thus unconditionally stable.



(c) Conditionally stable amplifier.

Fig. 13·33. Nyquist diagram of three arbitrary feedback amplifiers

The Nyquist diagram

Fig. 13·31 shows the polar diagram or Nyquist diagram obtained for an arbitrary amplifier. P is any point on the diagram. Then length $OP = A\beta$ and $\angle AOP = \phi$. A closed curve is obtained if diagram is drawn for all frequencies from zero to infinity.

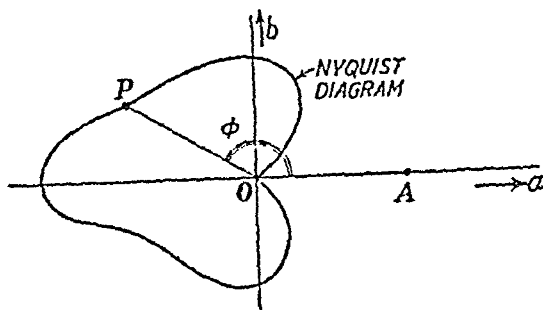


Fig. 13·31. Nyquist diagram of an arbitrary amplifier.

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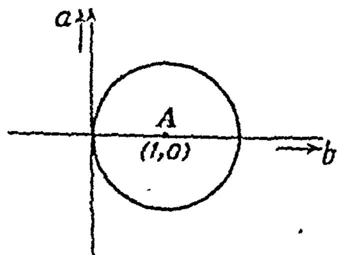


Fig. 13·32. Nyquist diagram for the relation $|1-A\beta|=1$.

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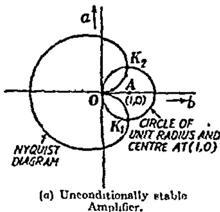
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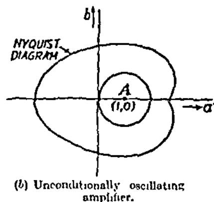
when $A\beta > +2$, amplifier is conditionally stable. To find the condition under which feedback amplifier with $A\beta > +2$, will be stable it is required to establish the condition of oscillation For oscillations to take place, $| 1-A\beta |$ should be equal to zero or $A\beta$ should be equal to one. This means that the vector $A\beta$ should terminate at point $A(1, 0)$. Thus if the polar diagram passes through the point A, oscillations get started at the corresponding frequency and the amplifier gets converted into an oscillator.

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To study the phenomenon of conditional stability of amplification, three specific cases of Nyquist diagram are studied.

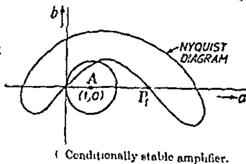


(a) Unconditionally stable Amplifier.



(b) Unconditionally oscillating amplifier.

[illegible] of Fig. 13.33 (a), for all frequencies [illegible] point from K_1 to O or from O to K_2, [illegible] hence feedback is positive. For all other frequencies, $| 1-A\beta | > 1$ and the feedback is negative. At no frequency, however, does the vector $A\beta$ either terminate at point $A(1, 0)$ or enclose it and hence there is no possibility of oscillation taking place. The amplifier is thus unconditionally stable.



(Conditionally stable amplifier.

Fig. 13.33. Nyquist diagram of three arbitrary feedback amplifiers.

The Nyquist diagram

shown in Fig. 13·33 (*b*) encloses the point $A(1, 0)$ and hence the amplifier will oscillate unconditionally.

With reference to the Nyquist diagram shown in Fig. 13·33 (c), such an amplifier should not oscillate normally since it does not enclose the point $A(1, 0)$. Further when $A\beta$ is real *i.e.* along the X-axis, its value is greater than two since $OP_1 > 2$. Feedback is then negative. But the operation of such an amplifier is conditionally stable, the condition being that the filament should be heated first to its normal operating temperature before the plate voltage is applied. If this condition is fulfilled, as soon as the plate voltage is applied, Nyquist diagram assumes its full size immediately. If, however, the above mentioned condition is not satisfied and plate voltage is applied before filament has reached its usual operating temperature, then Nyquist diagram starts expanding in dimensions from zero value to its final value but the final condition is never reached. During the process of expansion Nyquist diagram passes through the point $A(1, 0)$ and immediately oscillations are set up and sustained. Amplifier then works as an oscillator.

This conditional stability forms a serious limitation in an amplifier. Trouble is particularly serious, when the mains get switched off during operation and are then switched on subsequently. Obviously the above mentioned process of expansion of Nyquist diagram take place and the amplifier starts oscillating. To avoid this trouble, it is preferred to keep $A\beta$ negative rather than keeping it greater than two for inverse feedback.

Nyquist diagram for R.C. coupled feedback amplifier. Fig. 13·34 shows the circuit diagram of one stage of R.C. coupled amplifier with voltage inverse feedback.

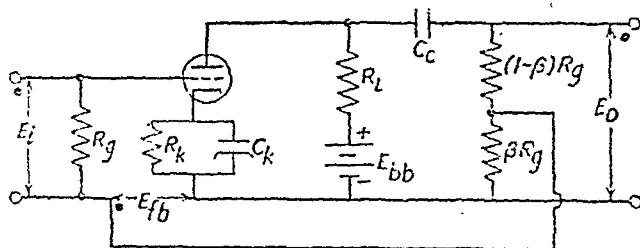


Fig. 13·34. Circuit diagram of one-stage R.C. coupled amplifier with voltage inverse feedback.

The voltage gain of this amplifier with feedback is given by the general feedback equation :

$$A_{fb} = \frac{A}{1 - A\beta} \qquad \ldots(13·6)$$

where **A** is the voltage gain of the amplifier without beedback and is given below in the case of R.C. coupled amplifier, for different frequency ranges :

For Middle Frequency Range, $A = -g_m . R_{pl g}$ ...(13·101)

For Low Frequency Range, $$A=\frac{-g_m R_{plg}}{1-j\frac{X_c}{R'_{eq}}} \qquad ...(13·102)$$

For High Frequency Range, $$A=\frac{-g_m R_{plg}}{1+j\frac{R_{plg}}{X_s}} \qquad ...(13·103)$$

where R_{plg} is parallel combination of r_p, R_l and R_g

X_c is reactance of C_c and is equal to $\frac{1}{2\pi f C_c}$

X_s is reactance of total shunt capacitance C_s

and $$R'_{eq}=R_g+\frac{r_p R_l}{r_p+R_l}$$

If we consider signal of different frequencies to be fed to this R.C. coupled amplifier, varying from zero to infinity, we find that in addition to changes in amplitude of voltage gain **A**, there occurs also a change in phase angle ϕ

Thus at zero frequency, phase angle ϕ of **A** is $=+270°$

In the true middle frequency range, $\phi=+180°$

and at infinitely large frequency, $\phi=-90°$

The phase angle ϕ_β of β network is zero for the feedback arrangement shown in Fig. 13·34. Hence if we plot the polar diagram of $A\beta$ i.e. plot the magnitude $A\beta$ against angle ϕ, we get the Nyquist diagram of the type shown in Fig. 13·35.

It is seen from Fig. 13·35 that the polar diagram for one-stage R.C. coupled amplifier never extends on the positive side and hence this amplifier can never oscillate so long as purely resistive feedback network is used.

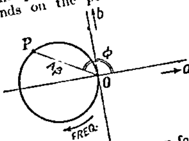


Fig. 13·35. Nyquist diagram for one-stage of R.C. coupled amplifier with voltage inverse feedback.

Nyquist diagram for one-stage transformer coupled amplifier with voltage inverse feedback. Fig. 13·36 shows the circuit diagram of one stage of a transformer coupled amplifier with voltage inverse feedback. The feedback may be positive or negative in the middle frequency range depending upon the transformer winding. For the winding directions shown in Fig. 13·36, feedback is negative.

Fig. 13·37 shows the nature of the polar diagram for [illegible] transformer coupled inverse voltage feedback amplifier of Fig. 1[illegible] [illegible] from this polar diagram that the circuit is degener[illegible]

for most of the frequency range and only at the highest and the lowest frequencies the circuit is regenerative at which values the

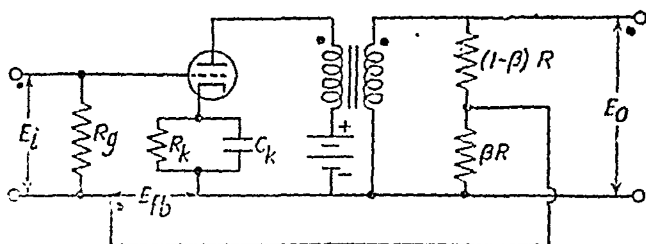


Fig. 13·36. One-stage transformer coupled amplifier with voltage inverse feedback.

gain has reduced to very low values. Further at no time the polar diagram encloses the point A (1, 0) so that the circuit can never oscillate.

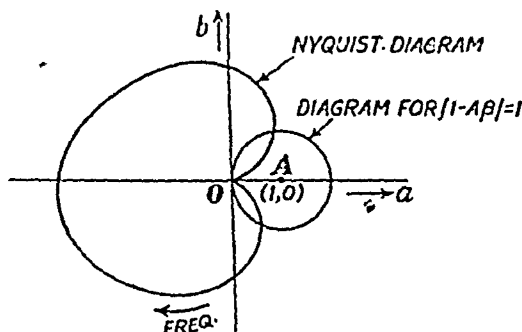
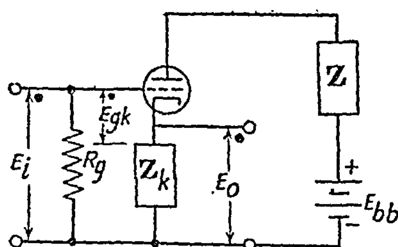
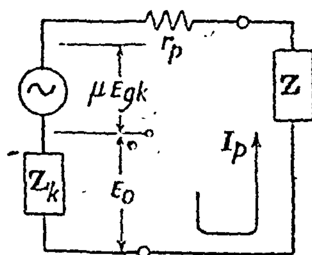


Fig. 13·37. Nyquist diagram of one-stage transformer coupled amplifier with voltage inverse feedback.

Cathode Coupled Amplifier. Fig. 13·38 (a) shows the circuit diagram of a cathode coupled amplifier. It is the same as circuit as that of Fig. 13·3 except that the output is now developed across the cathode impedance $\mathbf{Z}_k$.



(a) Circuit diagram.



(b) A.C. equivalent circuit.

Fig. 13·38. Cathode couple amplifier.

Fig. 13·38 (b) shows the a.c. equivalent circuit. Then from this equivalent circuit,

$$I_p\,[r_p+Z+Z_k]=\mu E_{gk} \qquad ...(13\cdot104)$$

$$=\mu[E_i-E_o] \qquad ...(13\cdot105)$$

or $$I_p[r_p+Z+Z_k]=\mu[E_i-I_p Z_k]$$

Rearranging Eqn. (13·105) we get,

$$I_p=\frac{\mu\,E_i}{r_p+(\mu+1)Z_k+Z} \qquad ...(13\cdot106)$$

Hence voltage gain $A_{fb}=\frac{E_o}{E_i}=\frac{I_p\,Z_k}{E_i}$

$$=\frac{\mu\,Z_k}{r_p+(\mu+1)Z_k+Z} \qquad ...(10\cdot107)$$

Eqn. (13·106) may be rewritten as,

$$I_p=\frac{\frac{\mu}{\mu+1}E_i}{\frac{r_p+Z}{\mu+1}+Z_k} \qquad ...(13\cdot108)$$

Eqn. (13·108) suggests that the circuit may be represented by its Thevenin's equivalent form shown in Fig. 13·39.

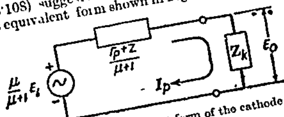


Fig. 13·39. Thevenin's equivalent form of the cathode coupled amplifier of Fig. 13·38.

From equivalent circuit of Fig. 13·39),

$$E_{tf}=\frac{\mu}{\mu+1}E_i \qquad ..(13\cdot109)$$

and $$Z_{tf}=\frac{r_p+Z}{\mu+1} \qquad (13\cdot110)$$

It may be noted here that by taking the output across Z_k, three changes from conventional amplifier working arise,

(1) the voltage is always less than unity.

(2) the effective internal impedance Z_{tf} is reduced to the value $\frac{r_p+Z}{\mu+1}$. This is very low compared with the value $r_p+(\mu+1)Z$ obtained in the case of cathode feedback amplifier

(3) Polarity of *amplified voltage* is not reversed by th[e] amplifier.

The output terminal impedance in this case is given by,

$$Z_{of}=\frac{Z_{tf}\,Z_k}{Z_{tf}+Z_k}. \qquad (13\cdot1$$

Single-tube Paraphase Amplifier. If in the cathode-coupled circuit of Fig. 13·38, $\mathbf{Z}$ is made equal to $\mathbf{Z}_k$ and each is equal to say a resistance R as shown in Fig. 13·40, then the amplifier so obtained is a single-tube paraphase amplifier.

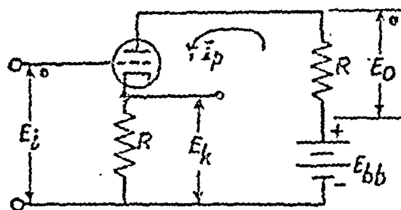


Fig. 13·40. Single tube Paraphase Amplifier.

From Eqn. (13·106) we may write for this circuit,

$$\mathbf{I}_p=\frac{\mu\mathbf{E}_i}{r_p+(\mu+1)R+R} \quad \dots \quad (13·112)$$

$$\text{or } \mathbf{I}_p=\frac{\mu\mathbf{E}_i}{r_p+(\mu+2)R} \quad \dots \quad (13·113)$$

This paraphase amplifier provides two equal potentials of opposite polarity from a single excitation source. When output potential is $\mathbf{E}_0$, then voltage gain is given by,

$$\mathrm{A}_{fb}=\frac{-\mathbf{I}_p\,.\,R}{\mathbf{E}_i}=-\frac{\mu R}{r_p+(\mu+2)R} \quad (13·114)$$

and effective internal impedance is given by,

$$\mathbf{Z}_{tt}=r_p+(\mu+1)\,.\,R \quad \dots \quad (13·115)$$

In this case circuit behaves as a cathode feedback amplifier.

When, however, output is taken from across cathode impedance R, the voltage gain is given by

$$\mathrm{A}'_{fb}=\frac{\mathbf{I}_p.R}{\mathbf{E}_i}=\frac{\mu R}{r_p+(\mu+2)R} \quad \dots \quad (13·116)$$

Eqn. (13·116) may be put as,

$$\mathrm{A}'_{fb}=\frac{\dfrac{\mu}{\mu+1}R}{\dfrac{r_p+R}{\mu+1}+R} \quad \dots \quad (13·117)$$

so that effective internal impedance in this case is given by,

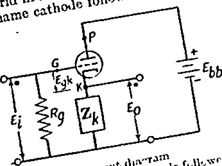
$$\mathbf{Z}'_{tt}=\frac{r_p+R}{\mu+1} \quad (13·118)$$

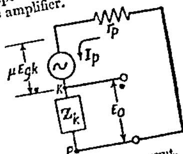
In this case, the circuit behaves as a cathode coupled amplifier.

CATHODE FOLLOWER

All the amplifiers we have dealt with so far are of the "grounded cathode type in which the signal is applied between grid and cathode and cathode is grounded. These amplifiers are most commonly used. There, however, exist two other circuit configurations for vacuum tube amplifiers. These are : (*a*) "grounded-anode amplifier", also called "cathode follower" and (*b*) "grounded-grid" amplifier, also sometimes called cathode input amplifier. In this article, we will discuss the working of a cathode follower.

Fig. 13·41 (a) shows the circuit arrangement of a cathode follower or grounded-anode amplifier. In this amplifier an increase in grid-to-ground voltage E_i increases the plate current I_p and hence increases the cathode-to-ground voltage. Thus the cathode follows the grid in its change of potential with respect to ground and hence the name cathode follower is given to this amplifier.





(a) Circuit diagram (b) A C equivalent circuit.

Fig 13 41 Cathode follower or grounded-anode amplifier

The circuit is the same as that of cathode coupled amplifier of Fig. 13 38 but without the impedance **Z** in the plate circuit. Equivalent circuit is given in F. 13 41 (b)

From the a c equivalent circuit,

$$I_p [r_p + Z_k] = \mu E_{gk} \quad \text{(13 119)}$$

$$= \mu [E_i - E_o] \quad \text{(13·120)}$$

or $$I_p [r_p + Z_k] = \mu [E_i - I_p Z_k] \quad \text{(11·121)}$$

Rearranging Eqn. (11 121) we get,

$$I_p = \frac{\mu E_i}{r_p + (\mu + 1) Z_k}$$

Hence voltage gain

$$A_{fb} = \frac{E_o}{E_i} = \frac{I_p \cdot Z_k}{E_i} = \frac{\mu Z_k}{r_p + (\mu + 1) Z_k} \quad \text{(13·122)}$$

Eqn. (11·122) may be put in the alternative form,

$$I_p = \frac{\frac{\mu}{\mu+1} E_i}{\frac{r_p}{\mu+1} + Z_k} \quad \text{(13·123)}$$

Eqn. (11·123) suggests the circuit of cathode follower may be represented by its Thevenin's equivalent form shown in Fig. 13 42.

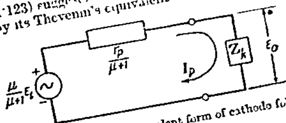


Fig. 13·42. Thevenin's equivalent form of cathode follower.

Thus the effective internal potential E_{if} and effective internal impedance Z_{if} are given by,

$$E_{if}=\frac{\mu}{\mu+1}E_i \qquad ...(13\cdot124)$$

and

$$Z_{if}=\frac{r_p}{\mu+1} \qquad ...(13\cdot125)$$

Output terminal impedance

$$Z_{of}=\frac{Z_{if}\cdot Z_k}{Z_{if}+Z_k} \qquad ...(13\cdot126)$$

Eqn. (13·126) may be put in the alternative form,

$$Z_{of}=\frac{\frac{r_p}{\mu+1}\cdot Z_k}{\frac{r_p}{\mu+1}+Z_k}=\frac{r_p\cdot Z_k}{r_p+(\mu+1)Z_k}$$

$$=\frac{\mu Z_k}{r_p+(\mu+1)Z_k}\times\frac{1}{g_m}$$

or

$$Z_{of}=\frac{A_{fb}}{g_m} \qquad ...(13\cdot127)$$

Also input impedance $Z_{if}=Z_i(1-A\beta)=Z_i(1+A)$ ...(13·128)

since $\beta=-1$

where Z_i is the input impedance of the value alone without feedback.

Salient features of the cathode follower are given below.

(*i*) **Voltage gain.** The voltage gain is always less than unity and in the limit approaches the value $\frac{\mu}{\mu+1}$ as the ratio $\frac{r_p}{Z_k}$ approaches zero. Thus for tubes with large values of amplification factor μ and with $Z_k \gg r_p$, the voltage gain approaches unity. For typical values of Z_k and r_p, voltage gain is of the order of 0·9. Since the gain is almost unity, cathode and grid rise and fall together in potential by almost equal amounts justifying the name cathode follower given to this amplifier.

(ii) Effective internal impedance. *Effective internal impedance is very low.* If $\mu \gg 1$, then Z_{if} is almost equal to $\frac{r_p}{\mu}=\frac{1}{g_m}$. g_m for most of the electron tubes lies in the range 1,000 to 10,000 μ mhos, so that Z_{if} lies in the range 1000 to 100 ohms. Thus cathode follower has a high input impedance and low output impedance and may, therefore, be conveniently used as a coupling device between a high impedance source and a low impedance load.

(*iii*) *The polarity of the amplified output voltage is the same as that of the input voltage.*

Graphical analysis of cathode follower. In the cathode follower circuit of Fig. 13·43, e_i, e_{gk}, e_p and e_o represent respectively the instantaneous a. c. values of input voltage, grid-to-cathode voltage, plate voltage and output voltage. Let E_{cn}, E_c, E_b and E_k represent respectively the d.c components of input voltage, grid-to-cathode voltage, plate voltage and output voltage. The corresponding instantaneous total values are e_{cn}, e_c, e_b and e_k respectively. The purpose of graphical analysis is to find a relation between the plate current i_b and the input voltage e_{cn} i e. *to draw the dynamic characteristic.* Let E_{bb} be the plate supply voltage Then

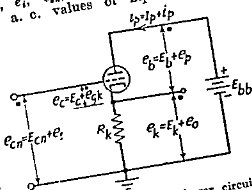


Fig 13·43. Basic cathode follower circuit.

$$e_b=E_{bb}-i_b\ R_k \qquad \text{...(13·129)}$$

and the path of operation on the plate characteristics is a straight line drawn as shown in Fig 13·44 Such a load line gives only the relation between i_b and e_c and not between i_b and e_{cn} But we know that,

$$e_{cn}=e_c+i_b\ R_k=e_c+e_k \qquad \text{..(13·130)}$$

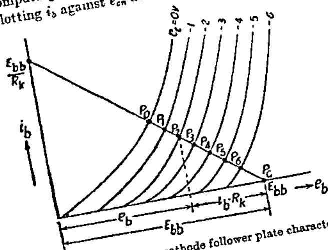
The procedure for graphical analysis, therefore, consists in

(i) selecting a value of e_c,

(ii) reading from the diagram, the corresponding values of i_b and $i_b \cdot R_k$

(iii) computing the value of e_{cn} using the Eqn. (13·130).

(iv) plotting i_b against e_{cn} as shown in Fig 13·45.



13·44 Load line on a cathode follower plate characteristics

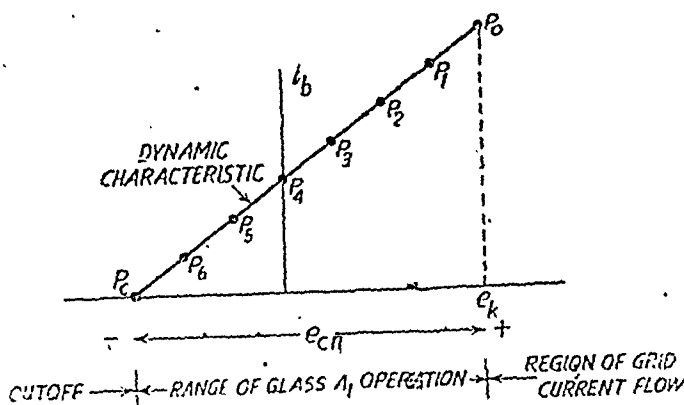


Fig. 13·45. Dynamic characteristic of a cathode follower giving relation between i_b and e_{cn}.

It is seen from the diagram that as e_{cn} increases in the positive direction starting from zero, the grid-to-cathode voltage approaches zero. When $e_c=0$ at point P_0, the output voltage equals the input voltage. Again as e_{cn} decreases from zero, the plate current i_b and output voltage e_k reduce approaching zero. i_b and e_k become zero when e_c equals the cutoff bias in which case, $e_c=e_{cn}=$cutoff bias. For class A_1 operation, e_{cn} must be restricted to be within these limits as shown in Fig. 13·45.

R.C. coupled cathode follower. Cathode follower circuit is idely used as a coupling amplifier between stages. Circuit for such a use is shown in Fig. 13·46.

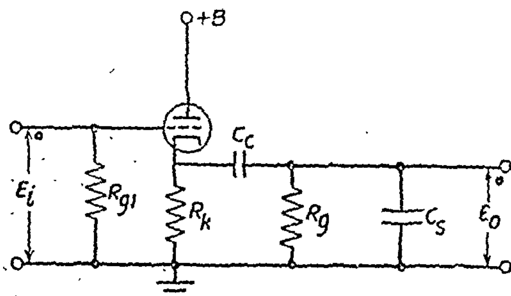


Fig. 13·46. R.C. coupled cathode follower.

In the circuit shown, C_c is the coupling condenser, R_g is the grid leak resistance of the next stage and C_s is the sum of effective output capacitance of the cathode follower stage and the effective input capacitance of the next stage.

Fig. 13·47 shows the Thevenin's form of a.c. equivalent circuit of this R.C. coupled cathode follower. This equivalent circuit directly follows the one in Fig. 13·42 provided $\mathbf{Z}_k$ is taken to represent the impedance of consisting of R_k, C_c, R_g and C_s. To find the output voltage, and voltage gain $\mathbf{A}_{fb}$, Millman's theorem may be applied or conventional circuit theory may be used in a manner similar to that done for R.C. coupled amplifier. It is however generally more convenient and useful to divide the entire frequency range into three

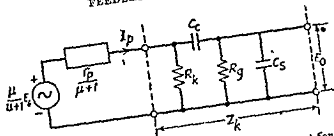


Fig. 13·47 Thevenin's form of a.c. equivalent circuit for R.C. coupled cathode follower amplifier.

arbitrary frequency ranges and to calculate the voltage gain etc. for each of these frequency ranges as given below :

(*a*) **Middle Frequency Range.** In this frequency range, frequency is small enough to make the reactance of C_s large compared to R_k and hence C_s may be omitted. Again the frequency is large enough to make the reactance of coupling condenser C_c small compared with R_g and hence C_c may also be omitted

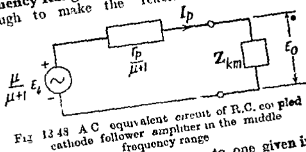


Fig 13·48 A C equivalent circuit of R.C. coupled cathode follower amplifier in the middle frequency range

A c. equivalent circuit of Fig 13·47 then reduces to one given in Fig. 13·48 where $\mathbf{Z}_{km}$ indicates parallel combination of R_k and R_g. $\mathbf{Z}_{km}$ is given by,

$$\mathbf{Z}_{km}=\frac{R_k \, . \, R_g}{R_k+R_g} \qquad \dots (13·131)$$

Then from the a c equivalent circuit of Fig. 13·48, the plate current in the middle frequency range is given by,

$$\mathbf{I}_p=\frac{\frac{\mu}{\mu+1}\mathbf{E}_i}{\frac{r_p}{\mu+1}+\mathbf{Z}_{km}} \qquad \dots (13·132)$$

Hence voltage gain in the middle frequency range is given by,

$$\mathbf{A}_m=\frac{\mathbf{E}_o}{\mathbf{E}_i}=\frac{\mathbf{I}_p \, . \, \mathbf{Z}_{km}}{\mathbf{E}_i}=\frac{\frac{\mu}{\mu+1}\mathbf{Z}_{km}}{\frac{r_p}{\mu+1}+\mathbf{Z}_{km}} \qquad \dots (13·13$$

or

$$\mathbf{A}_m=\frac{\mu\,\mathbf{Z}_{km}}{r_p+(\mu+1)\,\mathbf{Z}_{km}}=\frac{\mu}{\mu+1+\frac{r_p}{\mathbf{Z}_{km}}} \qquad \dots (13·1$$

From Eqn. (13·132), effective imternal impedance $\mathbf{Z}_{if}$ is equal to $\frac{r_p}{\mu+1}$ and the output terminal impedance $\mathbf{Z}_{of}$ is parallel combination of $\mathbf{Z}_{if}$ and $\mathbf{Z}_{km}$.

(*b*) **High Frequency Range.** In the high frequency range, frequency is so high that the reactance of coupling condenser C_c is negligibly small compared with R_g and hence C_c may be omitted. The a.c. equivalent circuit is the same as in Fig. 13·48 with the difference that $\mathbf{Z}_{km}$ is now replaced by $\mathbf{Z}_{kh}$ where $\mathbf{Z}_{kh}$ is the impedance of elements R_k, C_s and R_g in parallel. Then

$$\frac{1}{\mathbf{Z}_{kh}}=\frac{1}{R_k}+\frac{1}{R_g}+j\omega\, C_s \qquad \text{... (13·135)}$$

The plate current is given by,

$$\mathbf{I}_p=\frac{-\frac{\mu}{\mu+1}\mathbf{E}_i}{\frac{r_p}{\mu+1}+\mathbf{Z}_{kh}} \qquad \text{... (13·136)}$$

and voltage gain is given by,

$$\mathbf{A}_h=\frac{\mu\mathbf{Z}_{kh}}{r_p+(\mu+1)\mathbf{Z}_{kh}}=\frac{\mu}{\mu+1+\frac{r_p}{\mathbf{Z}_{kh}}}. \qquad \text{... (13·137)}$$

Hence
$$\frac{\mathbf{A}_h}{\mathbf{A}_m}=\frac{\mu+1+\frac{r_p}{\mathbf{Z}_{km}}}{\mu+1+\frac{r_p}{\mathbf{Z}_{kh}}}$$

$$=\frac{\mu+1+\frac{r_p}{\mathbf{Z}_{km}}}{\mu+1+r_p\left(\frac{1}{\mathbf{Z}_{km}}+j\omega C_s\right)}$$

or
$$\frac{\mathbf{A}_h}{\mathbf{A}_m}=\frac{1}{1+\frac{j\omega\, C_s r_p}{\mu+1+\frac{r_p}{\mathbf{Z}_{km}}}}$$

$$=\frac{1}{1+j\omega C_s r'_p}=\frac{1}{1+j\,\frac{f}{f_2}} \qquad \text{... (13·138)}$$

where
$$r'_p=\frac{r_p}{\mu+1+\frac{r_p}{\mathbf{Z}_{km}}} \qquad \text{... (13·139)}$$

and
$$f_2=\frac{1}{2\pi r'_p C_s} \qquad \text{... (13·140)}$$

Obviously f_2 is the upper half power frequency. Eqn. (13·138) is of same form as for conventional R.C. couple amplifier. But R_{pl} C_s is much smaller than the corresponding value R_{pl} C_s in R.C. coupled amplifier. Hence upper half power frequency f_2 in this case is much larger than in usual R.C. coupled amplifier. Thus high frequency limit is much higher in this case inspite of large input capacitance of next stage.

Effective internal impedance is again $\frac{r_p}{\mu+1}$ whereas output terminal impedance is parallel combination of $\frac{r_p}{\mu+1}$ and $\mathbf{Z}_{kk}$.

(c) **Low Frequency Range.** In the low frequency range, frequency is low enough to make the reactance of C_s very large compared with R_k and hence C_s may be omitted. The remaining elements that constitute the total load impedance $\mathbf{Z}_{kl}$ are R_k, R_g and C_c. The a.c. equivalent circuit is then the same as in Fig. 13.48 with the difference that $\mathbf{Z}_{km}$ is now replaced by $\mathbf{Z}_{kl}$ as shown in Fig. 13.49.

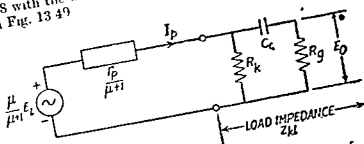


Fig. 13·38. Thevenin's form of a.c. equivalent circuit of R.C. coupled cathode follower in low frequency range.

Since $\mathbf{Z}_{kl}$ is the impedance of elements R_k, C_c and R_g as arranged in Fig. 13.49, we have,

$$\frac{1}{\mathbf{Z}_{kl}} = \frac{1}{R_k} + \frac{1}{R_g + \frac{1}{j\omega C_c}} \quad \text{... (13·141)}$$

$$= \frac{1}{\mathbf{Z}_{km}} - \frac{1}{R_g(1+j\omega C_c R_g)} \quad \text{... (13·142)}$$

The plate current $\mathbf{I}_p$ is given by,

$$\mathbf{I}_p = \frac{\frac{\mu}{\mu+1}\mathbf{E}_i}{\frac{r_p}{\mu+1} + \mathbf{Z}_{kl}}$$

Voltage gain in the low frequency range is given by,

$$A_l = \frac{\mathbf{I}_p \cdot \mathbf{Z}_{kl}}{\mathbf{E}_l} \times \frac{R_g}{R_g + \frac{1}{j\omega C_c}}$$

$$= \frac{\mu}{\mu + 1 + \frac{r_p}{\mathbf{Z}_{kl}}} \times \frac{R_g}{R_g + \frac{1}{j\omega C_c}} \quad \ldots (13\cdot 143)$$

or
$$\mathbf{A}_l = \frac{\mu}{\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}} - \frac{r_p}{R_g(1 + j\omega C_c R_g)}} \times \frac{R_g \cdot j\omega C_c}{1 + j\omega C_c R_g} \quad \ldots (13\cdot 144)$$

Hence
$$\frac{\mathbf{A}_l}{\mathbf{A}_m} = \frac{1}{\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}} - \frac{r_p}{R_g(1 + j\omega C_c R_g)}} \times \frac{j\omega C_c R_g}{1 + j\omega C_c R_g} \times \left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)$$

$$= \frac{1}{1 - \frac{r_p}{\mathrm{R}_g(1 + j\omega C_c R_g)\left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)}} \times \frac{j\omega C_c R_g}{1 + j\omega C_c R_g}$$

or
$$\frac{\mathbf{A}_l}{\mathbf{A}_m} = \frac{1}{1 + \frac{1}{j\omega C_c R_g} - \frac{(1 + j\omega C_c R_g)}{j\omega C_c R_g} \times \frac{r_p}{R_g\left(1 + j\omega C_c R_g\right)\left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)}}$$

$$= \frac{1}{1 + \frac{1}{j\omega C_c R_g}\left[1 - \frac{r_p}{R_g\left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)}\right]}$$

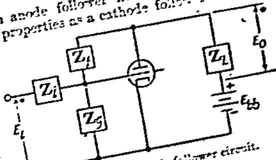
or
$$\frac{\mathbf{A}_l}{\mathbf{A}_m} = \frac{1}{1 - j\left(\frac{1}{\omega C_c \mathrm{R}_1}\right)} = \frac{1}{1 - j\frac{f_1}{f}} \quad \ldots$$

where
$$R_1 = \frac{R_g}{1 - \frac{r_p/R_g}{\left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)}} \quad \ldots (13\cdot 146)$$

and
$$f_1 = \frac{1}{2\pi C_c R_1} \quad \ldots (13\cdot 147)$$

Obviously f_1 is the lower half power frequency. For a typical cathode follower amplifier, the value of this lower half power frequency f_1 is of the same order of magnitude as in a conventional R.C. coupled amplifier.

[illegible]ode Follower. An anode follower has almost the same [illegible]gain and impedance properties as a cathode follower but has [illegible]versal between [illegible]d output voltage. In the case of [illegible] follower, cathode [illegible]al follows the [illegible] potential whereas [illegible] case of anode [illegible]er the anode potential falls by almost the same amount as the grid potential rises. Obvi[illegible] the anode does not follow the grid [illegible] thus no justification for the name anode follower. [illegible] is often referred to a "see-saw [illegible] almost equal and opposite changes [illegible] grid and [illegible] voltages. Fig. 13·5[illegible] the basic anode follower circuit.



[illegible] anode follower circuit.

The anode follower circuit of Fig. 13·50 is nothing but one stage of voltage feedback amplifier [illegible] feedback from anode to grid through an impedance $\mathbf{Z}$. [illegible] 13·51 shows the incremental equivalent circuit for the amplifier.

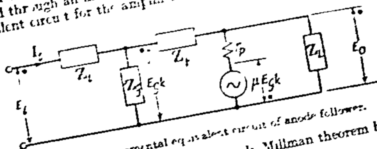


Fig. 13·51. Incremental equivalent circuit of anode follower.

To analyse this circuit, we first apply Millman theorem betw[illegible] the points G and K. Then,

$$\mathbf{E}_{gk}=\frac{\mathbf{E}_i\cdot\mathbf{Y}_i+\mathbf{E}_o\mathbf{Y}_f}{\mathbf{Y}_i+\mathbf{Y}_f+\mathbf{Y}_g} \qquad \ldots(13$$

where $\quad \mathbf{Y}_i=\dfrac{1}{\mathbf{Z}_i},\ \mathbf{Y}_f=\dfrac{1}{\mathbf{Z}_f}$, and $\mathbf{Y}_g=\dfrac{1}{\mathbf{Z}_g}$

Again applying Millman theorem between points P a[illegible] we get,

$$\mathbf{E}_o=\frac{\mathbf{E}_{gk}\mathbf{Y}_f+(-\mu\mathbf{E}_{gk})\mathbf{Y}_p}{\mathbf{Y}_f+\mathbf{Y}_p+\mathbf{Y}_l} \qquad \ldots(13\cdot45)$$

$$=\frac{\mathbf{E}_{gk}(\mathbf{Y}_f-g_m)}{\mathbf{Y}_f+\mathbf{Y}_p+\mathbf{Y}_l}$$

where $\quad \mathbf{Y}_p=\dfrac{1}{r_p}$

Voltage gain in the low frequency range is given by,

$$A_l = \frac{\mathbf{I}_p \cdot \mathbf{Z}_{kl}}{\mathbf{E}_i} \times \frac{R_g}{R_g + \frac{1}{j\omega C_c}}$$

$$= \frac{\mu}{\mu + 1 + \frac{r_p}{\mathbf{Z}_{kl}}} + \frac{R_g}{R_g + \frac{1}{j\omega C_c}} \qquad \text{... (13·143)}$$

or $$\mathbf{A}_l = \frac{\mu}{\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}} - \frac{r_p}{R_g(1 + j\omega C_c R_g)}} \times \frac{R_g \cdot j\omega C_c}{1 + j\omega C_c R_g} \qquad \text{... (13·144)}$$

Hence $$\frac{\mathbf{A}_l}{\mathbf{A}_m} = \frac{1}{\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}} - \frac{r_p}{R_g(1 + j\omega C_c R_g)}} \times \frac{j\omega C_c R_g}{1 + j\omega C_c R_g} \times \left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)$$

$$= \frac{1}{1 - \frac{r_p}{\mathrm{R}_g(1 + j\omega C_c R_g)(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}})}} \times \frac{j\omega C_c R_g}{1 + j\omega C_c R_g}$$

or $$\frac{\mathbf{A}_l}{\mathbf{A}_m} = \frac{1}{1 + \frac{1}{j\omega C_c R_g} - \frac{(1 + j\omega C_c R_g)}{j\omega C_c R_g} \times \frac{r_p}{R_g\left(1 + j\omega C_c R_g\right)\left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)}}$$

$$= \frac{1}{1 + \frac{1}{j\omega C_c R_g}\left[1 - \frac{r_p}{R_g\left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)}\right]}$$

or $$\frac{\mathbf{A}_l}{\mathbf{A}_m} = \frac{1}{1 - j\left(\frac{1}{\omega C_c \mathrm{R}_1}\right)} = \frac{1}{1 - j\frac{f_1}{f}} \qquad \text{...}$$

where $$R_1 = \frac{R_g}{1 - \frac{r_p/R_g}{\left(\mu + 1 + \frac{r_p}{\mathbf{Z}_{km}}\right)}} \qquad \text{... (13·146)}$$

and $$f_1 = \frac{1}{2\pi C_c R_1} \qquad \text{... (13·147)}$$

Obviously f_1 is the lower half power frequency. For a typical cathode follower amplifier, the value of this lower half power frequency f_1 is of the same order of magnitude as in a conventional R.C. coupled amplifier.

Anode Follower. An anode follower has almost the same voltage gain and impedance properties as a cathode follower but has phase reversal between input and output voltages. In the case of cathode follower, cathode potential follows the grid potential whereas in the case of anode follower the anode potential falls by almost the same amount as the grid potential rises. Obviously the anode does not follow the grid and there is thus no justification for the name "anode follower." The circuit is often referred to as a "see-saw circuit" because of almost equal and opposite changes in grid and anode voltages. Fig. 13·50 shows the basic anode follower circuit.

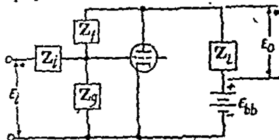


Fig. 13·50 Basic anode follower circuit.

The anode follower circuit of Fig 13·50 is nothing but one stage of voltage feedback amplifier with direct feedback from anode to grid through an impedance $\mathbf{Z}_f$. Fig 13·51 shows the incremental equivalent circuit for the amplifier.

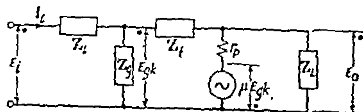


Fig. 13·51. Incremental equivalent circuit of anode follower.

To analyse this circuit, we first apply Millman theorem between the points G and K. Then,

$$\mathbf{E}_{gk}=\frac{\mathbf{E}_i.\ \mathbf{Y}_i+\mathbf{E}_o\ \mathbf{Y}_f}{\mathbf{Y}_i+\mathbf{Y}_f+\mathbf{Y}_g} \qquad \ldots(13\cdot148)$$

where $\mathbf{Y}_i=\dfrac{1}{\mathbf{Z}_i}$, $\mathbf{Y}_f=\dfrac{1}{\mathbf{Z}_f}$, and $\mathbf{Y}_g=\dfrac{1}{\mathbf{Z}_g}$

Again applying Millman theorem between points P and K, we get,

$$\mathbf{E}_o=\frac{\mathbf{E}_{gk}\ \mathbf{Y}_f+(-\mu\mathbf{E}_{gk})\mathbf{Y}_p}{\mathbf{Y}_f+\mathbf{Y}_p+\mathbf{Y}_L}$$

$$=\frac{\mathbf{E}_{gk}[\mathbf{Y}_f-g_m]}{\mathbf{Y}_f+\mathbf{Y}_p+\mathbf{Y}_L} \qquad \ldots(13\cdot149)$$

where $\mathbf{Y}_p=\dfrac{1}{r_p}$

Substituting the value of E_{gk} from Eqn. (13·148) into Eqn. (13·149), we get,

$$E_o = \frac{(Y_f - g_m)}{(Y_f + Y_p + Y_l)} \times \frac{E_i Y_i + E_o Y_f}{Y_i + Y_g + Y_f} \quad ...(13·150)$$

Rearranging Eqn. (13·150) we get,

$$E_o[(Y_f + Y_p + Y_l)(Y_i + Y_g + Y_f) - Y_f(Y_f - g_m)] = Y_i(Y_f - g_m)E_i$$

Hence voltage gain is given by,

$$A_{fb} = \frac{E_o}{E_i} = \frac{Y_i(Y_f - g_m)}{(Y_f + Y_p + Y_l)(Y_i + Y_g + Y_f) - Y_f(Y_f - g_m)} \quad ...(13·151)$$

or

$$A_{fb} = \frac{Y_i(Y_f - g_m)}{(Y_i + Y_f + Y_g)(Y_p + Y_l) + Y_f(Y_i + Y_g + g_m)} \quad ...(13·152)$$

The effective internal impedance Z_{tf} may be found by noting the current I_o that flows into the circuit on application of an a.c. potential E_o to the output terminals in the a.c. equivalent circuit. The input excitation potential is reduced to zero. The equivalent circuit is now drawn as shown in Fig. 13·52.

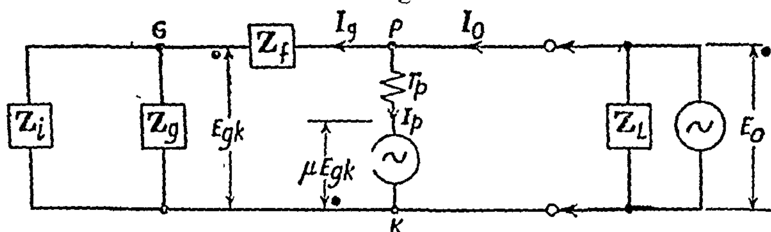


Fig. 13·52. Incremental equivalent circuit of an anode follower for calculating the effective internal impedance.

Then the effective internal admittance or output admittance of the tube alone is given by,

$$Y_{tf} = \frac{I_o}{E_o} = \frac{I_p + I_g}{E_o} \quad ...(13·153)$$

or

$$Y_{tf} = \frac{1}{E_o}\left[\frac{E_o + \mu E_{gk}}{r_p} + E_{gk}\left(Y_g + Y_i\right)\right]$$

$$= \frac{1}{E_o}[(Y_i + Y_g + g_m)E_{gk} + Y_p E_o] \quad ...(13·154)$$

where $Y_p = \dfrac{1}{r_p}$

But

$$\frac{E_{gk}}{E_o} = \frac{\dfrac{Z_i Z_g}{Z_i + Z_g}}{Z_f + \dfrac{Z_i Z_g}{Z_i + Z_g}} = \frac{Z_i Z_g}{Z_i Z_g + Z_i Z_f + Z_g Z_f}$$

or

$$\frac{E_{gk}}{E_o} = \frac{1/Z_f}{\dfrac{1}{Z_f} + \dfrac{1}{Z_i} + \dfrac{1}{Z_g}} = \frac{Y_f}{Y_f + Y_i + Y_g} \quad ...(13·1$$

Hence $$Y_{if} = Y_s + \frac{Y_s(Y_i + Y_f + g_m)}{(Y_f + Y_s + Y_l)} \qquad ...(13\cdot156)$$

Expression for the input admittance may be found from the a.c. equivalent circuit of Fig. 13·51. The input admittance is given by,

$$Y_{if} = \frac{I_i}{E_i} = \frac{E_i - E_{rk}}{Z_i E_i} = Y_i \left[1 - \frac{E_{rk}}{E_i} \right] \qquad ...(13\cdot157)$$

But from Eqn. (13·148),

$$E_r = \frac{E_i Y_i - E_o Y_f}{Y_i + Y_f + Y_s} = \frac{E_i Y_i - A_o E_i Y_f}{Y_i + Y_f + Y_s}$$

$$= E_i \frac{Y_i - A_o Y_f}{Y_i + Y_f + Y_s} \qquad ...(13\cdot158)$$

Hence $$Y_{if} = Y_i \left[1 - \frac{Y_i - A_o Y_f}{Y_i + Y_f + Y_s} \right]$$

$$= \frac{Y_i}{Y_i + Y_f + Y_s} \left[Y_s + (1 - A_o) Y_f \right] \qquad ...(13\cdot159)$$

GROUNDED-GRID AMPLIFIER

Ground-grid amplifier also sometimes called cathode input amplifier, is also essentially a [illegible]. As stated previously it is one of the three amplifier configurations [illegible] grounded cathode (or grid-input) amplifier, grounded plate (or cathode follower) amplifier and grounded-grid (or cathode input) amplifier. Grounded-grid amplifier has certain advantages over the conventional grounded cathode amplifier for certain specific uses. Fig. 13·53 shows the circuit of a grounded-grid amplifier. Input voltage is fed from a voltage source of voltage E_i and internal impedance Z_i.

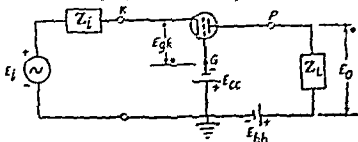


Fig. 13·53. Basic circuit of grounded-grid amplifier.

Fig. 13·54 shows the a.c. equivalent circuit obtained by replac-

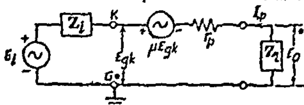


Fig. 13·54. A.C. equivalent circuit of grounded-grid amplifier of Fig.

ing the tube by its equivalent voltage generator. It is seen that the a.c. plate current $\mathbf{I}_p$ flows through the input source impedance $\mathbf{Z}_i$ as well as the load impedance $\mathbf{Z}_l$.

From the equivalent circuit, plate current $\mathbf{I}_p$ is given by,

$$\mathbf{I}_p = -\frac{\mathbf{E}_i - \mu\,\mathbf{E}_{gk}}{\mathbf{Z}_i + r_p + \mathbf{Z}_l} \qquad ...(13{\cdot}160)$$

But $\qquad \mathbf{E}_{gk} = -(\mathbf{E}_i + \mathbf{I}_p\,\mathbf{Z}_i) \qquad ...(13{\cdot}161)$

Substituting the values of $\mathbf{E}_{gk}$ from Eqn. (13·161) into Eqn. (13·160),

$$\mathbf{I}_p = \frac{-(\mu+1)\,\mathbf{E}_i}{r_p + (\mu+1)\,\mathbf{Z}_i + \mathbf{Z}_l} \qquad ...(13{\cdot}162)$$

Output voltage

$$\mathbf{E}_o = -\mathbf{I}_p\,.\,\mathbf{Z}_l \qquad ...(13{\cdot}163)$$

Hence the voltage gain of the amplifier is given by,

$$\mathbf{A}_{fb} = \frac{\mathbf{E}_o}{\mathbf{E}_i} = \frac{-\mathbf{I}_p\,\mathbf{Z}_l}{\mathbf{E}_i}$$

$$= \frac{(\mu+1)\,\mathbf{Z}_l}{r_p + (\mu+1)\,\mathbf{Z}_i + \mathbf{Z}_l} \qquad ...(13{\cdot}164)$$

Eqn. (13·162) suggests that the input circuit may be represented by its Thevenin's equivalent form shown in Fig. 13·55.

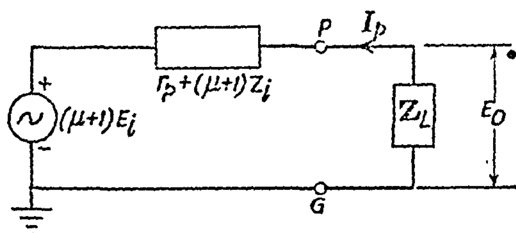


Fig. 13·55. Thevenin's form of a.c. equivalent circuit of grounded grid amplifier.

Thus the effective internal potential $\mathbf{E}_{tf}$ and effective internal impedance $\mathbf{Z}_{tf}$ are given by,

$$\mathbf{E}_{tf} = (\mu+1)\,\mathbf{E}_i \qquad ...(13{\cdot}165)$$

and $\qquad \mathbf{Z}_{tf} = r_p + (\mu+1)\,\mathbf{Z}_i \qquad ...(13{\cdot}166)$

The output terminal impedance $\mathbf{Z}_{of}$ is the parallel combination of $\mathbf{Z}_{tf}$ and $\mathbf{Z}_l$ and hence is given by,

$$\mathbf{Z}_{of} = \frac{\mathbf{Z}_{tf}\,.\,\mathbf{Z}_l}{\mathbf{Z}_{tf} + \mathbf{Z}_l} \qquad ...(13{\cdot}167)$$

Eqn. (13·164) shows that the voltage gain of the amplifier approaches the value $(\mu+1)$ as the load impedance $\mathbf{Z}_l$ approaches infinity.

Eqn. (13·162) may be written as,

$$-I_p = \frac{E_i}{Z_i + \frac{r_p + Z_l}{\mu + 1}} \quad \text{...(13·168)}$$

The equivalent circuit for the input circuit is then given by Fig. 13·56.

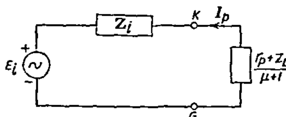


Fig. 13·56. Equivalent input circuit of grounded-grid amplifier of Fig. 13·53

Thus the input impedance presented by the grounded-grid amplifier to source is equal to $\frac{r_p + Z_l}{\mu + 1}$.

Advantages of grounded grid amplifier.

(*i*) Grounded-grid acts as an electrostatic screen between the input and output circuits of the amplifier It thus improves the stability of operation as an amplifier and reduces the tendency of oscillation.

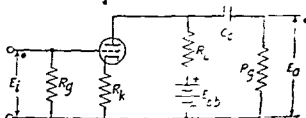
This advantage, normally obtained in screen grid tubes only, is now obtained even in a triode. Triode is superior to tetrodes and pentodes in ultra high frequency operation since it introduces less random noise. Thus grounded-grid triode becomes an ideal amplifier in earlier stages of ultra high frequency amplifiers and receivers.

(*ii*) There is no phase reversal between the output and input voltages.

Example 17. *In the following amplifier circuit let* $\mu = 50$, $g_m = 1250$ μ℧, $R_l = 50$ $k\Omega$, $R_g = 1$ $M\Omega$, $R_k = 1$ $k\Omega$ *Assume the reactance of coupling condenser* C_c *to be much smaller than* R_g. *Calculate*

(*a*) *voltage gain of the amplifier,*
(*b*) *effective internal impedance,*
(*c*) *output terminal impedance,*

Repeat the calculations when R_k *has been reduced to zero*



Solution. Load impedance $\mathbf{Z}_l$ is parallel combination of R_l and R_g. Hence

$$\mathbf{Z}_l=\frac{(50\times10)^3\times10^6}{(50\times10^3)+10^6}$$

$$=\frac{50000}{1050}\times10^3=47{\cdot}6\times10^3 \text{ ohms.}$$

$$r_p=\frac{\mu}{g_m}=\frac{50}{1250\times10^{-6}}=40000 \text{ ohms.}$$

Voltage gain with feedback through R_k is given by,

$$\mathbf{A}_{fb}=\frac{-\mu\ \mathbf{Z}_l}{r_p+(\mu+1)\ R_k+\mathbf{Z}_l}$$

$$=\frac{-50\times47600}{(40\times10^3)+(50+1)\times10^3+(47{\cdot}6\times10^3)}$$

$$=-17{\cdot}2.$$

Effective internal impedance with feedback is given by,

$$\mathbf{Z}_{if}=r_p+(\mu+1)\ R_k=(40\times10^3)+(50+1)\times10^3$$

$$=91\ k\Omega.$$

Output terminal impedance $\mathbf{Z}_{of}$ is given by,

$$\mathbf{Z}_{of}=\frac{\mathbf{Z}_{if}\ .\ \mathbf{Z}_l}{\mathbf{Z}_{if}+\mathbf{Z}_l}$$

$$=\frac{(91\times10^3)\times(47{\cdot}6\times10^3)}{(91+47{\cdot}6)\times10^3}=31{\cdot}25\times10^3 \text{ ohms.}$$

When R_k is reduced to zero,

$$\mathbf{A}=\frac{-\mu\ \mathbf{Z}_l}{r_p+\mathbf{Z}_l}$$

$$=\frac{-50\times47{\cdot}6\times10^3}{(40\times10^3)+(47{\cdot}6\times10^3)}=-27{\cdot}2$$

Effective internal impedance

$$\mathbf{Z}_i=r_p=40\ k\Omega.$$

Output terminal impedance

$$\mathbf{Z}_0=\frac{\mathbf{Z}_i\ .\ \mathbf{Z}_l}{Z_i+Z_l}$$

$$=\frac{(40\times10^3)(47{\cdot}6\times10^3)}{(40\times47{\cdot}6)\times10^3}=21{\cdot}74\times10^3 \text{ ohms.}$$

Example 18. *Following data relate to the amplifier of Example 10. $\mu=50$, $r_p=25\ k\Omega$, $R_l=20\ k\Omega$, $R_1=1000\ \Omega$, $R_2=99\ k\Omega$. Reactance of $C_c \ll (R_1+R_2)$. Calculate,*

(1) voltage gain of the amplifier,

(2) effective internal impedance of the amplifier,

(3) output terminal impedance.

Repeat the calculations for no feedback conditions. Hence verify the relation ;

$$\frac{Z_{of}}{Z_o}=\frac{1}{1-A\beta}.$$

Solution. Z_l=parallel combination of R_l and (R_1+R_2)

$$=\frac{(20\times10^3)(100\times10^3)}{(20+100)\times10^3}=16.7\times10^3 \text{ ohms.}$$

Without Feedback :

Voltage gain without feedback is given by,

$$A=-\frac{\mu Z_l}{r_p+Z_l}=\frac{-50\times16.7\times10^3}{(25+16.7)10^3}=-20$$

Effective internal impedance

$$Z_i=r_p=25\times10^3 \text{ ohms.}$$

Output terminal impedance

$$Z_o=\frac{Z_i\,.\,Z_l}{Z_i+Z_l}$$

$$=\frac{(25\times10^3)(16.7\times10^3)}{(25+16.7)\times10^3}\quad 10000\,\Omega$$

With Feedback :

$$A_{f\beta}=\frac{A}{1-A\beta}=\frac{-\mu Z_l}{r_p+Z_l(1+\beta\mu)}$$

$$\beta=\frac{1000}{100\times10^3}=-0.01$$

Hence $\quad A_{f\beta}=\frac{-50\times16.7\times10^3}{(25\times10^3)+16.7\times10^3(1+.01\times50)}=-16.7$

Effective internal impedance with feedback is given by,

$$Z_{if}=\frac{r_p}{1+\mu\beta}=\frac{25\times10^3}{1+.01\times50}$$

$$=16.66\times10^3 \text{ ohms}$$

Output terminal impedance with feedback is given by,

$$Z_{of}=\frac{Z_{if}Z_l}{Z_{if}+Z_l}=\frac{(16.66\times10^3)(16.7\times10^3)}{(16.66+16.7)\times10^3}$$

$$=8.33\times10^3 \text{ ohms.}$$

But $\quad Z_{of}=\frac{Z_o}{1-A\beta}=\frac{10000}{1+20\times0.01}=8333 \text{ ohms.}$

This value agrees with the value of Z_{of} [illegible]ped previously [illegible]

Example 19. *In the following* [illegible] *feedback* [illegible] *circuit, $R_1=2$ kΩ, $R_2=5$ kΩ, $h_{fe}=49$* [illegible] *small* [illegible] $(h_{ie}+h_{fe})$.

Solution. Load impedance $\mathbf{Z}_l$ is parallel combination of R_l and R_g. Hence

$$\mathbf{Z}_l = \frac{(50\times 10)^3 \times 10^6}{(50\times 10^3)+10^6}$$

$$= \frac{50000}{1050}\times 10^3 = 47{\cdot}6\times 10^3 \text{ ohms.}$$

$$r_p = \frac{\mu}{g_m} = \frac{50}{1250\times 10^{-6}} = 40000 \text{ ohms.}$$

Voltage gain with feedback through R_k is given by,

$$\mathbf{A}_{fb} = \frac{-\mu\ \mathbf{Z}_l}{r_p+(\mu+1)\ R_k+\mathbf{Z}_l}$$

$$= \frac{-50\times 47600}{(40\times 10^3)+(50+1)\times 10^3+(47{\cdot}6\times 10^3)}$$

$$= -17{\cdot}2.$$

Effective internal impedance with feedback is given by,

$$\mathbf{Z}_{if} = r_p+(\mu+1)\ R_k = (40\times 10^3)+(50+1)\times 10^3$$

$$= 91\ k\Omega.$$

Output terminal impedance $\mathbf{Z}_{of}$ is given by,

$$\mathbf{Z}_{of} = \frac{\mathbf{Z}_{if}\ .\ \mathbf{Z}_l}{\mathbf{Z}_{if}+\mathbf{Z}_l}$$

$$= \frac{(91\times 10^3)\times(47{\cdot}6\times 10^3)}{(91+47{\cdot}6)\times 10^3} = 31{\cdot}25\times 10^3 \text{ ohms.}$$

When R_k is reduced to zero,

$$\mathbf{A} = \frac{-\mu\ \mathbf{Z}_l}{r_p+\mathbf{Z}_l}$$

$$= \frac{-50\times 47{\cdot}6\times 10^3}{(40\times 10^3)+(47{\cdot}6\times 10^3)} = -27{\cdot}2$$

Effective internal impedance

$$\mathbf{Z}_i = r_p = 40\ k\Omega.$$

Output terminal impedance

$$\mathbf{Z}_0 = \frac{\mathbf{Z}_i\ .\ \mathbf{Z}_l}{Z_i+Z_l}$$

$$= \frac{(40\times 10^3)(47{\cdot}6\times 10^3)}{(40\times 47{\cdot}6)\times 10^3} = 21{\cdot}74\times 10^3 \text{ ohms.}$$

Example 18. *Following data relate to the amplifier of Example 10.* $\mu=50$, $r_p=25\ k\Omega$, $R_l=20\ k\Omega$, $R_1=1000\ \Omega$, $R_2=99\ k\Omega$. *Reactance of* $C_0 \ll (R_1+R_2)$. *Calculate,*

(1) *voltage gain of the amplifier,*

(2) *effective internal impedance of the amplifier,*

(3) *output terminal impedance.*

Repeat the calculations for no feedback conditions. Hence verify the relation ;

$$\frac{Z_{of}}{Z_o}=\frac{1}{1-A\beta}.$$

Solution. Z_l=parallel combination of R_l and (R_1+R_2)

$$=\frac{(20\times10^3)(100\times10^3)}{(20+100)\times10^3}=16\cdot7\times10^3 \text{ ohms.}$$

Without Feedback :

Voltage gain without feedback is given by,

$$A=-\frac{\mu Z_l}{r_p+Z_l}=\frac{-50\times16\ 7\times10^3}{(25+16\cdot7)\ 10^3}=-20$$

Effective internal impedance

$$Z_i=r_p=25\times10^3 \text{ ohms.}$$

Output terminal impedance

$$Z_o=\frac{Z_i\ .\ Z_l}{Z_i+Z_l}$$

$$=\frac{(25\times10^3)(16\cdot7\times10^3)}{(25+16\ 7)\times10^3}=10000\ \Omega.$$

With Feedback :

$$A_{fb}=\frac{A}{1-A\beta}=\frac{-\mu Z_l}{r_p+Z_l\ (1+\beta\mu)}$$

$$\beta=\frac{1000}{100\times10^3}=+0\ 01.$$

Hence $$A_{fb}=\frac{-50\times16\cdot7\times10^3}{(25\times10^3)+16\cdot7\times10^3\ (1+\ 01\times50)}=-16\ 7$$

Effective internal impedance with feedback is given by,

$$Z_{if}=\frac{r_p}{1+\mu\beta}=\frac{25\times10^3}{1+\ 01\times50}$$

$$=16\cdot66\times10^3 \text{ ohms.}$$

Output terminal impedance with feedback is given by,

$$Z_{of}=\frac{Z_{if}.Z_l}{Z_{if}+Z_l}=\frac{(16\ 66\times10^3)(16\ 7\times10^3)}{(16\ 66+16\ 7)\times10^3}$$

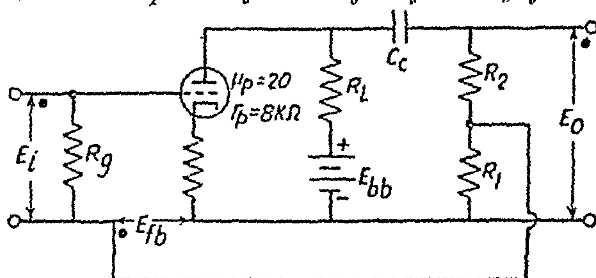
$$=8\cdot33\times10^3 \text{ ohms.}$$

But $$Z_{of}=\frac{Z_0}{1-A\beta}=\frac{10000}{1+20\times0\cdot01}=8333 \text{ ohms.}$$

This value agrees with the value of Z_{of} obtained previously.

Example 19. *In the following compound feedback amplifier circuit, R_l=20 kΩ, R_1=5 kΩ, R_2=495 kΩ. Reactance of C_e is very small compared with (R_1+R_2).*

Calculate (1) voltage gain (2) effective internal impedance and (3) output terminal impedance for each of the following four conditions



(a) $R_k=0$ and no voltage feedback (b) $R_k=0$ and voltage feedback used (c) $R_k=1000$ ohms and no voltage feedback and (d) $R_k=1000$ ohms and voltage feedback used.

Solution. *Case (a).* $R_k=0$; No voltage feedback.

$$\mathbf{Z}_l=\frac{R_l(R_1+R_2)}{R_l+(R_1+R_2)}=\frac{(20\times10^3)(500\times10^3)}{(20+500)\times10^3}$$
$$=19{\cdot}23\times10^3 \text{ ohms}$$

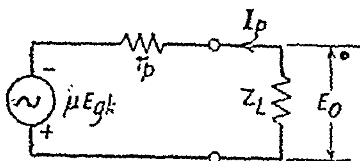
$$\text{Voltage gain } \mathbf{A}=\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l}=\frac{-20\times19{\cdot}23\times10^3}{(8\times10^3)+(19{\cdot}23\times10^3)}=-14{\cdot}12$$

$$Z_t=r_p=8\times10^3 \text{ ohms.}$$

$$\text{Output terminal impedance } \mathbf{Z}_o=\frac{\mathbf{Z}_t\,.\,\mathbf{Z}_l}{\mathbf{Z}_t+\mathbf{Z}_l}$$
$$=\frac{8000\times19{\cdot}23\times10^3}{(8+19{\cdot}23)\times10^3}=5{\cdot}648\times10^3 \text{ ohms.}$$

Case (b). $R_k=0$; voltage feedback used.

A.C. equivalent is given below.



$$\beta=\frac{5000}{500\times10^3}=+0{\cdot}01$$

$$\mathbf{I}_p(r_p+\mathbf{Z}_l)=\mu\mathbf{E}_{gk}=\mu(\mathbf{E}_i+\beta\mathbf{E}_o)$$
$$=\mu[\mathbf{E}_i+\beta(-\mathbf{I}_p.\mathbf{Z}_l)]$$

or
$$\mathbf{I}_p=\frac{\mu\mathbf{E}_i}{r_p+\mathbf{Z}_l(1+\mu\beta)}$$

Voltage gain

$$\mathbf{A}_{fb}=\frac{\mathbf{E}_o}{\mathbf{E}_i}=\frac{-\mathbf{I}_p.\mathbf{Z}_l}{\mathbf{E}_i}=\frac{-\mu\,\mathbf{Z}_l}{r_p+\mathbf{Z}_l(1+\mu\beta)}$$
$$=\frac{-20\times19{\cdot}23\times10^3}{(8\times10^3)+(19{\cdot}23\times10^3)(1+20\times0{\cdot}01)}=-12{\cdot}38$$

Effective internal impedance is given by,

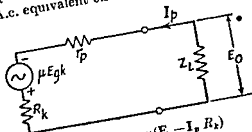
$$\mathbf{Z}_{tf}=\frac{r_p}{1+\mu\beta}=\frac{8000}{1+(20\times0{\cdot}01)}=6666 \text{ ohms}$$

Output terminal impedance is given by,

$$\mathbf{Z}_{of}=\frac{\mathbf{Z}_{tf}\,.\,\mathbf{Z}_l}{\mathbf{Z}_{tf}+\mathbf{Z}_l}=\frac{6666\times 19{\cdot}23\times 10^3}{(6{\cdot}666+19{\cdot}23)\times 10^3}=5007 \text{ ohms.}$$

Case (*c*). $R_k=1000$ ohms ; No. voltage feedback.

A.c. equivalent circuit is given by,



$$\mathbf{I}_p(R_k+r_p+\mathbf{Z}_l)=\mu\mathbf{E}_{gk}=\mu(\mathbf{E}_i-\mathbf{I}_p\,R_k)$$

or
$$\mathbf{I}_p=-\frac{\mu\mathbf{E}_i}{r_p+R_k(\mu+1)+\mathbf{Z}_l}$$

Hence voltage gain

$$\mathbf{A}_{fb}=\frac{\mathbf{E}_0}{\mathbf{E}_i}=\frac{-\mathbf{I}_p\,\mathbf{Z}_l}{\mathbf{E}_i}=\frac{-\mu\mathbf{Z}_l}{r_p+(\mu+1)R_k+\mathbf{Z}_l}$$

$$=\frac{-20\times 19\;23\times 10^3}{(8\times 10^3)+(20+1)\times 10^3+(19\;23\times 10^3)}$$

$$=-7\;977$$

Effective internal impedance

$$\mathbf{Z}_{tf}=r_p+(\mu+1)R_k=(8+21)\times 10^3=29\times 10^3 \text{ ohms}$$

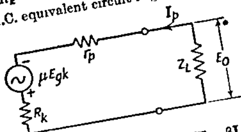
Output terminal impedance

$$\mathbf{Z}_{of}=\frac{\mathbf{Z}_{tf}\quad\mathbf{Z}_l}{\mathbf{Z}_{tf}+\mathbf{Z}_l}=\frac{(29\times 10^3)(19\;23\times 10^3)}{(29+19\;23)\times 10^3}$$

$$=11\;54\times 10^3 \text{ ohms}$$

Case (*d*). $R_k=1000$ ohms , voltage feedback used.

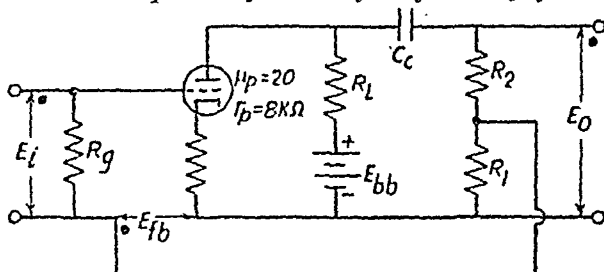
A.C. equivalent circuit is given below.



$$\mathbf{I}_p(R_k+r_p+\mathbf{Z}_l)=\mu\,\mathbf{E}_{gk}$$
$$=\mu[\mathbf{E}_i+\beta\mathbf{E}_0-\mathbf{I}_p\,R_k]=\mu[\mathbf{E}_i-\beta\mathbf{I}_p\mathbf{Z}_l-\mathbf{I}_p\ldots$$

$$\mathbf{I}_p=\ldots\frac{\mu\mathbf{E}_i}{\ldots+R_k(\mu+1)+\mathbf{Z}_l(1+\mu\beta)}$$

Calculate (1) voltage gain (2) effective internal impedance and (3) output terminal impedance for each of the following four conditions



(a) $R_k=0$ *and no voltage feedback (b)* $R_k=0$ *and voltage feedback used (c)* $R_k=1000$ *ohms and no voltage feedback and (d)* $R_k=1000$ *ohms and voltage feedback used.*

Solution. *Case (a).* $R_k=0$; No voltage feedback.

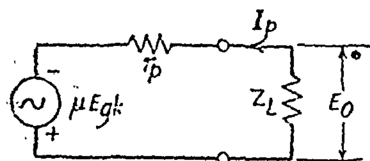
$$Z_l=\frac{R_l(R_1+R_2)}{R_l+(R_1+R_2)}=\frac{(20\times10^3)(500\times10^3)}{(20+500)\times10^3}$$
$$=19{\cdot}23\times10^3 \text{ ohms}$$

$$\text{Voltage gain } \mathbf{A}=\frac{-\mu\mathbf{Z}_l}{r_p+\mathbf{Z}_l}=\frac{-20\times19{\cdot}23\times10^3}{(8\times10^3)+(19{\cdot}23\times10^3)}=-14{\cdot}12$$
$$Z_i=r_p=8\times10^3 \text{ ohms.}$$

$$\text{Output terminal impedance } \mathbf{Z}_o=\frac{\mathbf{Z}_i\,.\,\mathbf{Z}_l}{\mathbf{Z}_i+\mathbf{Z}_l}$$
$$=\frac{8000\times19{\cdot}23\times10^3}{(8+19{\cdot}23)\times10^3}=5{\cdot}648\times10^3 \text{ ohms.}$$

Case (b). $R_k=0$; voltage feedback used.

A.C. equivalent is given below.



$$\beta=\frac{5000}{500\times10^3}=+0{\cdot}01$$
$$\mathbf{I}_p(r_p+\mathbf{Z}_l)=\mu\mathbf{E}_{gk}=\mu(\mathbf{E}_i+\beta\mathbf{E}_o)$$
$$=\mu[\mathbf{E}_i+\beta(-\mathbf{I}_p.\mathbf{Z}_l)]$$

or

$$\mathbf{I}_p=\frac{\mu\mathbf{E}_i}{r_p+\mathbf{Z}_l(1+\mu\beta)}$$

Voltage gain

$$A_{fb}=\frac{\mathbf{E}_o}{\mathbf{E}_i}=\frac{-\mathbf{I}_p.\mathbf{Z}_l}{\mathbf{E}_i}=\frac{-\mu\,\mathbf{Z}_l}{r_p+\mathbf{Z}_l(1+\mu\beta)}$$
$$=\frac{-20\times19{\cdot}23\times10^3}{(8\times10^3)+(19{\cdot}23\times10^3)(1+20\times0{\cdot}01)}=-12{\cdot}38$$

Effective internal impedance is given by,

$$Z_{if}=\frac{r_p}{1+\mu\beta}=\frac{8000}{1+(20\times0{\cdot}01)}=6666 \text{ ohms}$$

$$=\frac{\frac{\mu}{\beta\mu+1}\mathbf{E}_i}{\frac{r_p+R_k(\mu+1)}{\beta\mu+1}+\mathbf{Z}_l}$$

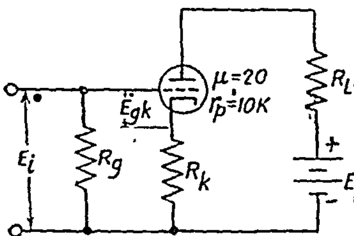
$$\mathbf{Z}_{if}=\frac{r_p+R_k(\mu+1)}{1+\mu\beta}=\frac{(8\times10^3)+10^3(20+1)}{(1+20\times\cdot01)}$$
$$=24\cdot166\times10^3 \text{ ohms.}$$

$$\mathbf{Z}_{of}=\frac{\mathbf{Z}_{if}.\mathbf{Z}_l}{\mathbf{Z}_{if}+\mathbf{Z}_l}=\frac{(24\cdot166\times10^3)(19\cdot23\times10^3)}{(24\cdot166+19\cdot23)\times10^3}$$
$$=10\cdot71\times10^3 \text{ ohms.}$$

Voltage gain

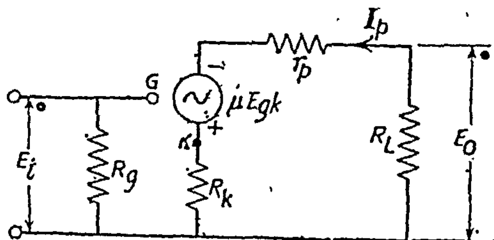
$$\mathbf{A}_{fb}=\frac{\mathbf{E}_o}{\mathbf{E}_i}=\frac{-\mathbf{I}_p.\mathbf{Z}_l}{\mathbf{E}_i}=\frac{-\mu\,\mathbf{Z}_l}{r_p+(\mu+1)R_k+\mathbf{Z}_l(1+\mu\beta)}$$
$$=\frac{-20\times19\cdot23\times10^3}{[8+29+(1+20\times\cdot01)\times19\cdot23]\times10^3}=-6\cdot4.$$

Example 20. *In the amplifier circuit shown, cathode resistance R_k=1000 ohms and resistance R_l=20 kΩ. Calculate (a) voltage gain of the amplifier (b) effective internal impedance and (c) output terminal impedance for the following two conditions (i) output developed across R_l and (ii) output developed across R_k.*



Solution. *Case (i).* Output developed across R_l

A.C. equivalent circuit is shown



$$\mathbf{I}_p\,(r_p+R_k+R_l)=\mu\mathbf{E}_{gk}=\mu(\mathbf{E}_i-\mathbf{I}_p.R_k)$$

or
$$\mathbf{I}_p=\frac{\mu\,\mathbf{E}_i}{r_p+(\mu+1)R_k+R_l}$$

$$\mathbf{A}_{fb}=\frac{\mathbf{E}_o}{\mathbf{E}_i}=\frac{-\mathbf{I}_p.R_l}{\mathbf{E}_i}=\frac{-\mu R_l}{r_p+(\mu+1)R_k+R_l}$$
$$=\frac{-20\times20\times10^3}{(10\times10^3)+(20+1)\times10^3+(20\times10^3)}$$
$$=-7\cdot843$$

Effective internal impedance is given by

$$\mathbf{Z}_{if}=r_p+(\mu+1)R_k=(10\times10^3)+(20+1)\times10^3$$
$$=31\times10^3 \text{ ohms.}$$

or $$I_p = \frac{\mu E_i}{r_p + (\mu+1)R_k}$$

Hence voltage gain

$$A_{fb} = \frac{E_o}{E_i} = \frac{I_p \cdot R_k}{E_i}$$

$$= \frac{\mu R_k}{r_p + (\mu+1)R_k} = \frac{50 \times 10^3}{(20 \times 10^3) + (50+1)10^3}$$

$$= 0{\cdot}7042$$

Expression for I_p may be put in the form,

$$I_p = \frac{\frac{\mu}{\mu+1} E_i}{\frac{r_p}{\mu+1} + R_k}$$

Hence effective internal impedance is given by,

$$Z_{if} = \frac{r_p}{\mu+1} = \frac{20 \times 10^3}{50+1} = 392{\cdot}1 \text{ ohms.}$$

Output terminal impedance is given by,

$$Z_{of} = \frac{Z_{if} \times R_k}{Z_{if} + R_k} = \frac{392 \times 1000}{392 + 1000} = 281{\cdot}6 \text{ ohms.}$$

Example 22. *A triode employed as a cathode follower has amplification factor of 40 and mutual conductance of 4 ma/volt. The load consists of a capacitance of 1000 μμF. Calculate the magnitude and phase angle of the voltage gain of the amplifier at a frequency of 79·6 kc/s.*

Solution. $g_m = 4 \times 10^{-3}$ mho. ; $\mu = 40$

Hence $$r_p = \frac{\mu}{g_m} = \frac{40}{4 \times 10^{-3}} = 10^4 \text{ ohms.}$$

Reactance of condenser

$$X_c = \frac{1}{2\pi f_c}$$

$$= \frac{1}{2\pi \times 79{\cdot}6 \times 10^3 \times 1000 \times 10^{-12}}$$

$$= 2000 \text{ ohms.}$$

$$I_p = \frac{\mu E_i}{r_p + (\mu+1)Z_k}$$

Hence voltage gain

$$A_{fb} = \frac{E_o}{E_i} = \frac{I_p \cdot Z_k}{E_i} = \frac{\mu Z_k}{r_p + (\mu+1)Z_k}$$

Substituting the values,

$$A_{fb} = \frac{40 \times (-j\ 2000)}{10^4 + (40+1)(-j\ 2000)} = \frac{-j\ 80 \times 10^3}{10^4 - j\ 82000}$$

But $\beta=\frac{R_k}{R_l}=\frac{1000}{100000}=0{\cdot}01$

Hence $\mathbf{A}=\frac{-0{\cdot}585}{{\cdot}01}=-58{\cdot}5$

Hence $-58{\cdot}5=\frac{-\mu R_l}{r_p+R_k+R_l+R}$

$$=\frac{-100\times100\times10^3}{(50\times10^3)+10^3+(100\times10^3)+R}$$

or $151\times10^3+R=\frac{+10^7}{58{\cdot}5}=170{\cdot}9\times10^3$

Hence $R=(170{\cdot}9-151)\times10^3=19{\cdot}9\times10^3$ ohms.

Example 24. *Derive expression for the voltage gain of a cathode follower.*

A cathode follower uses a pentode having mutual conductance of 4 ma/volt and the output is required to be matched to a cable of characteristic impedance 75 ohms. Calculate the load resistance necessary. Also calculate the gain of the amplifier, given that the amplification factor of the pentode used is 100.

Solution. $r_p=\frac{\mu}{g_m}=\frac{100}{4\times10^{-3}}=25\times10^3$ ohms.

$$\mathbf{I}_p=\frac{\mu\,\mathbf{E}_i}{r_p+(\mu+1)\,R_k}=\frac{\frac{\mu}{\mu+1}\,\mathbf{E}_i}{\frac{r_p}{\mu+1}+R_k}$$

Effective internal impedance

$$\mathbf{Z}_{if}=\frac{r_p}{\mu+1}$$

Output terminal impedance

$$\mathbf{Z}_{of}=\frac{\mathbf{Z}_{if}\,.\,R_k}{\mathbf{Z}_{if}+R_k}=\frac{\frac{r_p}{\mu+1}\,.\,R_k}{\frac{r_p}{\mu+1}+R_k}$$

$$=\frac{r_p\,.\,R_k}{r_p+(\mu+1)\,R_k}$$

For proper matching $\mathbf{Z}_{of}$ should be 75 ohms.

Hence $75=\frac{r_p\,.\,R_k}{r_p+(\mu+1)\,R_k}$

$$=\frac{(25\times10^3)\,.\,R_k}{(25\times10^3)+(100+1)\,R_k}$$

But $\qquad \beta=\dfrac{R_k}{R_l}=\dfrac{1000}{100000}=0{\cdot}01$

Hence $\qquad \mathbf{A}=\dfrac{-0{\cdot}585}{\cdot 01}=-58{\cdot}5$

Hence $\quad -58{\cdot}5=\dfrac{-\mu R_l}{r_p+R_k+R_l+R}$

$$=\frac{-100\times 100\times 10^3}{(50\times 10^3)+10^3+(100\times 10^3)+R}$$

or $151\times 10^3+R=\dfrac{+10^7}{58{\cdot}5}=170{\cdot}9\times 10^3$

Hence $\qquad R=(170{\cdot}9-151)\times 10^3=19{\cdot}9\times 10^3$ ohms.

Example 24. *Derive expression for the voltage gain of a cathode wer.*

A cathode follower uses a pentode having mutual conductance of /volt and the output is required to be matched to a cable of charactic impedance 75 ohms. Calculate the load resistance necessary. calculate the gain of the amplifier, given that the amplification r of the pentode used is 100.

Solution. $r_p=\dfrac{\mu}{g_m}=\dfrac{100}{4\times 10^{-3}}=25\times 10^3$ ohms.

$$\mathbf{I}_p=\frac{\mu\,\mathbf{E}_i}{r_p+(\mu+1)\,R_k}=\frac{\dfrac{\mu}{\mu+1}\,\mathbf{E}_i}{\dfrac{r_p}{\mu+1}+R_k}$$

Effective internal impedance

$$\mathbf{Z}_{if}=\frac{r_p}{\mu+1}$$

Output terminal impedance

$$\mathbf{Z}_{of}=\frac{\mathbf{Z}_{if}\,.\,R_k}{\mathbf{Z}_{if}+R_k}=\frac{\dfrac{r_p}{\mu+1}\,.\,R_k}{\dfrac{r_p}{\mu+1}+R_k}$$

$$=\frac{r_p\,.\,R_k}{r_p+(\mu+1)\,R_k}$$

For proper matching $\mathbf{Z}_{of}$ should be 75 ohms.

Hence $\qquad 75=\dfrac{r_p\,.\,R_k}{r_p+(\mu+1)\,R_k}$

$$=\frac{(25\times 10^3)\,.\,R_k}{(25\times 10^3)+(100+1)\,R_k}$$

$$=\frac{1}{2000}+\frac{1}{100\times10^3}+j\times2\pi\times796\times10^3\times500\times10^{-12}$$

$$=[510+j\ 2500]\times10^{-6}$$

Hence voltage gain at 796 kc/s is given by,

$$A_h=\frac{\mu\ Z_{kh}}{r_p+(\mu+1)\ Z_{kh}}=\frac{\mu}{(\mu+1)+\frac{r_p}{Z_{kh}}}$$

$$=\frac{50}{(50+1)+12500\times10^{-6}\ (510+j\ 2500)}$$

$$=\frac{50}{57{\cdot}375+j\ 31{\cdot}25}$$

Magnitude $A_h=\frac{50}{\sqrt{(57{\cdot}375)^2+(31{\cdot}25)^2}}=0{\cdot}7654.$

Example 26. *Derive expression for the voltage gain of a grounded-grid amplifier. A grounded-grid amplifier uses a triode having $\mu=20$ and $r_p=8000$ ohms and has load resistance of 10,000 ohms. The impedance of the input voltage source is 1000 ohms. Calculate the voltage gain and output terminal impedance.*

Solution. Voltage gain is given by,

$$A_{fb}=\frac{(\mu+1)\ R_l}{r_p+(\mu+1)\ R_i+R_l}$$

$$=\frac{(20+1)\times10^4}{(8\times10^3)+(20+1)+10^3+10^4}=5{\cdot}384.$$

Effective internal impedance

$$Z_{if}=r_p+(\mu+1)\ R_i$$

$$=(8+20+1)\times10^3=29\times10^3 \text{ ohms.}$$

Output terminal impedance

$$Z_{of}=\frac{Z_{if}\times R_l}{Z_{if}+R_l}$$

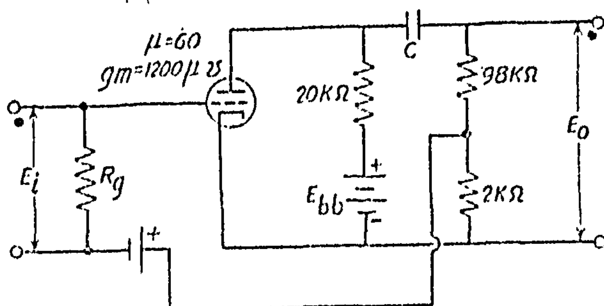
$$=\frac{(29\times10^3)+(10\times10^3)}{(29+10)\times10^3}=7435 \text{ ohms.}$$

EXERCISES

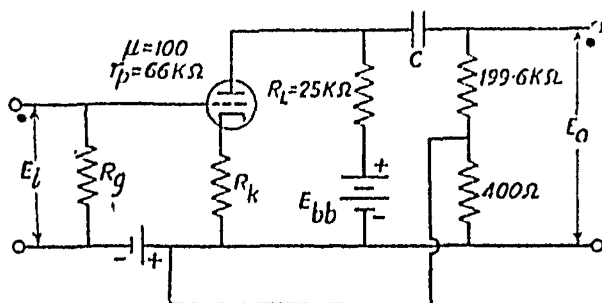
1. An amplifier has voltage gain of -400. If the voltage gain is reduced to -100 by negative feedback, determine the feedback ratio β.

2. In a feedback amplifier voltage gain with negative feedback is $500\ \angle 0$ and feedback ratio is $-0{\cdot}002$. Calculate the voltage in case the feedback ratio is made $-0{\cdot}004$.

3. An amplifier has voltage gain of 6000 with normal plate supply voltage and a gain of 4000 with reduced plate supply voltage.



9. For the feedback amplifier circuit shown using both current and voltage inverse feedback, calculate the value of R_k for which overall voltage gain with compound feedback is 30-*db* below the volatge gain without any feedback.



10. In a four-stage amplifier, voltage gain of first two stages is -50 each and that of last two stage is -100 each. A negative feedback network with feedback ratio of $-\frac{1}{50}$ may be connected across any two consecutive stages. Find the overall voltage amplification in all the three cases.

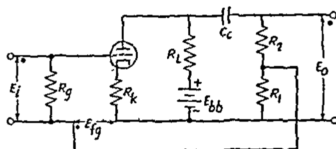
11. A resistance-capacitance coupled amplifier has load resistance $R_l=50$ kilo-ohms, grid leak resistance of next stage$=500$ kilo-ohms, coupling condenser$=\cdot01$ micro-farad and total shunt capacitance equal to 1000 micro-micro farad. If the amplifier tube has amplification factor of 20 and dynamic plate resistance of 8000 ohms, calculate the voltage gain of the amplifier at frequencies of 10, 100, 1,000 and 10,000 cycles per second. If negative feedback is provided by feeding $\frac{1}{50}$th of the output voltage back to the input, calculate the voltage gain of the amplifier at the above mentioned frequencies.

12. A three-stage R.C. coupled feedback amplifier has all stages exactly identical. Each stage has a load resistance of 50 kilo-ohms, coupling condenser of 0·1 micro-farad and grid leak resistance of 500 kilo-ohms. Amplification factor and anode slope resistance of amplifier tube are respectively equal to 20 and 10,000 ohms. Total effective shunt capacitance in each stage is 500 μμF. One hundredth of the output of last stage is fed to the input of the first

stage, resulting in negative feedback at midband frequencies. Calculate the voltage gain of the amplifier with and without feedback at frequencies of 10 cycles and 100 kc/s.

13. ... amplifier
$=20$ kΩ,
C_1 and C_2 to have negligible reactance at the frequency of operation. A.C. impedance of cathode biasing circuit may be neglected. Calculate the overall gain of the 2-stage amplifier with feedback and also the effective internal impedance.

14. Following particulars relate to the composite feedback amplifier shown in following diagram : $\mu=50$, $r_p=20$ kΩ, $R_l=50$ kΩ, $R_1=1$ kΩ, $R_2=49$ kΩ.



Reactance of coupling condenser C_c is negligible at operating f... n and effective internal imped... four conditions :—(a) $R_k=0$, ... voltage feedback (c) $R_k=1000$ ohms, no voltage feedback (d) $R_k=1000$ ohms, with voltage feedback.

15. An amplifier uses triode with amplification factor $\mu=15$, and dynamic plate resistance $r_p=7500$ ohms. The load resistance is 10,000 ohms and there is cathode resistance of 500 ohms. Calculate the voltage gain and output terminal impedance for the following two cases (i) output developed across R_l and (ii) output developed across R_k.

16. A valve having dynamic plate resistance of 12 kilo-ohms and amplification factor of 24 is used as a cathode follower working into a resistive load of 3,000 ohms Calculate (a) the voltage gain and (b) output terminal impedance Prove any formula used

CHAPTER XIV

UNTUNED POWER AMPLIFIERS

The primary function of voltage amplifiers is to increase the signal voltage from a low level to a reasonably high voltage level without distortion *i. e.* keeping the signal wave-form unaltered. Distortionless amplification requires that operation be linear class *A*. It is further necessary to keep grid current negligibly small by suitably adjusting the value of grid bias and grid signal amplitude. If grid current is allowed to flow during part of the signal cycle grid-to-cathode circuit offers non-linear resistance and results in waveform distortion of the grid signal particularly when there exists an appreciable impedance in series with the grid. The amplified output waveform will then also be distorted. Hence it is necessary in voltage amplifiers to disallow grid current *i.e.* use class A_1 operation. Class A_1 operation is obtained by allowing operation of the tube over linear portion of the tube characteristics by proper adjustment of bias and allowing small excursions of the signal.

Primary function of power amplifier, on the other hand, is to increase the signal power from a low level to such high level as may be required by the load with very small grid driving power demand. To get the necessary high power output it is often required to allow excursion of the operating point beyond absolutely linear range of tube characteristics. By so doing the ratio of average value of plate current I_b to the value of varying component I_p is reduced *i.e.* larger fraction of power from the plate supply source is utilized in producing useful a.c. outp t power. Stated otherwise, plate circuit efficiency of the amplifier is increased This results in lesser wastage of power. This factor is of great significance at higher power levels. But the above mentioned increase in plate circuit efficiency is obtained at the cost of distortion which is caused by the operation over non-linear region of tube characteristics. Harmonic components so introduced increase in magnitude as the amplitude of signal is increased to larger and larger values Methods must, therefore, be deviced in power amplifiers to reduce the distortion terms. Two techniques are available for reducing the harmonic distortion (*i*) push-pull operation of amplifiers and (*ii*) use of tuned circuit as plate load.

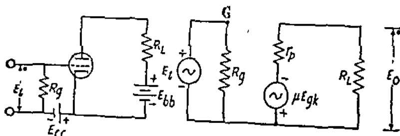
The push-pull amplifier removes even harmonic terms and thus the total harmonic distortion may be reduced to reasonably small magnitude. When power amplifier uses a parallel tuned circuit as the plate load, it responds to only a narrow band of frequencies centred about the resonant frequency and hence all harmonic terms are automatically removed. But such a technique can be used only in Radio Frequency amplifiers because of the restricted frequency range in which the amplifier operates.

Derivations in subsequent articles will show that series-fed class A_1 amplifiers have small plate circuit efficiency, theoretical maximum being 50% for idealized pentode and 25% for idealized triode. Practical values of plate circuit efficiency are much smaller, being about 10-15 % for triode amplifiers and about 20-25% for pentode-amplifiers. This means that in triode amplifiers, as much as 85% or more of the plate supply power is wasted and only 10-15% is available as useful a c. output power. This wastage is evidently excessive at high power levels. To get better plate circuit efficiency amplifier operation is changed from A to B or C. Class B operation gives maximum theoretical plate circuit efficiency of 78·5% and practical maximum value of about 6 % while class C gives [illegible] usually exceeding 70%. Thus class [illegible] ate circuit efficiencies in the increa[illegible] harmonic distortion also increases in the similar order. Radio frequency power amplifiers can conveniently utilize B or C operation with high plate circuit efficiency because the tuned circuit eliminates all distortion. But n audio frequency power amplifiers class A and B operations alone can be used. To reduce distortion push pull operation is always used in class B audio frequency power amplifiers. But class C audio frequency operation introduces such a large distortion that even push-pull operation cannot handle it. Consequently class C operation of untuned amplifiers is never permitted. In this chapter we will deal with the untuned power amplifiers alone.

CLASS A_1 UNTUNED POWER AMPLIFIERS

Series-fed Power Amplifiers.

Fig. 14 1 (*a*) shows the basic schematic diagram of an untuned class A_1 series-fed power amplifier using triode. Its a.c equivalent circuit is shown in Fig. 14 1 (*b*). It may be readily seen that this circuit is the same as that of a simple class A_1 untuned voltage amplifier. We further assume that the operation is confined to the linear portion of tube characteristics similar to that in linear A_1 voltage amplifiers.



(*a*) Circuit diagram. (*b*) A.C. equivalent circuit.

Fig. 14 1. Schematic diagram and a.c equivalent circuit of a simple class A_1 series fed untuned power amplifiers.

In general, in any amplifier, power is fed to the vacuum tube from the following three sources :—(i) from the grid circuit (ii) from plate circuit and (iii) from cathode heating circuit. In class A_1 operation, power from grid circuit is negligible. The grid voltage simply controls the plate current and thus controls the a.c. power output. As the amplitude of grid signal voltage is increased, amplitude of alternating plate current component is correspondingly increased and hence the a.c. power output is increased. The power consumed in cathode heating remains constant and is given by $E_f.I_f$ wher E_f and I_f are respectively the effective values of filament voltage an filament current. The power supplied from the plate power supply is more important and requires careful study.

Here we assume grid signal to be a pure sine wave having no d.c. component. Plate load is assumed to be a pure resistance R_L. Fig. 14·2 shows the load line, zero-signal operating point and output current and output voltage wave-forms for the pure sinusoidal grid signal.

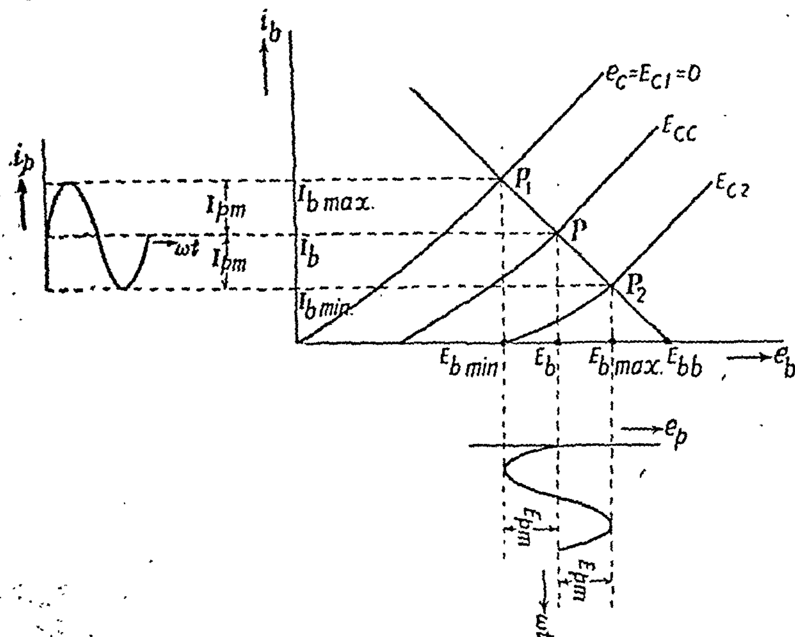


Fig 14·2. Output current and voltage waveforms in a series-fed A_1 triode power amplifier.

Let us first consider the zero signal condition. I_b and E_b are respectively zero signal plate current and plate voltage. Then total power supplied from the plate power supply is given by,

$$P_{bb} = E_{bb}.I_b \qquad \dots (14\cdot1)$$

Out of this total power P_{bb}, a part is consumed in the tube and the remaining in the plate load. The power consumed in the tube at

zero signal is given by

$$P_{po} = E_b I_b \qquad \text{... (14·2)}$$

This power P_{po} is transformed into heat at the plate and gets dissipated there. This, therefore, is zero-signal plate dissipation.

The power absorbed in the plate load is given by,

$$P_{dc} = I_b^2 R_l \qquad \text{... (14·3)}$$

But for series-fed amplifier,

$$E_{bb} = E_b + I_b . R_l \qquad \text{... (14·4)}$$

Hence $$E_{bb} . I_b = E_b . I_b + I_b^2 . R_l \qquad \text{... (14·5)}$$

or $$P_{bb} = P_{po} + P_{dc} \qquad \text{... (14·6)}$$

Next consider the situation when grid-signal is applied. Plate current i_b then has d c. component as well as a c. component. Let the grid-signal be a pure sine wave given by,

$$e_{gk} = E_{gm} \cos \omega t \qquad \text{... (14·7)}$$

The average value of total power input from the plate power supply is,

$$P_{bs} = \frac{1}{2\pi}\int_0^{2\pi} E_{bb} . i_b \, d(\omega t) \qquad \text{... (14·8)}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} E_{bb}(I_b + i_p) \, d(\omega t) \qquad \text{... (14·9)}$$

In general,

$$i_p = I_{po} + I_{p1m} \cos \omega t + I_{p2m} \cos 2\,\omega t + \ldots$$

But since the operation is linear A_1 and the grid signal e_{gk} as given by equation (14·7) contains no d c. component, then,

$$I_{po} = I_{p2m} = I_{p3m} = 0 \qquad \text{...(14·10)}$$

Since all harmonics are absent, let I_{p1m} be indicated simply by I_{pm}.

Hence power from plate supply source is,

$$P_{bs} = E_{bb} I_b + \frac{1}{2\pi}\int_0^{2\pi} {}_{bb}(I_{pm} \cos \omega t) \, d(\omega t) \qquad \text{.. (14·11)}$$

$$= E_{bb} . I_b .$$

Thus for linear A_1 operation $P_{bs} = P_{bb}$ i.e. the power drainage from B-battery remains constant at value $P_{bb} = E_{bb} I_b$.

Power delivered to the load is given by,

$$P_l = \frac{1}{2\pi}\int_0^{2\pi} i_b^2 R_l \, d(\omega t) \qquad \text{...(14·12)}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (I_b + I_{pm} \cos \omega t)^2 R_l \, d(\omega t) \qquad \text{[illegible]}$$

neglecting harmonic components I_{p2}, I_{p} [illegible]
average component I_{po} etc. [illegible]

or $$P_l = I_b^2 R_2 + \frac{1}{2\pi}\int_0^{2\pi} 2I_b.I_{pm} \cos \omega t \, d(\omega t) + \frac{1}{2\pi}\int_0^{2\pi} I^2_{pm} \cos^2 \omega t \, d(\omega t) \quad ...(14\cdot14)$$

or $$P_l = P_{dc} + P_{ac} \quad ...(14\cdot15)$$

where P_{ac} = a.c. power output

$$= \frac{1}{2\pi}\int_0^{2\pi} I^2_{pm} \cos^2 \omega t \, R_l \, d(\omega t)$$

$$= I^2_p \, R_l \quad ...(14\cdot16)$$

where I_p is the r.m.s. value of fundamental component.

The second term in right hand side of equation (14·14) is zero.

From equation (14·15) we conclude that with the application of grid-signal, power delivered to the load increases by an amount equal to P_{ac} but the total power taken from the plate supply source remains the same. The power delivered to the tube *i.e.* the plate dissipation must, therefore, reduce by the same amount in accordance with the principle of Conservation of Energy.

The same result may, however, also be obtained by integrating the instantaneous power $e_b.i_b$ given to the tube. Accordingly plate dissipation in the presence of grid-signal is given by,

$$P_p = \frac{1}{2\pi}\int_0^{2\pi} e_b \, i_b \, d(\omega t) \quad ...(14\cdot17)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (E_{bb} - I_b.R_l) \, i_b \, d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \left[E_{bb} - I_b \, R_l - I_{pm} \, R_l \cos \omega t\right]\left[I_b + I_{pm} \cos \omega t\right] d(\omega t) \quad ...(14\cdot18)$$

$$= E_{bb}.I_b - I_b^2 \, R_l - \frac{1}{2\pi}\int_0^{2\pi} I^2_{pm} \, R_l \cos^2 \omega t.d(\omega t)$$

The remaining terms in equation (14·18) become zero

or $$P_p = P_{bb} - (P_{dc} + P_{ac}) \quad ...(14\cdot19)$$

But $$P_{bb} = P_{po} + P_{dc} \quad ...(14\cdot20)$$

Hence $$P_p = P_{po} - P_{ac} \quad ...(14\cdot21)$$

Thus on application of grid-signal plate dissipation reduces by an amount equal to a.c. power output. Since power P_{bb} from plate-supply source remains the same for linear A_1 operation, tube simply removes an amount of power P_{ac} from plate dissipation and makes it available as the useful a.c. output power across the load resistance. This function is performed by the tube under the controlling force

power is, however, derived from the grid circuit. Complete power
simply converts a part of
into useful a.c. power
ximum with zero signal
and reduces as the signal amplitude increases is of considerable
iers Application of grid-
Tubes normally operate at
If accidently the grid-
exceeds the rated value
and temperature exceeds the maximum permitted value thereby damaging the tube. Similarly in radio frequency power amplifier such as are used in radio transmitters, if the tuned circuit acting as the plate load gets detuned perchance, then the alternating power output P_{ac} becomes almost zero and plate dissipation again may exceed the permissible limit. Considerable care is, therefore, required in the operation of such a high power tube.

Out of the total power E_{bb} received from the plate-supply source, only a part equal to P_{ac} is available as the useful a.c output power. Plate circuit efficiency is defined as the ratio of a.c. power output P_{ac} to total d c. power input P_{bb} from the plate-supply source.

Thus percentage plate circuit efficiency is given by,

$$\eta_p = \frac{P_{ac}}{P_{bb}} \times 100 = \frac{P_{ac}}{E_{bb}\, I_b} \times 100 \text{ per cent} \quad ...(14\cdot22)$$

Normally only a.c power across the load is taken as the useful power. D. C. power in the load $I_b^2.R_l$ does not provide useful output

From Eq.(14·22) we see that in order to have greater plate circuit efficiency d.c. plate current I_b should be as small as possible. High value of I_b is the main reason for low plate circuit efficiency of class A amplifiers. To increase the plate circuit efficiency, operation is changed from A to B or C which have low values of I_b.

Maximum Power Sensitivity of Series-fed power Amplifier.

The following analysis applies to the series-fed power amplifier of Fig. 14·1 using a load resistance of R_l but it applies with only a slight modification to shunt-fed amplifier or transformer-coupled power amplifier. Fig. 14·1 shows a triode as the amplifier tube but it applies equally well to any multi-grid tube.

In the last article we have seen that useful a.c. power output must be maximized in order to obtain a high plate circuit efficiency.
I nfronted
as, and
assume
that the following things are prescribed : (*a*) tube (*b*) linear class A_1 operation (*c*) grid signal voltage and (*d*) zero-signal operating point. The only variable then left is the load impedance and we propose to establish the condition for maximum power output and to find the value of this maximum power output.

For the sake of generality let us consider a complex load impedance given by,

$$Z_l = R_l + jX_l \quad \text{...(14·23)}$$

With reference to a.c. equivalent circuit of Fig. 14·1(b), a.c. plate current I_p is given by,

$$I_p = \frac{\mu E_{gk}}{r_p + Z_l} = \frac{\mu E_{gk}}{(r_p + R_l) + jX_l} \quad \text{...(14·24)}$$

Magnitude $$I_p = \frac{\mu E_{gk}}{\sqrt{(r_p + R_l)^2 + X_l^2}} \quad \text{...(14·25)}$$

A.C. power output is given by,

$$P_{ac} = I_p^2 . R_l = E_p . I_p \cos \theta \quad \text{...(14·26)}$$

where θ is the impedance angle of load impedance.

Hence $$P_{ac} = \frac{\mu^2 E_{gk}^2 R_l}{(r_p + R_l^2 + X_l)^2} \quad \text{...(14·27)}$$

The term power sensitivity is defined as the ratio of useful power output to square of grid signal voltage E_{gk}.

The power sensitivity $$= \frac{P_{ac}}{E^2_{gk}} = \frac{\mu^2 R_l}{(r_p + R_l)^2 + X_l^2} \quad \text{...(14·28)}$$

Since load impedance consists of both real and imaginary components R_l and X_l, either or both of them may be variable. Accordingly three conditions are possible, (a) R_l alone is variable (b) X_l alone is variable and (c) both R_l and X_l are variable. If both R_l and X_l vary independently condition becomes very complex and hence we impose the condition that R_l and X_l vary in such a manner that impedance angle θ of Z_l remains constant.

We first consider the condition that R_l alone is variable. Then in order to get the condition of maximum power output we differentiate Eq. (14·27) with respect to R_l and equate it to zero. We then get,

$$\frac{dP_{ac}}{dR_l} = \mu^2 E_{gk}^2 \left[\frac{(r_p + R_l)^2 * X_l^2 - 2R_l (R_p + R_l)}{[(r_p + R_l)^2 + X_l^2]^2} \right] = 0 \quad \text{...(14·29)}$$

or $$(r_p + R_l)^2 + X_l^2 = 2 R_l (r_p + R_l$$

or $$R_l = \sqrt{r_p^2 + X_l^2} \quad \text{...(14·30)}$$

When X_l alone is variable, obviously P_{ac} is maximum when $X_l = 0$.

When Z_l is variable, keeping impedance angle θ constant, P_{ac} may be put as,

$$P_{ac} = \mu^2 E_{gk} \frac{R_l}{(r_p + R_l)^2 + (QR_l)^2} \quad \text{...(14·31)}$$

where $Q = \frac{X_l}{R_l}$.

If impedance angle θ remains constant, Q also remains constant

To get the condition for maximum a.c. power output, we differentiate P_{ac} with respect to R_l.

$$\frac{dP_{ac}}{dR_l} = \mu^2 E^2_{st}\left[\frac{(r_p+R_l)^2+(QR_l)^2-R_l\,[2Q^2R_l+2(r_p+R_l)]}{[(r_p+R_l)^2+(QR_l)^2]}\right]$$

$$= 0 \qquad \text{...(14·32)}$$

or $\quad r_p^2+R_l^2+2r_pR_l+Q^2R_l^2 = 2Q^2R_l^2+2R_lr_p+2R_l^2$

or $\quad r_p = \sqrt{R_l^2+X_l^2} = Z_l \qquad \text{...(14·33)}$

Often however, load impedance is a pure resistance. Expression for P_{ac} then is,

$$P_{ac} = \mu^2E^2_{st}\frac{R_l}{(r_p+R_l)^2} \qquad \text{...14·34)}$$

Obviously P_{ac} is maximum when $r_p = R_l$ i.e. source resistance r_p is equal to load resistance R_l. This power output, obtained when load resistance is equal to source resistance, is the maximum power available from any energy source, and hence is referred to as "available power" and may be indicated by P_{av}

Hence $P_{av} = \frac{(\mu E_{sx})^2}{4r_p} = (g_m E_{sx})^2\frac{r_p}{4} \qquad \text{(14·35)}$

Maximum power sensitivity, then is given by

Maximum power sensitivity $= \frac{P_{av}}{E^2_s} = \frac{\mu^2}{4r_p} \qquad \text{(14·36)}$

At any value of load resistance R_l, a.c. power output is given by,

$$\frac{P_{ac}}{P_{av}} = \frac{R_l}{(r_p+R_l)^2}\times\frac{4r_p}{1} = \frac{4}{\frac{r_p}{R_l}+2+\frac{R_l}{r_p}} \qquad \text{(14·37)}$$

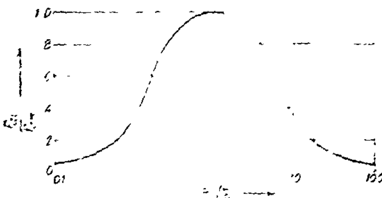


Fig. 14·3. Variation a.c. power output as a function of relative load resistance $\frac{R_l}{r_p}$.

Fig. 14·3 shows the variation of ratio $\frac{P_{ac}}{P_{av}}$ with ratio $\frac{R_l}{r_p}$.

The curve is seen to be symmetrical on either side of value $\frac{R_l}{r_p} = 1$. This follows directly from the symmetry of the denominator of Eq. (12·37).

It may be noted that though the output power is maximum at $R_l/r_p = 1$, this maximum is very broad. As the resistance ratio R_l/r_p varies from 0·5 to 2·0, P_{ac}/P_{av} does not reduce below 0·88 *i.e.* about 2·25 *db.*

Further from Eq. (14·36), $\frac{\mu^2}{4r_p}$ $\left(\text{or } \frac{\mu g_m}{4}\right)$ is a figure of merit of the tube from the consideration of maximum power output. This quantity $\frac{\mu g_m}{4}$ depends entirely on the tube construction. Since r_p is required to be equal to R_l, it is preferred to use low r_p tube in order to obtain reasonable amount of power output with usual value of plate supply voltage. Further g_m of an electron tube cannot be designed to vary over a large range and hence in general low r_p tubes will also have low μ. Hence in order to get sufficient a.c. power output, grid excitation voltage must be large.

For multigrid tubes condition of $R_l = r_p$ is not economical and hence factor $\frac{\mu g_m}{4}$ cannot be taken as a figure of merit of the tube.

Shunt-fed and transformer-coupled linear A_1 power amplifiers.

Fig. 14·5 shows the static and dynamic load lines for the shunt-fed triode amplifier of Fig. 14·4. In this case zero-signal plate

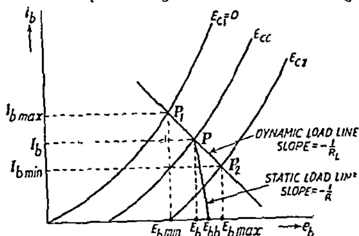


Fig. 14·5. Dynamic and static load lines of a shunt-fed power amplifier.

current [illegible] of [illegible] R [illegible] For [illegible] is equal to E_{bb}.

Often it is preferred to use transformer coupling. This has the advantage over inductance-capacitance coupling of facility of impedance matching. Thus a small load resistance may be matched to large plate resistance r_p and optimum power transfer condition may be attained. This is particularly useful when very low impedance such as the voice coil of loud speaker having resistance of a few ohms is to be fed from a power amplifier. Fig. 14·6 shows the transformer coupled power amplifier.

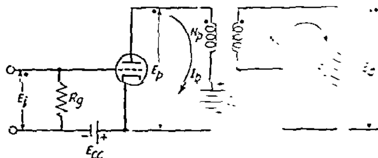


Fig. 14·6. Transformer coupled power amplifier

If N_p and N_s indicate the [illegible] primary and secondary turns, then primary voltage E_p [illegible] secondary voltage [illegible] the relation,

$$E_p = \frac{N_p}{N_s} E_s$$

Also $$I_p = \frac{N_s}{N_p} . I_s \qquad ...(14.39)$$

Hence $$\frac{E_p}{I_p} = \left(\frac{N_p}{N_s}\right)^2 \frac{E_o}{I_s} \qquad ...(14.40)$$

Hence load impedance reflected to the primary become,

$$R_p = \left(\frac{N_p}{N_s}\right)^2 R_l \qquad ...(14.41)$$

Equation (14.41) applies to an ideal transformer with no leakage inductances and winding resistances. In practical transformers input impedance Z_p is given by,

$$Z_p = R_l\left(\frac{N_p}{N_s}\right)^2 + R_p + R_s\left(\frac{N_p}{N_s}\right)^2 + j\omega\left[L_p + L_s\left(\frac{N_p}{N_s}\right)^2\right] \qquad ...(14.42)$$

where L_p and L_s are primary and secondary leakage inductances and R_p and R_s are primary and secondary winding resistances.

Static and dynamic load lines of Fig. 14.5 apply to transformer coupled amplifier as well provided that the static load line is drawn for R_p, the primary winding resistance while the dynamic load line corresponds to $\left(\frac{N_p}{N_s}\right)^2 \left(R_l + R_s \right)$ neglecting the leakage inductances.

Maximum undistorted power output in triode amplifier.

In foregoing analysis for maximum power sensitivity, distortion caused by non-linearity of tube characteristic has been ignored. Considerable curvature of the plate characteristics exists in the region of small plate current. Hence in distortion-free class A_1 operation, while attempting to obtain maximum a.c. power output, care has to be taken that plate current does not become less than a certain minimum value I_d during the most negative part of the grid signal cycle and the total instantaneous grid voltage e_c does not become positive during the most positive part of the grid cycle. Such a condition can be attained by a careful selection of the following quantities : grid bias E_{cc}, load impedance R_l, plate supply voltage E_{bb} and dynamic plate resistance r_p.

Fig. 14.7 shows the graphical construction for determination of operating conditions for maximum undistorted a.c. output power. I_d indicates the minimum plate current below which operation is not to be allowed in order to avoid distortion. Obviously for class A_1 linear operation and undistorted output, maximum swing of the point of operation may be allowed between zero grid bias and a negative grid bias which yields plate current equal to I_d. We here assume that above the horizontal line corresponding to plate current of I_d, plate characteristics are almost linear with a slope corresponding to r_p and hence amplification is distortion-free. For a given plate supply voltage (or alternatively for a prescribed zero-signal plate voltage E_b) we are required to find a value of load resistance R_l which provides the maximum output power.

From Fig. 14·7, we note that

$$E_b = E_d + 2I_{pm} r_p + E_{pm} \qquad ...(14·43)$$

where E_{pm} and I_{pm} are respectively peak values of alternating plate voltage and plate current.

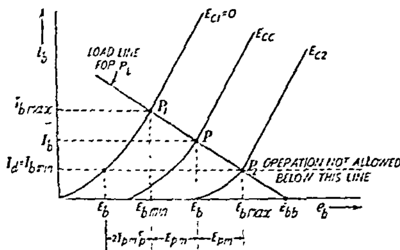


Fig. 14·7. Graphical construction for determining maximum undistorted a.c. power output and the corresponding operating condition in linear A_1 triode amplifier.

But $\qquad E_{pm} = I_{pm} R_l \qquad .. (14·44)$

Hence $\qquad E_b = E_d + 2 I_{pm} r_p + I_p R_l$

$$= E_d + I_{pm} (2 r_p + R_l) \qquad .. (14·45)$$

or $\qquad I_{pm} = \dfrac{E_b - E_d}{R_l + 2r_p} \qquad .. (14·46)$

Hence a.c. power output is given by,

$$P_{ac} = \frac{I^2_{pm} R_l}{2}$$

$$= \frac{(E_b - E_d)^2}{16\, r_p} \times \frac{4(R_l/2r_p)}{(1 + R_l/2r_p)^2} \qquad ..(14·47)$$

Obviously P_{ac} is maximum when $R_l = 2r_p$. Let this maximum value be indicated by $P_{ac\,max}$

Hence $\qquad P_{ac\,max} = \dfrac{(E_b - E_d)^2}{16 r_p} \qquad ...(14·48)$

so that $\qquad \dfrac{P_{ac}}{P_{ac\,max}} = \dfrac{4\left(\dfrac{R_l}{2r_p}\right)}{\left(1 + \dfrac{R_l}{2r_p}\right)^2} \qquad ...(14·49)$

Fig. 14·8 shows the variation of ratio $P_{ac}/P_{ac\,max}$ with the relative load resistance R_l/r_p

From Fig. 14·8 we see that the output power reaches a maximum at $R_l = 2\,r_p$ but in this case again the maximum is very broad.

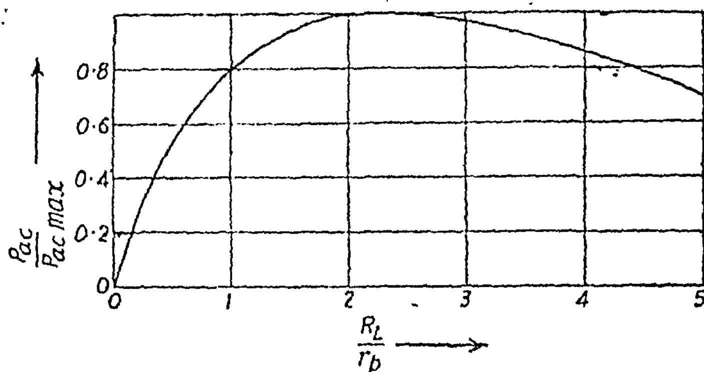


Fig. 14·8. Variation of relative power output $\frac{P_{ac}}{P_{ac\,max}}$ with relative load resistance $\frac{R_l}{r_p}$ for undistorted amplification condition.

As the point of operation moves from zero signal position P to zero grid-voltage point P_1 grid voltage changes by an amount equal to $E_{cc} = E_{gm}$ and the plate current changes by an amount I_{pm}.

Hence $$I_{pm} = \frac{\mu E_{cc}}{R_l + r_p} \qquad (14·50)$$

Combining equations (14·50) and (14·46 and putting $R_l = 2r_p$ we get,

$$E_{cc} = \frac{3}{4\mu}\left(E_b - E_d\right) \qquad ...(14·51)$$

E_d may be obtained directly from the plate characteristic curves. This is the point where characteristic for $e_c = 0$ cuts horizontal line for I_d, the minimum allowed plate current. Further if zero signal plate voltage is prescribed, the value of E_{cc} which satisfies the condition of $R_l = 2r_p$ for maximum a.c. power output as given by equation (14·51).

Equation (14·51) may be utilized to obtain expression for $P_{ac\,max}$ in terms of grid bias E_{cc}. Thus combining equations (14·51) and (14·48) we get,

$$P_{ac\,max} = \left(\frac{4\mu}{3}E_{cc}\right)^2 \times \frac{1}{16\,r_p} = \frac{\mu^2 E_{cc}^2}{9\,r_p} \qquad ...(14·52)$$

But amplitude of grid signal for maximum power output equals E_{cc}. Hence,

$$P_{ac\,max} = \frac{\mu^2 E^2_{gm}}{9r_p} = \frac{2}{9}\,\mu_{gm}\,E^2_{gk} \qquad ...(14·53)$$

Hence maximum power sensitivity

$$= \frac{P_{acmax}}{E^2_{g}} = \frac{2}{9}\mu_{gm} \quad \ldots(14{\cdot}54)$$

... ltage E_{bb} or zero signal
plate supply voltage of
... sis will prove that for
maximum power output, R_L should be far greater than $2r_p$.

In the above analysis a series-fed amplifier has been shown but the analysis applies equally well to either a shunt-fed power amplifier or a transformer-coupled power amplifier provided load line is drawn for load impedance transferred to the primary side.

Theoretical plate circuit efficiency of triode power amplifier.

An approximate expression for theoretical value of plate circuit efficiency η_p may be found by considering the tube to have ideal characteristic i.e. the characteristics are perfectly linear, parallel and equispaced for equal increment of the parameter.

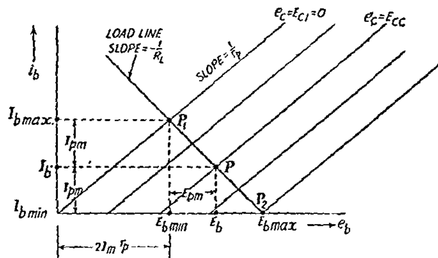


Fig. 14·9. Ideal plate characteristic triode operated for maximum power output for given r_p and R_L

Fig. 14·9 shows the ideal plate characteristic of a triode. Operation is linear A_1 so that grid voltage e_c never exceeds zero and at the same time plate current is never cut off. In Fig. 14·9, therefore, grid voltage may vary from point P_1 corresponding to zero ... zero plate current. This ... and hence gives maximum ... the same. Then $I_b = I_{pm}$.
Hence plate circuit efficiency is given by,

$$\eta_p = \frac{E_p . I_p}{E_{bb} . I_b} = \frac{E_{pm} \, I_{pm}}{2 \, E_{bb} . I_{pm}} = 50 \times \frac{E_{pm}}{E_{bb}} \% \quad \ldots(14{\cdot}55)$$

For series-fed triode amplifier :

$$E_{bb} = E_{b\ max} \text{ for maximum power output operation.}$$

so that $E_{bb} = E_{b\ max} = 2E_{pm} + 2I_p.r_p$ ...(14·56)

$$\text{Hence } \eta_p = 50 \frac{E_{pm}}{2[E_{pm}+I_m.r_p]} = \frac{25}{\left[1+\frac{I_{pm}\ r_p}{E_{pm}}\right]}$$

But $I_{pm} = \frac{E_{pm}}{R_l}$ so that

$$\eta_p = \frac{25}{1+\frac{r_p}{R_l}} \% \qquad ...(14·57)$$

This theoretical plate circuit efficiency is maximum when $r_p/R_l = 0$. Thus theoretical maximum plate circuit efficiency of series-fed triode power amplifier is only 25 per cent. But if maximum power is to be obtained, R_l must be adjusted to be equal to r_p. With $r_p = R_l$ plate efficiency is only 12·5%. Such is the case when operation has no restraint *i.e.* operation may extend upto zero plate current. But if operation is taking place in a limited range, efficiency is hardly ever greater than 10% proving that linear A_1 triode amplifier is a very inefficient device.

In shunt-fed power amplifier, d.c. losses in the load resistance are eliminated so that plate circuit efficiency is higher. In that case,

$$E_{bb} = E_b = E_{pm} + 2\ I_{pm}\ r_p \qquad ...(14·58)$$

$$\text{Hence } \eta_p = 50 \frac{E_{pm}}{E_{bb}} = 50 \frac{E_{pm}}{[E_{pm}+2\ I_{pm}.r_p]} \% \qquad ...(14·59)$$

But $E_{pm} = I_{pm}.R_l$

$$\text{Hence } \eta_p = \frac{50}{1+\frac{2r_p}{R_l}} \% \qquad ...(14·60)$$

Thus theoretical maximum plate circuit efficiency of shunt-fed or transformer coupled triode amplifier is 50% twice that for series-fed triode amplifier. For condition of maximum undistorted power output with prescribed zero-signal plate voltage, $R_l = 2r_p$ so that plate circuit efficiency η_p is 25%. But operation cannot be extended to zero plate current in order to avoid distortion and also d.c. resistance of inductor is not zero. Hence practical plate circuit efficiency in shunt-fed or transformer-coupled amplifier is less than 25%.

A_1 power amplifier using power pentodes and beam power tubes.

The graphical analysis given so far apply to triodes alone and are not valid for pentode because the plate characteristics of a power pentode differ widely from those of a triode. In the case of pentodes,

plate characteristics are essentially linear and horizontal beyond knees. Same is the case with beam power tubes. Fig. 14·10

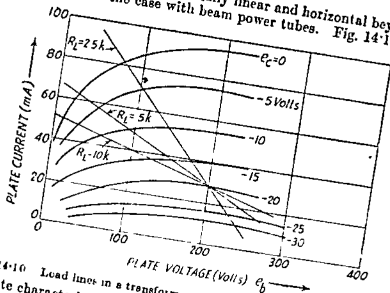


Fig. 14·10 Load lines in a transformer coupled pentode A_1 power amplifier.

the plate characteristics of a power pentode along with three load lines for a transformer-coupled load. Transformer is assumed ideal so that zero signal plate voltage E_b is equal to E_{bb} for each of the three load resistance

Linearity of amplification depends upon the linearity of dynamic characteristic. In the case of a triode dynamic transfer

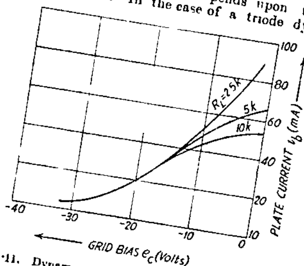


Fig. 14·11. Dynamic transfer characteristics of a power pentode amplifier for the three values of load resistance.

characteristic is excessively curved in the region of small plate current but for all higher values of plate current dynamic characteristic is more or less linear, the order of linearity increasi--

load resistance increases. Hence in the operating region, as resistance in a tricde amplifier is increased, distortion decrea- the case of power pentodes, on the other hand, dynamic char- in the operating region has excessive curvature for both low a values of load resistance as shown in Fig. 14·11. These c characteristics have been drawn by picking up the values o current for different values of grid voltage for each of the thre lines shown in Fig. 14·10. It is not possible in power pentode, t fore, to use either low or high values of load resistance. Hence not possible to chose a critical value of R_l relative to r_p that give maximum power output. In pentodes effective R_l is always less than dynamic plate resistance r_p. One is tempted to conclude, therefore, that the output power sensitivity of pentode would be extremely low but such is not the case. Power pentodes using usual values of R_l provide output power that is much larger than the output obtained when the same tube is used as a triode in conjunction with suitable load resistance. This results voltage of the large μg_m product of a pentode. Further grid signal voltage required in a pentode amplifier is much smaller.

An approximate expression for theoretical plate circuit efficiency of a pentode power amplifier may be derived as below. Assumptions made are :—(*i*) operation is class *A* (*ii*) plate characteristics are ideal *i.e.* horizontal and parallel straight lines extending upto current axis. (*iii*) zero signal plate voltage is prescribed.

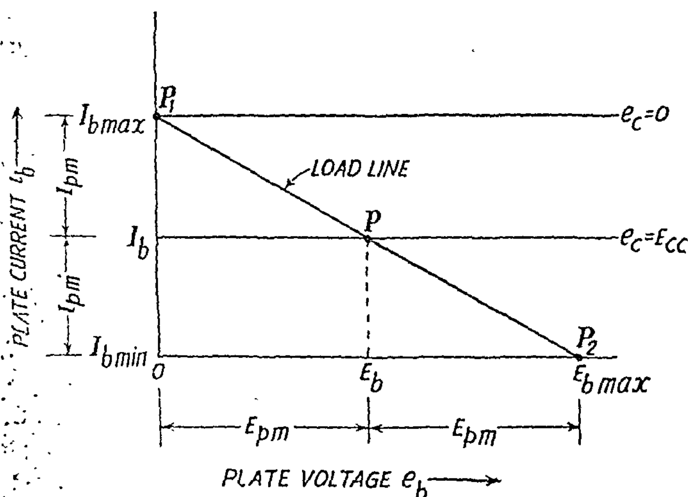


Fig. 14·12. Operation of A_1 pentode power amplifier for maximum power output and prescribed E_b.

Fig. 14·12 shows the idealized plate characteristics of pentode used as A_1 power amplifier together with the load line. P is the zero signal operating point. For maximum a.c. power output point of operation for class A_1 amplification may move between point P_1 corresponding to zero grid voltage at the most positive value of grid

... zero plate current at the ... For a given value of ... tance R_l should be so ... eration i.e. zero grid voltage and zero plate current at extremes of grid signal cycle, are simultaneously achieved. This load resistance R_l then gives the maximum output power. Obviously from the geometry of the diagram for optimum load resistance R_l,

$$E_{pm} = E_b \text{ and } I_{pm} = I_b$$

Hence plate circuit efficiency

$$\eta_p = \frac{E_p I_p}{E_{bb} I_b} = \frac{\frac{E_b}{\sqrt{2}} \cdot \frac{I_b}{\sqrt{2}}}{E_{bb} I_b} = 50 \frac{E_b}{E_{bb}} \% \qquad \dots (14.61)$$

For series-fed pentode amplifier, $E_{bb} = E_{b\,max} = 2E_b$ so that

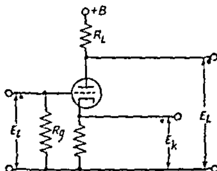
$$\eta_p = 50 \frac{E_b}{2E_b} = 25\%.$$

For shunt-fed pentode amplifier or transformer-coupled pentode amplifier, $E_{bb} = E_b$ (assuming an ideal transformer) so that

$$\eta_p = 50 \frac{E_b}{E_b} = 50\%.$$

Thus we see that plate circuit efficiencies of series-fed and transformer-coupled pentode power amplifiers are twice the corresponding plate circuit efficiencies of triode amplifier with load resistance adjusted for maximum power output.

Example 1. *Linear A_1 amplifier shown in the diagram below uses plate supply voltage $E_{bb} = 300$ volts, grid bias $E_{cc} = -25$ volts, grid signal voltage $E_{gk} = 10$ volts r.m.s. at an angular frequency of 400 radians per second. Load resistance R_l - 2000 ohms. At zero signal operating point, plate current $I_b = 50$ mA, amplification factor $\mu = 6$ and dynamic plate resistance $r_p = 1000$ ohms. Calculate (i) voltage gain (ii) a.c. output voltage (iii) power supplied from the plate supply source (iv) zero signal plate dissipation P_{po} (v) a.c. power output (vi) plate dissipation with signal applied and (vii) plate circuit efficiency.*



Solution.

(*i*) Voltage gain $= \dfrac{-\mu R_l}{r_p+R_l} = \dfrac{-6\times 2000}{1000+2000} = -4$

(*ii*) A.c. voltage output $= e_p = \dfrac{-\mu\, E_{gk}\, R_l}{r_p+R_l}$

$$= \frac{-6\times 10\times 2000}{1000+2000} = -40 \text{ volts.}$$

(*iii*) Power supplied by plate supply source is given by

$$P_{bb} = E_{bb}.I_b = 300(50\times 10^{-3}) = 15 \text{ watts.}$$

(*vi*) Zero signal plate voltage E_b

$$= E_{bb} - I_b.R_l$$
$$= 300 - 50\times 10^{-3}\times 2000$$
$$= 200 \text{ volts}$$

Hence P_{po} = zero signal plate dissipation

$$= E_b.I_b = 200\times 50\times 10^{-3} = 10 \text{ watts.}$$

(*v*) A.c. power output $P_{ac} = \left(\dfrac{\mu\, E_{gk}}{r_p+R_l}\right)^2 \times R_l$

$$= \left(\frac{6\times 10}{1000+2000}\right)^2 \times 2000$$
$$= 0{\cdot}8 \text{ watt}$$

(*vi*) Plate dissipation with signal applied is given by

$$P_p = P_{po} = P_{ac} = 10 - 0{\cdot}8 = 9{\cdot}2 \text{ watts.}$$

(*vii*) % plate circuit efficiency $\eta_p = \dfrac{P_{ac}}{P_{bb}}\times 100$

$$= \frac{0{\cdot}8}{15}\times 100 = 5{\cdot}33\%.$$

Example 2. *A triode power amplifier is operated with a bias = −24 volts, load resistance = 400 ohms and plate supply voltage = 350 volts. Zero signal plate voltage is 250 volts. Amplification factor of the tube = 8 and dynamic plate resistance = 4000 ohms. Safe dissipations of the valve and load resistor are respectively 5·75 and 3·5 watts. Find the permissible range of applied a.c. grid signal voltage without exceeding the ratings of the tube and resistor dissipation.*

Solution.

$$I_b = \frac{E_{bb}-E_b}{R_l} = \frac{350-250}{4000} \text{ amp.} = 25\ mA$$

Zero signal plate dissipation

$$P_{po} = E_b.I_b = 250\times 25\times 10^{-3} \text{ watts}$$
$$= 6{\cdot}25 \text{ watts}$$

Zero signal power in load resistor

$$= I_b^2 R_l$$
$$= (25 \times 10^{-3})^2 \times 4000$$
$$= 2.5 \text{ watts}$$

On application of signal, total dissipation in the load resistor increases by an amount equal a.c. output power P_{ac}.

Hence P_{ac} must not exceed $(3.5 - 2.5) = 1.0$ watt.

But $$P_{ac} = \left(\frac{\mu E_{gk}}{r_p + R_l}\right)^2 R_l$$

Hence $E_{gk\,(max)}$ is given by,

$$1 = \left[\frac{8 \times E_{gk(max)}}{4000 + 4000}\right]^2 4000$$

Hence $E_{gk\,(max)} = 15.81$ volts.

On application of signal, plate dissipation reduces by an amo equal to P_{ac}. Hence P_{ac} must not be less than $(6.25 - 5.75)$ wa $= 0.5$ watt.

$$\text{Hence } 0.5 = \left\{\frac{\mu E_{gk\,(min)}}{r_p + R_l}\right\}^2 R_l$$

$$\text{or} \qquad 0.5 = \left[\frac{8 \times E_{gk(min)}}{4000 + 4000}\right]^2 \times 4000$$

Hence $E_{gk(min)} = 11.18$ volts

Example 3. *A linear A_1 power amplifier uses triode hav amplification factor $\mu = 6$ and dynamic plate resistance $r_p = 4000$ oh Amplitude of grid signal voltage is 20 volts. Calculate the maxim power output obtainable.*

Solution.

$$\text{A.c. power output } P_{ac} = \left(\frac{\mu E_{gk}}{r_p + R_l}\right)^2 . R_l$$

P_{ac} is maximum when $r_p = R_l$

Hence $$P_{ac\,(max)} = \frac{\mu^2 E_{gk}^2}{4 r_p^2} \times r_p = \frac{\mu^2 E_{gk}^2}{4 r_p}$$

$$= \frac{\left(6 \times \frac{20}{\sqrt{2}}\right)^2}{4 \times 4000} = 0.45 \text{ watt.}$$

Example 4 *A class A_1 amplifier uses triode having $\mu = 15$, $r_p = 12000$ ohms. A signal of 20 volts r.m.s. is applied at the in; Calculate the available power output of the amplifier. Express the output power as a fraction of available power if the load impedanc (a) 6,000 ohms resistive, (b) 24,000 ohms resistive and (c) $3000 + j3$ ohms;*

Solution. $P_{ac} = \left(\frac{\mu E_{gk}}{r_p + R_l}\right)^2 R_l$

(a) $Z_l = 6000\ \Omega$ resistive.

$$P_{ac} = \left[\frac{15 \times 2}{12000 + 6000}\right]^2 \times 6000 = 1{\cdot}666 \text{ watts}$$

$$\text{Available power } P_{av} = \frac{\mu^2 E_{gk}^2}{4r_p} = \frac{(15 \times 20)^2}{4 \times 12000} = 1{\cdot}875 \text{ watts.}$$

Hence $\frac{P_{ac}}{P_{av}} = \frac{1{\cdot}666}{1{\cdot}875} = 0{\cdot}888$

(b) $Z_l = 24000\ \Omega$ resistive

$$P_{ac} = \left[\frac{15 \times 20}{12000 + 24000}\right]^2 \times 24000$$

$$= 1{\cdot}666 \text{ watts}$$

Hence $\frac{P_{ac}}{P_{av}} = \frac{1{\cdot}666}{1{\cdot}875} = 0{\cdot}888$

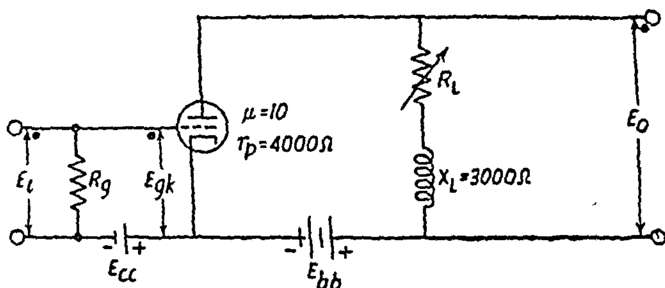
(c) $Z_l = 3000 + j\,3000$

$$P_{ac} = \frac{(\mu E_{gk})^2}{(r_p + R_l)^2 + (X_l)^2} \times R_l$$

$$= \frac{(15 \times 20)^2 \times 3000}{(12000 + 3000)^2 + (3000)^2} = 1{\cdot}155$$

Hence $\frac{P_{ac}}{P_{av}} = \frac{1{\cdot}155}{1{\cdot}875} = 0{\cdot}615$

Example 5. *In the A_1 power amplifier shown in the diagram, load resistance R_l is variable. Calculate the value of R_l which will give maximum power output. Also calculate the maximum output power so obtained if grid signal E_{gk} is $10\sqrt{2} \sin 2\pi \times 1000t$.*



Solution. $P_{ac} = \frac{\mu^2 E_{gk}^2 R_l}{(r_p + R_l)^2 + (X_l)^2}$

It may be proved that P_{ac} is maximum when

$$R_l = \sqrt{r_p^2 + X_l^2}$$

Hence $R_l = \sqrt{(4000)^2+(3000)^2} = 5000$ ohms

Hence $P_{ac(max)} = \dfrac{(10\times10)^2\times5000}{(4000+5000)^2+(3000)^2} = 0.555$ watt.

Example 6. *A class A_1 transformer-coupled power amplifier uses triode having $\mu = 8$ and $r_p = 2000$ ohms. Load is a loudspeaker of resistive impedance 2 ohms. Turns ratio is 30. Calculate the r.m.s. signal voltage necessary to deliver a.c. power of 0.5 watt to the loudspeaker.*

Solution. Load impedance transferred to the primary side becomes

$$2\times(30)^2 = 1800\ \Omega \text{ resistive}$$

Output power $P_{ac} = \dfrac{\mu^2 E_{gk}^2}{(r_p+R_l)^2}\cdot R_l$

or $$0.5 = \frac{(8)^2 E_{gk}^2}{(2000+1800)^2}\times1800$$

Hence $$E_{gk}^2 = \left(\frac{3800}{8}\right)^2\times\frac{0.5}{1800}$$

and $$E_{gk} = \frac{3800}{8\times60} = 7.92 \text{ volts}$$

Example 7. *In order to amplify without distortion a class A_1 triode amplifier is operated above a minimum plate current of mA. Zero signal plate voltage E_b is prescribed to be 200 volts. The tube has amplification factor $\mu=10$ and anode slope resistance that will give maximum undistorted output power. If plate characteristic for $e_c=0$ has plate current $i_b = 5mA$ at plate voltage $e_b = 20$ volts, calculate the value of grid bias and grid signal voltage to get maximum undistorted output power. Calculate this maximum power sensitivity.*

Solution. To obtain maximum undistorted output power R must be equal to $2\,r_p$.

Thus load resistance $R_l = 2r_p = 2\times4000 = 8000$ ohms.

Again for maximum undistorted output power

grid bias $E_{cc} = \sqrt{2}\,E_{gk} = \dfrac{3}{4}\cdot\dfrac{E_b-E_d}{\mu}$

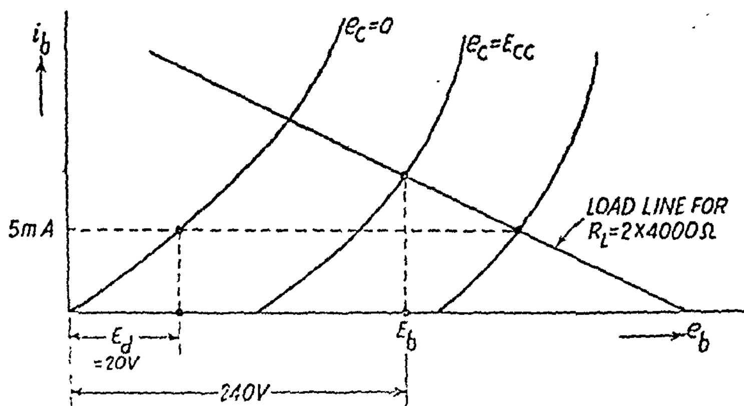
$$= \frac{3(240-20)}{4\times10} = 16.5 \text{ volts.}$$

Hence $$E_{gk} = \frac{16.5}{\sqrt{2}} = 11.7 \text{ volts}$$

Maximum a.c. output power $P_{ac\,max} = \dfrac{\mu^2E_{gk}^2}{(r_p+R_l)^2}\times R_l$

$$= \frac{2\,\mu^2}{9\,r_p}E_{gk}^2 \qquad (\text{Since } R_l = 2r_p$$

$$= \frac{2\times(10)^2\times(11.7)^2}{9\times4000} = 0.757 \text{ watt}$$



$$\text{Maximum power sensitivity} = \frac{P_{ac(max)}}{E^2_{gk}} = \frac{2\mu^2}{9\ r_p}$$

$$= \frac{2\times(10)^2}{9\times4000}\ \text{mho}$$

$$= 5{\cdot}556\times10^{-3}\ \text{mho.}$$

Example 8. *A class A_1 transformer-coupled power amplifier uses triode having $\mu = 10$ and $r_p = 3200$ ohms in the operating range. The load is a loudspeaker of resistance 4 ohms. Plate supply voltage is prescribed at 300 volts. Distortionless amplification is to be achieved by not permitting plate current to fall below a certain minimum value I_d. Maximum output power is obtained by suitably selecting the effective load resistance and grid bias. Calculate the turns ratio necessary to get this proper effective load resistance on the primary side. If grid bias so used is -25 volts, calculate maximum power output and maximum power sensitivity. If the zero signal plate current I_b is 40 mA, calculate plate circuit efficiency. Assume the transformer to be ideal.*

Solution. For undistorted amplification in triode with prescribed zero-signal plate voltage, in order to get maximum output power, effective load resistance must be equal to $2\ r_p$.

Hence
$$2r_p = \left(\frac{N_p}{N_s}\right)^2 \times R_l$$

or
$$\left(\frac{N_p}{N_s}\right)^2 = \frac{2r_p}{R_l} = \frac{2\times3200}{4} = 1600$$

Hence
$$\frac{N_p}{N_s} = 40$$

For maximum power output $\sqrt{2}E_{gk} = E_{cc} = 25$ volts

Hence
$$E_{gk} = \frac{25}{\sqrt{2}} = 17{\cdot}7\ \text{volts}$$

Hence $\quad R_l = \sqrt{(4000)^2+(3000)^2} = 5000$ ohms

Hence $P_{ac(max)} = \dfrac{(10\times 10)^2 \times 5000}{(4000+5000)^2+(3000)^2} = 0{\cdot}555$ watt.

Example 6. *A class A_1 transformer-coupled power amplifier uses triode having $\mu = 8$ and $r_p = 2000$ ohms. Load is a loudspeaker of resistive impedance 2 ohms. Turns ratio is 30. Calculate the r.m.s. signal voltage necessary to deliver a.c. power of 0·5 watt to the loudspeaker.*

Solution. Load impedance transferred to the primary side becomes

$$2\times(30)^2 = 1800\ \Omega \text{ resistive}$$

$$\text{Output power } P_{ac} = \frac{\mu^2 E_{rg}^2}{(r_p+R_l)^2}\cdot R_l$$

or
$$0{\cdot}5 = \frac{(8)^2 E_{rg}^2}{(2000+1800)^2}\times 1800$$

Hence
$$E_{rg}^2 = \left(\frac{3800}{8}\right)^2 \times \frac{0{\cdot}5}{1800}$$

and
$$E_{rg} = \frac{3800}{8\times 60} = 7{\cdot}92 \text{ volts.}$$

Example 7. *In order to amplify without distortion a class A_1 triode amplifier is operated above a minimum plate current of [illegible] mA. Zero signal plate voltage E_b is prescribed to be 200 volts. The tube has amplification factor $\mu = 10$ and anode slope resistance that will give maximum undistorted output power. If plate characteristic for $e_c = 0$ has plate current $i_b = 5$ mA at plate voltage $e_b = 20$ volts, calculate the value of grid bias and grid signal voltage to get maximum undistorted output power. Calculate this maximum power similarly.*

Solution. To obtain maximum undistorted output power R must be equal to $2\,r_p$.

Thus load resistance $R_l = 2r_p = 2\times 4000 = 8000$ ohms.

Again for maximum undistorted output power

$$\text{grid bias } E_{cc} = \sqrt{2}\,E_{rg} = \frac{3}{4}\cdot\frac{E_b - E_e}{\mu}$$

$$= \frac{3(240-20)}{4\times 10} = 16{\cdot}5 \text{ volts.}$$

Hence
$$E_{rg} = \frac{16{\cdot}5}{\sqrt{2}} = 11{\cdot}7 \text{ volts}$$

$$\text{Maximum a.c. output power } P_{ac\,max} = \frac{\mu^2 E_{rg}^2}{(r_p+R_l)^2}\times R_l$$

$$= \frac{2\mu^2}{9\,r_p}E_{rg}^2 \qquad (\text{Since } R_l = 2r_p)$$

$$= \frac{2\times 10^2\times(11{\cdot}7)^2}{9\times 4000} = 0{\cdot}760 \text{ watt.}$$

Solution. $P_{ac} = \left(\frac{\mu E_{gk}}{r_p + R_l}\right)^2 R_l$

(a) $Z_l = 6000\ \Omega$ resistive.

$$P_{ac} = \left[\frac{15 \times 2}{12000 + 6000}\right]^2 \times 6000 = 1{\cdot}666 \text{ watts}$$

Available power $P_{av} = \frac{\mu^2 E_{gk}^2}{4r_p} = \frac{(15 \times 20)^2}{4 \times 12000} = 1{\cdot}875$ watts.

Hence $\frac{P_{ac}}{P_{av}} = \frac{1{\cdot}666}{1{\cdot}875} = 0{\cdot}888$

(b) $Z_l = 24000\ \Omega$ resistive

$$P_{ac} = \left[\frac{15 \times 20}{12000 + 24000}\right]^2 \times 24000$$

$$= 1{\cdot}666 \text{ watts}$$

Hence $\frac{P_{ac}}{P_{av}} = \frac{1{\cdot}666}{1{\cdot}875} = 0{\cdot}888$

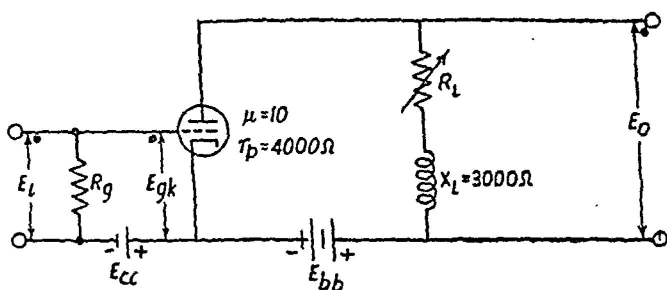
(c) $Z_l = 3000 + j\,3000$

$$P_{ac} = \frac{(\mu E_{gk})^2}{(r_p + R_l)^2 + (X_l)^2} \times R_l$$

$$= \frac{(15 \times 20)^2 \times 3000}{(12000 + 3000)^2 + (3000)^2} = 1{\cdot}155$$

Hence $\frac{P_{ac}}{P_{av}} = \frac{1{\cdot}155}{1{\cdot}875} = 0{\cdot}615$

Example 5. *In the A_1 power amplifier shown in the diagram, load resistance R_l is variable. Calculate the value of R_l which will give maximum power output. Also calculate the maximum output power so obtained if grid signal E_{gk} is $10\sqrt{2} \sin 2\pi \times 1000t$.*



Solution. $P_{ac} = \frac{\mu^2 E_{gk}^2 R_l}{(r_p + R_l)^2 + (X_l)^2}$

It may be proved that P_{ac} is maximum when

$$R_l = \sqrt{r_p^2 + X_l^2}$$

Hence $R_t = \sqrt{(4000)^2+(3000)^2} = 5000$ ohms

Hence $P_{ac(max)} = \dfrac{(10\times10)^2\times5000}{(4000+5000)^2+(3000)^2} = 0.555$ watt.

Example 6. *A class A_1 transformer-coupled power amplifier uses triode having $\mu = 8$ and $r_p = 2000$ ohms. Load is a loudspeaker of resistive impedance 2 ohms. Turns ratio is 30. Calculate the r.m.s. signal voltage necessary to deliver a.c. power of 0·5 watt to the loud-speaker.*

Solution. Load impedance transferred to the primary side becomes

$$2\times(30)^2 = 1800\ \Omega \text{ resistive}$$

$$\text{Output power } P_{ac} = \frac{\mu^2 E_{gk}^2}{(r_p+R_l)^2} \cdot R_l$$

or

$$0{\cdot}5 = \frac{(8)^2 E_{gk}^2}{(2000+1800)^2}\times1800$$

Hence

$$E_{gk}^2 = \left(\frac{3800}{8}\right)^2\times\frac{0{\cdot}5}{1800}$$

and

$$E_{gk} = \frac{3800}{8\times60} = 7{\cdot}92 \text{ volts.}$$

Example 7. *In order to amplify without distortion a class A_1 triode amplifier is operated above a minimum plate current of mA. Zero signal plate voltage E_b is prescribed to be 200 volts. The tube has amplification factor $\mu = 10$ and anode slope resistance that will give maximum undistorted output power. If plate characteristic for $e_c = 0$ has plate current $i_b = 5mA$ at plate voltage $e_b = 20$ volts, calculate the value of grid bias and grid signal voltage to get maximum undistorted output power. Calculate this maximum power sesitivity.*

Solution. To obtain maximum undistorted output power R must be equal to 2 r_p.

Thus load resistance $R_l = 2r_p = 2\times4000 = 8000$ ohms.

Again for maximum undistorted output power

$$\text{grid bias } E_{cc} = \sqrt{2}\,E_{gk} = \frac{3}{4}\cdot\frac{E_b-E_s}{\mu}$$

$$= \frac{3(210-20)}{4\times10} = 16{\cdot}5 \text{ volts.}$$

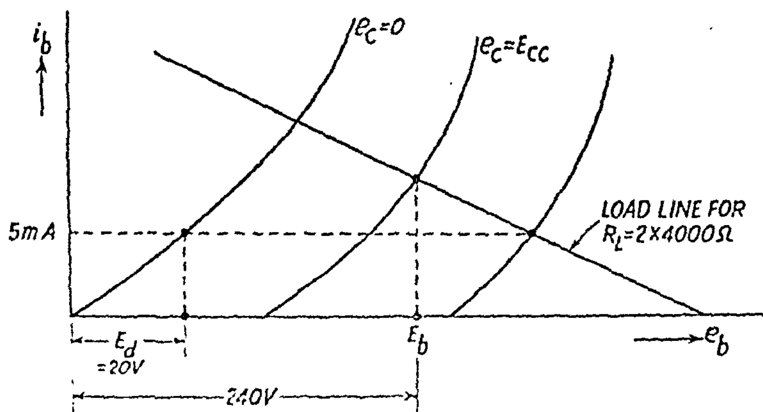
Hence

$$E_{gk} = \frac{16{\cdot}5}{\sqrt{2}} = 11{\cdot}7 \text{ volts}$$

$$\text{Maximum a.c. output power } P_{ac(max)} = \frac{\mu^2 E_{gk}^2}{(r_p+R_l)^2}\times R_l$$

$$= \frac{2\,\mu^2}{9\,r_p} E_{gk}^2 \qquad (\text{Since } R_l = 2r_p$$

$$= \frac{2\times(10)^2\times(11{\cdot}7)^2}{9\times4000} = 0{\cdot}757 \text{ watt.}$$



$$\text{Maximum power sensitivity} = \frac{P_{ac(max)}}{E^2_{gk}} = \frac{2\mu^2}{9\,r_p}$$

$$= \frac{2\times(10)^2}{9\times4000}\text{ mho}$$

$$= 5{\cdot}556\times10^{-3}\text{ mho.}$$

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Solution. For undistorted amplification in triode with prescribed zero-signal plate voltage, in order to get maximum output power, effective load resistance must be equal to 2 r_p.

Hence $$2r_p = \left(\frac{N_p}{N_s}\right) \times R_l$$

or $$\left(\frac{N_p}{N_s}\right)^2 = \frac{2r_p}{R_l} = \frac{2\times3200}{4} = 1600$$

Hence $$\frac{N_p}{N_s} = 40$$

For maximum power output $\sqrt{2}E_{gk} = E_{cc} = 25$ volts

Hence $$E_{gk} = \frac{25}{\sqrt{2}} = 17{\cdot}7\text{ volts}$$

Hence maximum power output

$$P_{ac\,(max)} = \frac{2\mu^2 E_{gk}^2}{9 r_p}$$

$$= \frac{2\times(10\times17\cdot7)^2}{9\times3200} = 2\cdot18 \text{ watts}$$

Maximum power sensitivity

$$= \frac{P_{ac\,max}}{E_{gk}^2} = \frac{2\mu^2}{9 r_p}$$

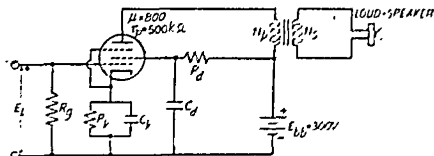
$$= \frac{2\times100}{9\times3200} = 6\cdot94\times10^{-3} \text{ mho}$$

$$P_{bb} = \text{Power from plate supply source}$$
$$= E_{bb}.I_b = 300\times40\times10^{-3} = 12 \text{ watts}$$

Hence plate circuit efficiency at maximum output is given by

$$\eta_p = \frac{2\cdot18}{12}\times100 = 18\cdot17\%.$$

Example 9. *In class A output power amplifier using pentode, shown in the diagram, impedance offered by loud-speaker is 5 ohms resistive, zero signal plate current is 40mA and input grid signal is 10 volts r.m.s. Calculate output a.c. power and plate circuit efficiency. Primary to the secondary turns ratio is 30. A.C. impedance of $R_k - C_k$ biasing circuit may be considered to be zero. Assume the transformer to be ideal.*



Solution : Effective load resistance

$$R_l' = \left(\frac{N_p}{N_s}\right)^2 R_l$$

$$= (30)^2\times5 = 4500 \text{ ohms.}$$

$$P_{ac}' = \left[\frac{\mu E_{gk}}{r_p + R_l'}\right]^2 R_l' = \left[\frac{800\times10}{500+4\cdot5}\right]^2\times\frac{4500}{10^6} = 1\cdot121 \text{ watts}$$

Power supplied from plate supply source is given by

$$P_{bb} = E_{bb}.I_b = 300\times40\times10^{-3} = 12 \text{ watts.}$$

Plate circuit efficiency $\eta_p = \dfrac{1\cdot121}{12}\times100 = 9\cdot34\%.$

Hence maximum power output

$$P_{ac\,(max)} = \frac{2\mu^2 E_{gk}^2}{9r_p}$$

$$= \frac{2\times(10\times17{\cdot}7)^2}{9\times3200} = 2{\cdot}18 \text{ watts}$$

Maximum power sensitivity

$$= \frac{P_{ac(max)}}{E_{gk}^2} = \frac{2\mu^2}{9r_p}$$

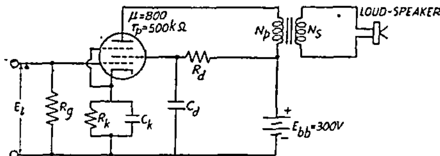
$$= \frac{2\times100}{9\times3200} = 6{\cdot}94\times10^{-3} \text{ mho}$$

$$P_{bb} = \text{Power from plate supply source}$$
$$= E_{bb}.I_b = 300\times4'\times10^{-3} = 12 \text{ watts}$$

Hence plate circuit efficiency at maximum output is given by

$$\eta_p = \frac{2{\cdot}18}{12}\times100 = 18{\cdot}17\%.$$

Example 9. *In class A output power amplifier using pentode, shown in the diagram, impedance offered by loud-speaker is 5 ohms resistive, zero signal plate current is 40mA and input grid signal is 10 volts r.m.s. Calculate output a.c. power and plate circuit efficiency. Primary to the secondary turns ratio is 30. A.C. impedance of $R_k - C_k$ biasing circuit may be considered to be zero. Assume the transformer to be ideal.*



Solution : Effective load resistance

$$R'_l = \left(\frac{N_p}{N_s}\right)^2 R_l$$

$$= (30)^2\times5 = 4500 \text{ ohms.}$$

$$P_{ac}' = \left[\frac{\mu E_{gk}}{r_p+R_l'}\right]^2 R_l' = \left[\frac{800\times10}{500+4{\cdot}5}\right]^2\times\frac{4500}{10^6} = 1{\cdot}134 \text{ watts}$$

Power supplied from plate supply source is given by

$$P_{bb} = E_{bb}.I_b = 300\times40\times10^{-3} = 12 \text{ watts.}$$

Plate circuit efficiency $\eta_p = \dfrac{1{\cdot}134}{12}\times100 = 9{\cdot}45\%.$

Class A, Pushpull Amplifier

Every power amplifier has a limited output power capacity usually depending upon the maximum permissible plate dissipation in the tube. If greater output power is required to be obtained there are two methods that may be used. One consists in parallel operation of two tubes *i.e.* operation in which the plates of two tubes are connected together, so also are the grids and cathodes. Such a parallel operation permits a.c. output power twice as large as provided by a single tube. Alternatively two tubes of identical characteristics may be connected in "push-pull" as shown in Fig. 14·13. This push-pull amplifier also gives output power twice as large as in a single tube but there are additional advantages accruing from the use of push-pull connection, the most important being the elimination of even harmonic distortion terms from the output of the amplifier. Hence for the same maximum permitted harmonic distortion, output power much greater than twice that of a single tube may be obtained. Further class *AB* and *B* operations of audio frequency power amplifier with resultant high plate circuit efficiencies may be used without excessive distortion. Class *AB* and class *B* operations of single tube audio frequency amplifiers is not possible because of excessive distortion. Another advantage that results from push-pull operation is the fact that zero signal plate currents in a push-pull amplifier flow through the primary of output transformer in opposite directions and hence magnetic saturation of the core is avoided. A smaller and lighter core may, therefore, be used for the output transformer.

In the circuit of Fig. 14·13, t_1 is the input push-pull transformer. Input signal voltage is applied to the primary while the secondary is centre-tapped so that voltage e_{gk} and e'_{gk} are exactly equal in magnitude and opposite in phase *i.e.* while one is increasing the other is decreasing by the same amount. This type of connection

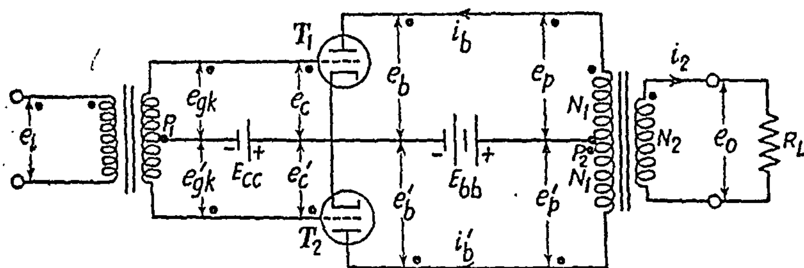


Fig. 14·13. Push-pull untuned power amplifier.

is referred to as "push-pull" connection. A similar push-pull transformer is used at the output, the centre-tapped winding being used as the primary. Obviously then as the plate current i_b of one tube increases, the plate current i_b' of the other tube decreases by the same amount. In other words, a.c. plate currents i_p and i'_p are equal and 180 degrees out of phase. By using push-pull output transformer, output voltages produced by these a.c. plate currents i_p

and i_p' get added. A.c. power output is, therefore, twice as large as would be obtained with a single tube.

Assumption made in the analysis of A_1 push-pull amplifier are :—

(i) Grid bias and grid-drive are so adjusted that plate current flows for the entire a.c. cycle.

(ii) Operation is restricted to negative-grid region of the tube characteristics and hence grid current does not flow.

(iii) Transformers are ideal.

(iv) Points P_1 and P_2 are true electrical centres of the windings.

(v) Load is a pure resistance.

(vi) Tubes T_1 and T_2 have indentical characteristics.

Zero signal operation of the amplifier circuit will be considered first followed by discussion of operation with a.c. signal applied.

Zero signal operation. With no signal applied at the input, following relations become apparent :—

$$e_i = 0$$

$$e_{gk} = e'_{gk} = 0$$

$$e_g = e_g' = E_{cc}$$

$$e_p = e_p' = 0$$

$e_b = e_b' = E_b = E_{bb}$ (assuming zero d. c. voltage drop in transformer winding)

$$i_b = 2i'_b = I_b.$$

We observe that zero-signal plate currents in the two tubes have the same value I_b. The total zero-signal plate current through the plate supply source is then $2I_b$. Hence in zero-signal operation, conditions are similar to those in parallel operation.

Further under zero-signal condition, plate currents of value I_b each flow through the two halves of the primary of the output [illegible]

But since these currents are equal, and the number of turns are also equal, the net magnetisation of the core resulting from the zero [illegible]

the core is thus avoided. For a prescribed power output and permissible harmonic distortion caused by nonlinearity of transformer characteristic, it is possible, therefore, to use a smaller lighter and comparatively inexpensive output transformer core in a pushpull amplifier.

Operation over linear region of tube character

Having considered zero-signal operation of an ideal pushpull we may next study the effectof application of such a sn

signal voltage that the operating point travels over essentially linear region of the tube characteristics. Obviously then,

$$e_{gk} = -e'_{gk} \qquad \ldots(14\cdot57)$$

Total instantaneous grid voltages are given by,

$$e_c = E_{co} + e_{gk} \qquad \ldots(14\cdot58)$$

and $$e'_c = E_{co} + e'_{gk} = E_{co} - e_{gk}. \qquad \ldots(14\cdot59)$$

Because of the linearity of operation and symmetry of amplifier circuit,

$$i'_p = -i_p \qquad \ldots(14\cdot60)$$

and instantaneous total plate voltages are given by,

$$i_b = I_b + i_p \qquad \ldots(14\cdot61)$$

and $$i'_b = I_b + i'_p = I_b - i_p \qquad \ldots(14\cdot62)$$

Total current through plate power source

$$= i_b' + i_b = (I_b + i_p) + (I_b - i_p) = 2\,I_b \quad \ldots(14\cdot63)$$

Thus current through plate power source has current constant value of 2 I_b and has no a.c. component. Hence it is not necessary for the plate power source in pushpull amplifier with linear A_1 operation to have low internal impedance. Further if cathode-bias is used as shown in Fig. 14·14, it is not necessary to use the bypass condenser C_k since in linear A_1 operation there is no a.c. component of plate-current through plate supply source.

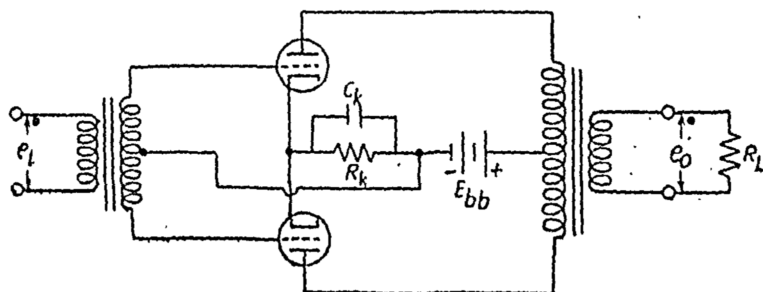


Fig. 14·14. Pushpull amplifier using cathode-bias.

A.C. equivalent circuit of linear A_1 pushpull untuned amplifier is shown in Fig. 14·15.

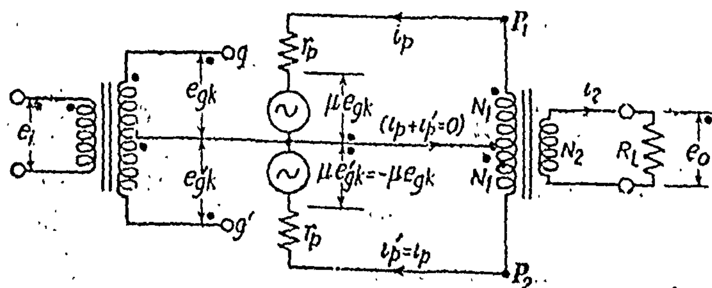


Fig. 14·15. A.C. equivalent circuit of linear A_1 pushpull untuned power amplifier.

Load impedance R_l introduces between points P_1 and P_2 on the primary side, an apparent resistance equal to

$$\left(\frac{2N_1}{N_2}\right) R_l = 4\left(\frac{N_1}{N_2}\right)^2 R_l.$$

Further the centre connection carries no a.c. component of plate current so that a.c. equivalent circuit of Fig. 14·15. re uces to one shown in Fig. 14·16. Apparent resistance of $4\left(\frac{N_1}{N_2}\right)R_l$ between plates of two tubes is referred to as "plate-to-plate resistance" R_{pp} so that $R_{pp} = 4\left(\frac{N_1}{N_2}\right)^2 R_l$. ...(14·63)

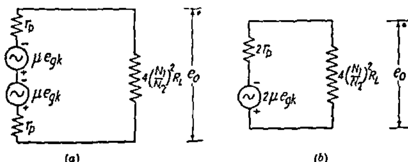


Fig. 14·16. Modified a c. equivalent circuit of linear A_1 pushpull amplifier

Equivalent circuit of Fig. 14·16. (a) may further be put in the form shown in Fig. 14·16 (b) in which generator voltages and internal impedances have been grouped together. From these equivalent circuits, we may calculate a.c. plate current, a.c. output voltage, a.c. output power etc.

Let us assume the input signal voltage to the input of tube T_1 may be put as,

$$e_{gk} = E_{gm} \cos \omega t = \sqrt{2} E_{gk} \cos \omega t \quad ...(14·64)$$

All alternating quantities will then be of the same sinusoidal [illegible] or r.m.s. values [illegible] h this modification [illegible]

$$\text{Plate current } \mathrm{I}_p = \frac{2\mu \mathrm{E}_{gk}}{2r_p + R_{pp}} \quad ...(14·65)$$

or

$$I_p = \frac{\mu E_{gk}}{r_p + \frac{R_{pp}}{2}} \quad ...(14·66)$$

Total power delivered to the load is given by,

$$P_{ac} = I_p^2 . R_{pp} = \left(\frac{\mu E_{gk}}{r_p + \frac{R_{pp}}{2}}\right)^2 . R_{pp} \quad ...(14·67)$$

$$=2\left[\frac{\mu E_{gk}}{r_p+\frac{R_{pp}}{2}}\right]^2 \cdot \frac{R_{pp}}{2} \qquad \ldots(14{\cdot}68)$$

Equation (14·68) indicates that the total output power of class A pushpull amplifier is twice that of each tube considered to be operating into the equivalent load resistance $\frac{R_{pp}}{2}$.

Equation (14·68) may also be put in the following form,

$$P_{ac} = \left[\frac{\mu E_{gk}}{\frac{r_p}{2}+\frac{R_{pp}}{4}}\right]^2 \cdot \frac{R_{pp}}{4} \qquad \ldots(14{\cdot}69)$$

From equation (14·69) we conclude that in so far as the output power is concerned class A pushpull amplifier may be equated to a

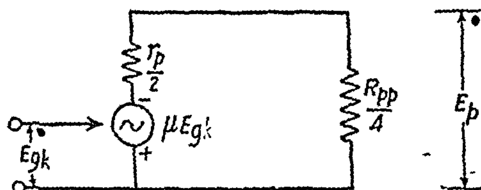


Fig. 14·17. Alternative form of a.c. equivalent.

single composite voltage generator having e.m.f. equal to μE_{gk} with internal resistance of $\frac{r_p}{2}$ and which works into a load resistance equal to $\frac{R_{pp}}{4}$ as shown in Fig. 14·17.

Large signal operation extending into non-linear region of tube characteristics.

When the signal is of large amplitude, the operation is not entirely confined to linear region of tube characteristics and results in generation of harmonics in the plate current. Let us again assume the grid signal voltage to be sinusoidal as given by equation (14·64). Then harmonics present in the plate current are given in the following expression,

$$i_b = I_b+I_{po}+I_{p1m}.\cos \omega t+I_{p2m} \cos 2\omega t+I_{p3} \cos 3\omega t+\ldots \qquad \ldots(14{\cdot}70)$$

But $e'_{gk} = -e_{gk}$, tubes are identical and circuit is symmetrical. Hence plate current i'_b is similar to i_b except that ωt should be replaced by $(\omega t+180°)$.

$$\text{Thus, } i'_b = I_b+I_{po}+I_{p1m} \cos (\omega t+180°)+I_{p2m} \cos (\omega t+360°) +I_{p3m} \cos (3\,\omega t+540°)\ldots\ldots \qquad \ldots(14{\cdot}71)$$

$$\text{or} \quad i'_b = I_b+I_{po}-I_{p1m} \cos (\omega t)+I_{p2m} \cos 2\omega t -I_{p3m} \cos 3\omega t+\ldots \qquad \ldots(14{\cdot}72)$$

Output transformer is assumed to be ideal so that leakage . . . le. The sum of . . . currents in the

Thus $i_b(N_1)-i_b'(N_1)-i_2(N_2) = 0$ ...(14·73)

or $i_2 = \frac{N_1}{N_2}\left(i_b-i'_b\right)$ ...(14·74)

$= 2\frac{N_1}{N_2}[I_{p1m}\cos\omega t+I_{p3}\cos 3\omega t+...]$ ...(14·75)

A very significant property of pushpull operation that becomes evident from equation (14·75) is that even harmonics *i.e.* second, fourth etc. harmonics generated within the tube are absent in the output. This property makes a pushpull amplifier far superior to a parallel connected amplifier.

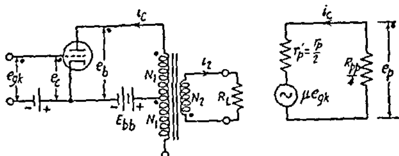
Current through the plate supply source is equal to

$$i_b+i'_b = 2I_b+2I_{po}+2[I_{p2m}\cos 2\omega t+I_{p4m}\cos 4\,\omega t+...]$$ ...(14·76)

Thus on application of signal voltage, current through the plate supply source contains in addition to the original d.c. current $2I_b$, odd harmonic terms and an additional average component $2I_{po}$. If a d.c. milliammeter is connected in series with the plate supply source, on application of grid signal voltage, milliammeter reading increases by an amount equal to $2I_{po}$ indicating that harmonic distortion is taking place.

Composite tube and composite static characteristics.

A.c. equivalent circuit of Fig. 14·17 enables us to determine the a.c. plate current of the composite tube which gives the same power output as the actual pushpull amplifier. Further the tube operates into an impedance of $\frac{R_{pp}}{4}$ *i.e.* into one-half of the output trans-



(a) Circuit diagram. (b) A.C. equivalent circuit.
Fig. 14·18. Circuit diagram of composite tube amplifier and its a.c. equivalent circuit.

former primary winding with the other half open-circuited. Thus a composite tube may be defined as a single tube which when operating

into one-half of the output transformer primary winding with the other half open circuited, yields the same load current and a.c. output power as the actual pushpull amplifier using two tubes. Fig. 14·18 shows the circuit diagram of composite tube amplifier and its a.c. equivalent circuit.

Although plate current of composite tube and the a.c. output power may be calculated from equivalent circuit of Fig. 14·18 (*b*), it is useful and necessary to study the characteristics of this composite tube. The composite-tube static plate characteristics may be obtained graphically from the static plate characteristics of the individual tubes. It is assumed, however, that the output transformer is ideal *i.e.* it has no winding resistances, leakage inductances and excitation current and that the centre tap is the true electrical centre of the windings. From Eqn. 14·74,

$$i_2 = \frac{N_1}{N_2}(i_b - i_b')$$

so that output voltage $e_o = i_2 . R_l$

$$= \frac{N_1}{N_2} R_l (i_b - i'_b) \qquad ...(14·77)$$

But $$e_o = \frac{N_2}{N_1} . e_p \qquad ...(14·78)$$

From Equations (14·77) and (14·78),

$$e_p = \left(\frac{N_1}{N_2}\right)^2 R_l (i_b - i'_b) \qquad ...(14·79)$$

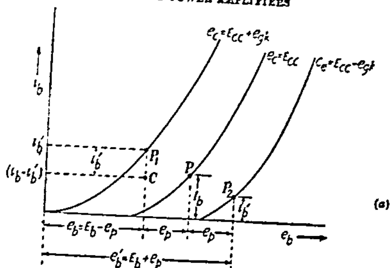
or $$e_p = (i_b - i'_b)\frac{R_{pp}}{4} \qquad (14·80)$$

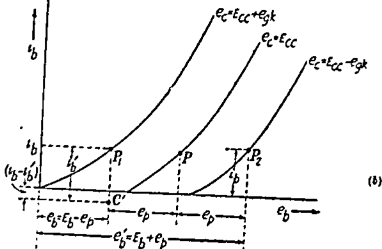
and $$i_c = i_b - i'_b \qquad ...(14·80\ a)$$

From the circuit diagram of Fig. 14·13,

$$\left.\begin{aligned} e_b &= E_b - e_p & ...(a)\\ e'_b &= E_b + e_p & ...(b)\\ e_c &= E_{cc} + e_{gk} & ...(c)\\ \text{and} \quad e'_c &= E_{cc} - e_{gk} & ...(d) \end{aligned}\right\} \qquad ...(14·81)$$

From Equations (14·81) we conclude that when grid-to-cathode voltage e_c of tube T_1 increases above E_{cc} by an amount equal to e_{gk}, the corresponding value of e'_c decreases below E_{cc} by the same amount e_{gk}. Similarly when plate-to-cathode votlage e_b for tube T_1 decreases below the zero signal voltage E_b by an amount equal to e_p, the corresponding plate voltage e'_b of tube T_2 increases above E_b by the same amount e_p. In Fig. 14·19 (*a*) three static characteristics for grid biases of E_{cc}, $E_{gk} + e_{gk}$ and $E_{cc} - e_{gk}$ are shown. P is the zero signal operating point. Then for grid bias of E_{cc}, with the application of signal e_{gk}, if the operating point P_1 of tube T_1 lies anywhere on static characteristic for $E_{cc} + e_{gk}$, the operating point P_2 for tube T_2 must lie somewhere on the static characteristic for $E_{cc} - e_{gk}$. The exact position of points P_1 and P_2 depends upon the value of incremental plate


$e_c = E_{cc} + e_{gk}$
$e_c = E_{cc}$
$e_c = E_{cc} - e_{gk}$
i_b
i_b'
$(i_b - i_b')$
P_1
C
P
P_2
$e_b = E_b - e_p$
e_p
$e_b' = E_b + e_p$
e_b
(a)


$e_c = E_{cc} + e_{gk}$
$e_c = E_{cc}$
$e_c = E_{cc} - e_{gk}$
i_b
i_b'
$(i_b - i_b')$
P_1
P
P_2
C'
$e_b = E_b - e_p$
e_p
$e_b' = E_b + e_p$
e_b
(b)

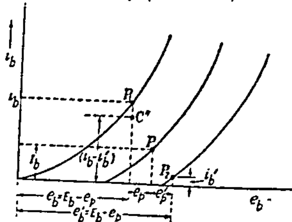

i_b
P_1
C''
P
$(i_b - i_b')$
P_2
i_b'
$e_b = E_b - e_p$
e_p
$e_b' = E_b - e_b$
e_b

Fig. 14·19 (c).

voltage e_p as shown in Fig. 14·19 (a). Values of instantaneous total plate currents i_b and i_b' may be found from positions of points P_1 and P_2. The ordinate of point C in Fig. 14·19 (a) gives the difference plate current $(i_b - i'_b)$. This construction is repeated for two other values of e_p in Figs. 14·19 (b) and (c) and let the corresponding points indicating $(i_b - i'_b)$ be C' and C''. The points C, C', C'' etc. so obtained are joined by a line as shown in Fig 14·19 (d). This line shows the variation of difference current $(i_b - i'_b)$ against e_b and hence gives the composite-tube static characteristic for the given value of grid-signal voltage e_{gk}.

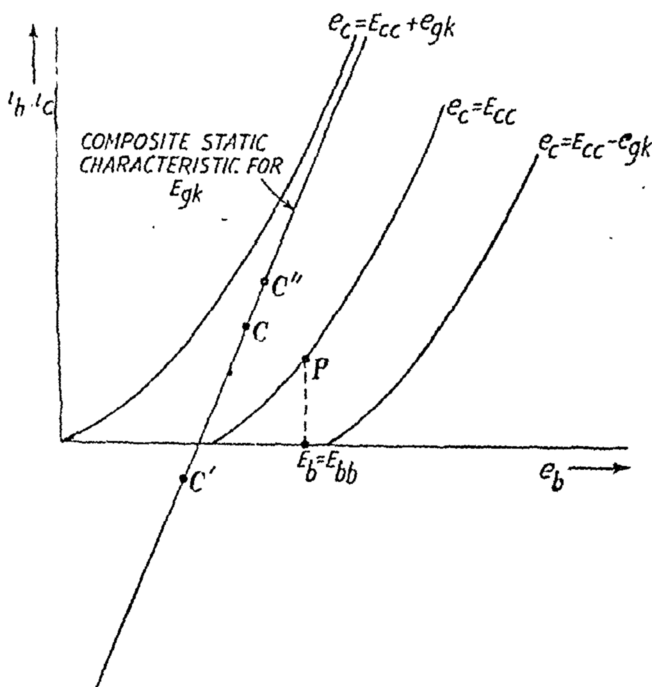


Fig. 14·19 (d). Construction of composite static characteristic from static plate characteristics of tubes.

It may be seen from Fig. 14·19 (d) that the composite static characteristic is far more linear than the static plate characteristic of the individual tubes. Further this composite static characteristic extends even into the negative plate current region. A family of such composite static characteristics may be drawn for different values of grid signal voltage e_{gk} as the parameter but for a fixed value of E_{cc} as shown in Fig. 14·20. If the value of E_{cc} is changed, the whole family of characteristics gets changed.

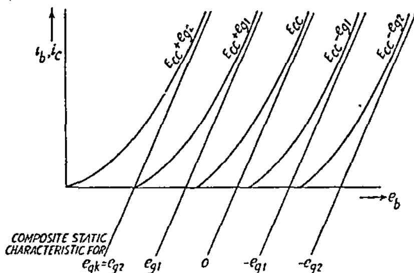


Fig. 14·20. Family of composite static characteristics.

An alternative and more common method of deriving composite static characteristics is due to Thompson. Validity and procedure for this method become apparent from the following consideration. Let current i_c of the composite tube be given by,

$$i_c(e_c, e_b) = i_b(e_c, e_b) - i'_b(e'_c - e'_b) \qquad ...(14·82)$$

where e_c and e_b refer to tube T_1 and e'_b and e'_c refer to tube T_2

But since $e'_{gk} = -e_{gk}$, we have

$$e'_c = E_{cc} + e'_{gk} = E_{cc} - e_{gk} = e_c - 2\, e_{gk} \qquad ...(14·83$$

Further $e_p' = -e_p$ and for ideal lossless output transformer

$$E_{bb} = E_b = E_b' \qquad ...(14·84$$

so that $e'_b = E_{bb} + e_p' = E_{bb} - e_p = 2\, E_{bb} - e_b \qquad ...(14·85$

Further tubes T_1 and T_2 are identical and hence currents i_b and i'_b are similar functions of different variables. Hence i'_b (e'_c, e'_b) may simply be written as $i_b(e_c, e'_b)$. Therefore, Eq. (14 82) may be put in the form,

$$i_c[E_{cc} + e_{gk}, e_b] = i_b[E_{cc} + e_{gk}, e_b] - i_b[E_{cc} - e_{gk}, 2E_{bb} - e_b] \qquad ...(14·86$$

Eq. (14·86) suggests the Thompson method of drawing the composite static characteristic. In accordance with this method and Eq. (14·86) the static plate characteristics of one tube say tube T_1 are plotted in the usual manner while the static plate characteristic of the other tube T_1 are plotted in an inverted manner with potential scale shifted by $2E_b$ [or $2E_{bb}$ for ideal output transformer] so that potentials E_b of both the set of curves are aligned with each other. Pairs of static characteristic of two tubes are [illegible]ked up. Thus if we pick up for tube T_1, static plate char[illegible]

of $E_{cc}+e_{gk}$, corresponding characteristic of tube T_2 picked up is for $E_{cc}-e_{gk}$ and *vice versa*. Thus while picking up the characteristics, it must be remembered that the sum of the parameters of these picked up characteristics is $2E_{cc}$. For any such pair picked up a number of vertical lines are drawn and the

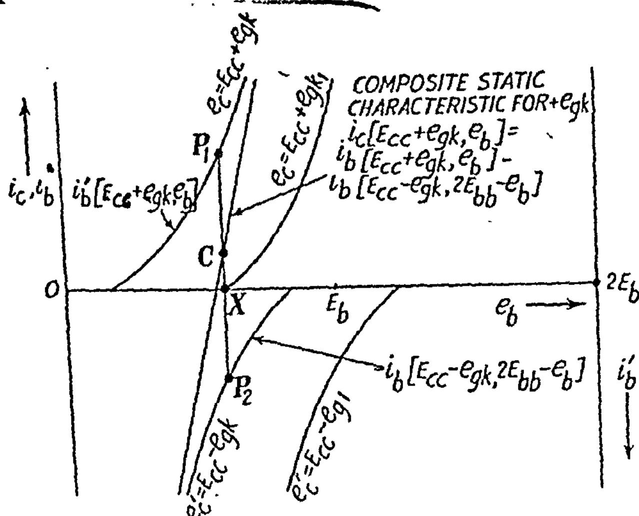
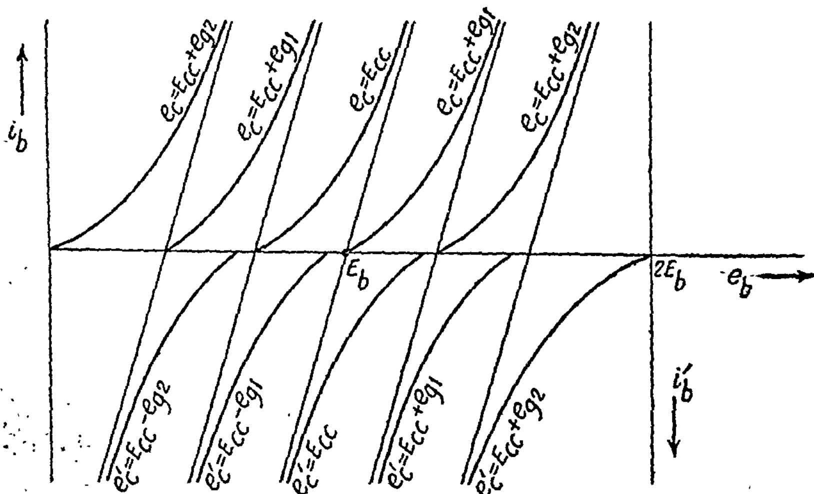


Fig. 14·21 (*a*). Constructio of single composite static characteristic.

ordinates of the two curves are algebraically added. Thus in Fig. 14·21 (*a*) length of the ordinate XC is equal to length XP_1



(*b*) Family of composite static characteristic for given values of E_{cc} and E_{bb}.

Fig. 14·21. Composite static characteristics by Thompson method.

minus XP_2. All points C so obtained, are joined to give the composite static characteristic for the two tubes T_1 and T_2 for given values of E_{bb} and E_{cc} and for the given grid signal voltage e_{gk}. Similarly other composite static characteristic for different grid signals may be drawn to produce the family of composite static characteristics for given values of E_{cc} and E_{bb}.

It is apparent from Fig. 14·21 (*b*) that,

(*i*) composite static characteristics are far too linear compared with the individual tube static characteristics.

(*ii*) zero-signal plate current in the composite tube is zero.

(*iii*) composite static characteristics are almost linear, parallel and equispaced for equal increments of grid voltage

The Thompson method and the first method given in Fig. 14 19 yield the same characteristics.

The above discussion regarding the graphical construction is general and, therefore, applies equally well to class A, AB and B operations of a pushpull amplifier. Composite static characteristics for pushpull amplifier using pentodes can be obtained in a similar manner. Fig. 14 22 gives the composite tube characteristics (shown dotted) for three values of grid signal voltage e_{gk} for grid bias $E_{cc} = -20$ volts. Construction method for $e_{gk} = +10$ volts is indicated. Composite static characteristics for pentodes are obviously more easy to draw since the individual tube characteristics are essentially horizontal. For the same reason zero signal composite static characteristic coincides with zero current axis

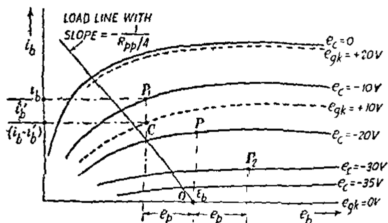


Fig. 14 22. Composite static characteristics of a pentode pushpull amplifier

Load lines and dynamic transfer characteristics of composite tube and individual tubes

In accordance with equation (14 8) effective load resistance for the composite tube is $\frac{R_{pp}}{4}$ and load line may be

slope of $-\frac{1}{(R_{pp}/4)}$ and passing through the point P on the voltage axis distant E_b from the origin as shown in Fig. 14·23. Let this load line cut the composite characteristics in points P_1, P_2 etc. Then these are the points on the composite dynamic transfer characteristic. Each point of intersection gives value of current i_c of composite tube for the corresponding grid signal e_{gk}. These points may be plotted to give the composite dynamic transfer characteristic. This is shown in Fig. 14·24.

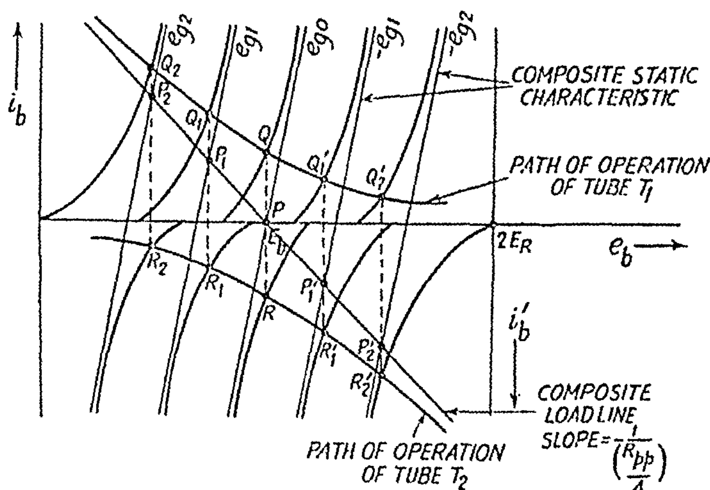


Fig. 14·23. Load lines of composite tube and individual tubes.

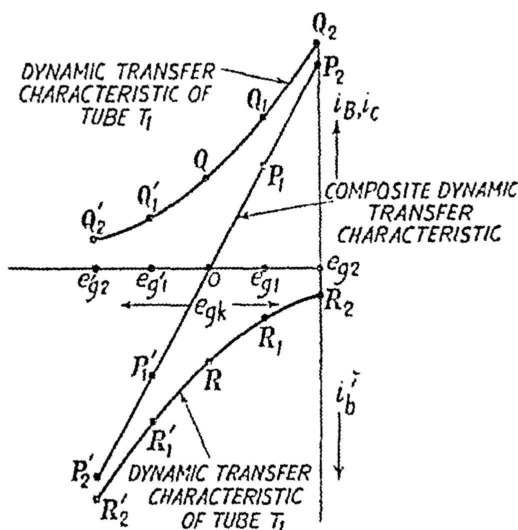


Fig. 14·24. Dynamic transfer characteristics of composite tube and individual tubes.

In order to find the path of operation or load line of individual ..., we draw vertical lines from points P_2, P_1, P, P_1' and P_2' on the composite load line. Let these vertical lines cut the ...sponding static plate characteristics for tube T_1 in points ...$_1$, Q, Q_1' and Q_2' and for tube T_2 in points R_2, R_1, R, R_1' and ... A line joining points Q_2, Q_1, Q, Q_1' etc. then gives the load line ...ube T_1. Similarly a line joining points R_2, R_1, R, R_1' etc. gives ...oad line for tube T_2.

Dynamic transfer characteristic of tube T_1 may be plotted by ...ng the values of plate current i_1 and grid voltage $e_{c1} = E_{cc}+e_g$ ... the points Q_2, Q_1, Q etc. on the load line of tube T_1 and plotting ...se points as shown in Fig. 14·24. Similarly dynamic transfer ...racteristic of tube T_2 may be plotted by noting the values of ...te current i'_1 and grid voltage $e_{c2} = E_{cc}+e_{g2}$ for tube T_2 and ...tting these points as shown in Fig. 14·24.

These dynamic transfer characteristics may subsequently be ...ed to get the waveform of plate currents of individual tubes and ...at of composite tube. It may be noted that composite dynamic ...ransfer characteristic is almost a straight line inspite of the fact ...hat the dynamic transfer characteristics of individual tubes are considerably curved. Hence if a pure sinusoidal signal is applied to the input of the amplifier, the composite plate current i_c is essentially sinusoidal while plate currents in each tube depart considerably from sinusoidal waveform.

It is not necessary to draw the individual tube dynamic transfer characteristic if total output current or output power is required to be found. If, however, plate current efficiency is required to be calculated, it is necessary to draw these individual tube dynamic characteristics in order to evaluate zero signal d.c. plate current I_b and average d.c. plate current I_p, caused due to distortion in individual tube current waveforms. The total d.c. power input to the pushpull amplifier plate circuit is $2E_{bb}(I_b+I_p)$.

It may be noted that the composite tube characteristics depend upon the values of E_{cc} and E_{bb} and thus they are functions of quantities external to the tube and thus they differ from static characteristics of tube which depend entirely on the tube alone. Another significant fact worth noticing is that while the individual tube plate currents i_1 and i'_1 for class AB operation may become zero and remain so far appreciable part of the a.c. cycle, no such thing happens with composite tube. The operation along the composite load line is truly symmetrical about zero signal operating point for both positive and negative values of grid signal voltage e_g. For the condition shown in Fig. 14·23, study of the path of operation of individual tube reveals that the individual tube currents never fall to zero as the grid signal voltage goes through a complete a.c. cycle from operating point P_2 corresponding to zero total grid voltage e_c of tube T_1 to operating point P_2' corresponding to zero total grid voltage e_c' of tube T_2 or corresponding to most negative value of grid voltage e_c of tube T_1. The operation is thus class A_1. Fig. 14·25 shows the composite tube characteristics and load lines in a limiting case of class A_1 operation. Hence the current i'_1 of tube T_2 just

becomes zero when grid voltage e_c of tube T_1 reaches zero and *vice versa.*

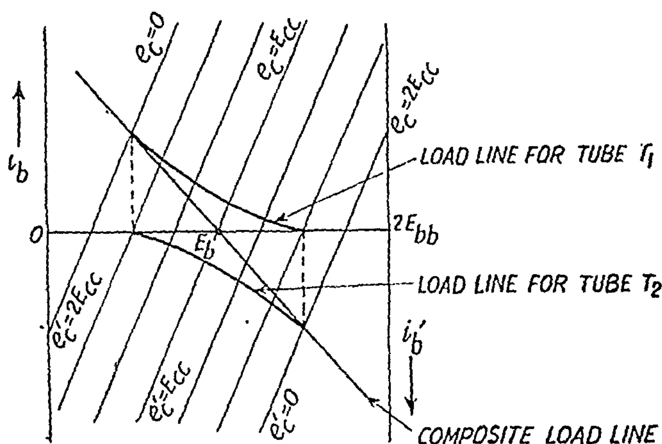


Fig. 14·25. Composite static characteristics and load lines for composite tubes and individual tubes in limiting case of class A1 operation.

Output power and distortion in pushpull amplifiers.

For input sinusoidal signal, output current may be found from composite dynamic transfer characteristic. Obviously with reference to the Fischer Hinnen method, studying the nature of composite dynamic transfer characteristic, $I_b = 0$, $i_{b\ max} = i_{b\ min}$ and $i_1 = -i_2$. Hence in accordance with this selected ordinate method or Fischer Hinnen method, from equation (11·111) to (11·116),

$$I_{bs} = I_{p2m} = I_{4m} \qquad \text{...(14·87)}$$

$$I_{p1m} = \tfrac{2}{3}(i_{max}+i_1) \qquad \text{...(14·88)}$$

$$I_{p3m} = \tfrac{1}{3}(i_{max}-2i_1) \qquad \text{...(14·89)}$$

where I_{p1m}, I_{p2m}, I_{p3m} and I_{p4m} are amplitudes of fundamental, second, third and fourth harmonics of plate current respectively.

Hence a.c. output power at fundamental frequency is given by,

$$P_1 = \left(\frac{I_{p1m}}{\sqrt{2}}\right)^2 \cdot \frac{R_{pp}}{4} = \frac{I_{p1m}{}^2 R_{pp}}{8} \qquad \text{...(14·90)}$$

If harmonic components of output power are neglected, total output power is given by,

$$P_{ac} = \frac{I_{pm} E_{pm}}{2} \qquad \text{(14·91)}$$

where I_{pm} and E_{pm} are respectively peak values of the a.c. output current and voltage respectively. These may be found directly from 14·23 where

$I_{pm} = I_{p2}$ (ordinate of point P_2) and

$E_{pm} = E_b - E_{P_2}$, E_{P_2} being abscissa of point P_2

Hence $$P_{ac} = \frac{(E_b - E_{p2})(I_{p2})}{2} \qquad \text{...(14·92)}$$

For maximum output power in class A triode pushpull amplifier, in the equivalent circuit of the composite tube given in Fig. 14·18 (*b*), the effective load resistance $R_{pp}/4$ must equal the source resistance $r_p/2$.

For pentode pushpull amplifier, above considerations of matching the effective load impedance with eff ctive internal impedance of the source does not arise. Instead, the optimum load is so chosen that it gives maximum power with low distortion. In *Fig. 14·22, optimum load line is drawn from point Q on the voltage axis at* $e_b = E_b$, at such an angle as to intersect the composite tube maximum grid signal characteristic near its knee.

Driver amplifier for pushpull amplifier.

Driver amplifier is the one which supplies to the grids of the pushpull power amplifier tubes, two voltages of equal magnitudes but opposite phases. This may be achieved by one of the following two methods :—(*i*) use of centre-tapped transformer and (*ii*) use of paraphase amplifier.

Driver amplifier using centre-tapped transformer.

This is one of the most common driver circuit and is shown in Fig. 14·26. It is a simple transformer-coupled amplifier with centre-tapped secondary. If no grid current flows in the pushpull amplifier that follows, then the design of this driver amplifier is quite simple. Often, however, pushpull amplifier tubes carry appreciable grid current, so that power as large as 15% of the pushpull amplifier output power may be required to be fed from the driver amplifier. Grid circuits of the pushpull amplifier then present non-linear grid-to-cathode impedances resulting in distortion of the signal waveform unless the effective internal impedance of the driven amplifier is very small. Hence in such cases, driver amplifier must be designed to have low internal impedance. Further a step-down transformer should be used to couple the driver-amplifier to reduce the effective internal impedance further.

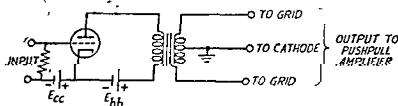


Fig. 14·26. Schematic diagram of transformer-coupled driver circuit.

Paraphase amplifiers or Phase Invertors.

A paraphase amplifier, also sometimes called a ph[...] is an amplifier which produces from a single source [...] equal in magnitude and 180° out of phase. Follo[...] of the more commonly used paraphase amplifiers :— [...] paraphase amplifier (*b*) two-tube paraphase amplifi[...] paraphase amplifier (*d*) Cathode-coupled paraphase a[...]

Single tube paraphase amplifier.

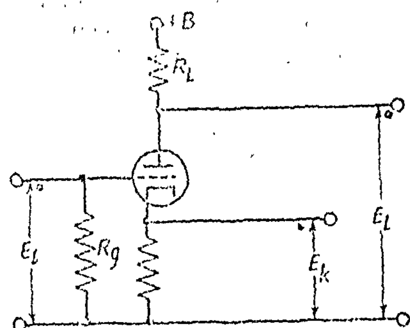


Fig. 14·27. Single tube paraphase amplifier

Fig. 14·27 shows the basic circuit. The amplifier has already been discussed in chapter XIII. Resistances R_l and R_k are kept equal. The voltage $\mathbf{E}_k$ and $\mathbf{E}_l$ are equal in mangnitude and opposite in phase. Analysis in chapter XIII has shown that voltage gain of the stage is less than unity and is given by,

$$\mathbf{A}_{fb} = \frac{\pm \mu R}{(\mu+2)R + r_p} \qquad ...(14·93)$$

where $R_k = R_l = R$

Since $(\mu+2)R >> r_p$ for triode,

$$\mathbf{A}_{fb} \approx \frac{\pm \mu}{\mu+2} \qquad ...(14·94)$$

Two-tube paraphase amplifier.

Fig. 14·28 shows a simple two-tube paraphase amplifier. Tube T_1 in conjunction with load resistance R_l, coupling condenser C_c

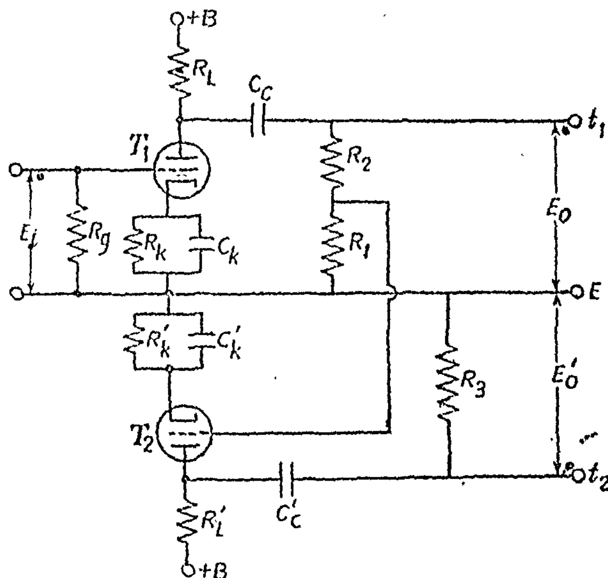


Fig. 14·28. Two-tube paraphase amplifier.

and shunt resistance (R_1+R_2) acts as a conventional one-stage R.C. coupled amplifier. $\mathbf{E}_o$ is the output voltage. Resistors. R_1 and R_2 constitute a potential divider. A fraction $\frac{R_1}{R_1+R_2}$ of output voltage $\mathbf{E}_o$ of this amplifier is fed between grid and cathode of second tube T_2. This tube T_2 in conjunction with load resistance R_l', coupling condenser C_c' and shunt resistor R_3 constitutes another stage of R.C.-coupled amplifier. The ratio of resistances R_1 and R_2 is so

chosen that the output $\mathbf{E}_o$ of tube T_1 equals the output voltage $\mathbf{E}_o'$ of the second tube T_2. This happens only when the ratio $\frac{R_1}{R_1+R_2}$ is equal to voltage gain A_2 of the amplifier using tube T_2. Further this second amplifier inverts the phase of amplified voltage so that $\mathbf{E}_o'$ is equal to $\mathbf{E}_o$ in magnitude and is opposite in phase. Thus it is possible to obtain two voltages equal in magnitude and [illegible] This circuit [illegible] of having [illegible] ng utilized [illegible] lifier stage. But it is necess- [illegible] g conditions such that the curvature of characteristics of tube T_2 just compensates for the curvature of characteristics of tube T_1. This ensures that the voltage between output terminal t_1 and t_2 is essentially undistorted [illegible] terminals and [illegible] of operating [illegible] e and consti-

Two-tube floating paraphase amplifier.

Fig. 14·29 shows the circuit diagram of one type of floating paraphase amplifier. It is a sort of feedback device. The difference

Fig. 14·29. Two-tube floating paraphase amplifier.

potential between the output voltages $\mathbf{E}_o$ and $\mathbf{E}_o'$ of two tubes is used as input signal to the second tube T_2 *i.e.* phase invertor tube. Tube T_1 along with load resistance R_l, coupling condenser C_c and shunt resistances R_1 and R_2, is a conventional R.C. coupled amplifier. Resistances R_1, R_2 and R_{fb} are kept equal. The voltage developed across R_{fb} constitutes the input voltage for tube T_2. Tube T_2 along with load resistance R'_l, coupling condenser C'_c and shunt resistance (R_2+R_{fb}) constitutes the phase invertor amplifier. Let $\mathbf{E}_o$ and $\mathbf{E}_o'$ be the output voltages of tubes T_1 and T_2 respectively. Then across R_{fb} is developed a voltage equal to $\frac{1}{2}(\mathbf{E}_o+\mathbf{E}'_o)$. But voltages $\mathbf{E}_o$ and $\mathbf{E}_o'$ are 180° out of phase and $\mathbf{E}_o$ is adjusted to be slightly larger than $\mathbf{E}_o'$. The voltage $\frac{1}{2}(\mathbf{E}_o+\mathbf{E}_o')$ is then quite small and drives the tube T_2 to maintain relative values of $\mathbf{E}_o$ and $\mathbf{E}_o'$. Any change in $\mathbf{E}_o'$ causes a change in $\mathbf{E}_{fb}$ which after amplification changes $\mathbf{E}_o'$ in such a way as to bring it back to the desired value. In order that $\mathbf{E}_{fb}$ be kept small, gain A_2 of tube T_2 must be large and for this purpose T_2 is usually a pentode.

Cathode-coupled Paraphase Amplifier.

Fig. 14·30 shows the basic circuit diagram of cathode coupled paraphase amplifier. It is basically a direct coupled amplifier and

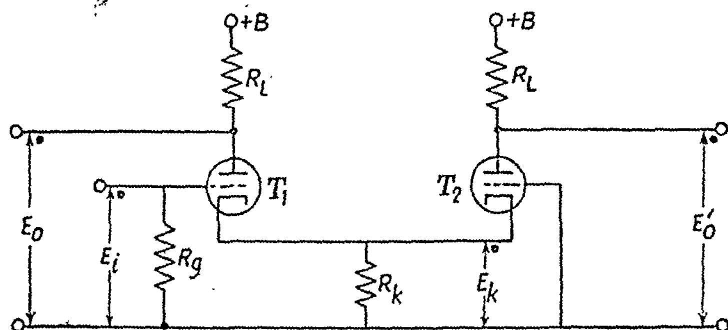


Fig. 14·30. Cathode-coupled paraphase amplifier.

is widely used to provide pushpull voltages for oscilloscopy and to pushpull amplifier. Tube T_1 along with load resistance R_l and cathode resistance R_k is a simple cathode feed-back amplifier. R_k is usually very small compared with R_l. Voltage $\mathbf{E}_k$ developed across R_k is 180° out of phase with output voltage $\mathbf{E}_o$ across R_l. This votage $\mathbf{E}_k$ constitutes the input voltage to the next tube T_2. Thus tube T_2 is cathode coupled to the first tube T_1. Output voltage $\mathbf{E}_o'$ of tube T_2 is in phase with R_k and is, therefore, 180° out of phase with $\mathbf{E}_o$.

Fig. 14·31 shows the Thevenin's form of a.c. equivalent circuit of the amplifier as seen from cathode-ground terminals. The two tubes T_1 and T_2 are similar.

From this a.c. equivalent circuit,

$$\mathbf{E}_k = -\mathbf{I}_{p2}.\frac{r_p+R_l}{\mu+1} \quad ...(14·95)$$

Also $$\mathbf{E}_k = \frac{\mathbf{I}_{p1}.R_k.\dfrac{r_p+R_l}{\mu+1}}{R_k+\dfrac{r_p+R_l}{\mu+1}} \qquad ...(14\cdot 96)$$

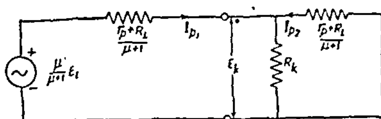


Fig. 14·31. Thevenin's form of a.c. equivalent circuit of cathode coupled paraphase amplifier as seen from cathode-ground terminals

Hence $$-\mathbf{I}_{p2}.\frac{r_p+R_l}{\mu+1} = \mathbf{I}_{p1}\ \frac{R_k.\dfrac{r_p+R_l}{\mu+1}}{R_k+\dfrac{r_p+R_l}{\mu+1}} \qquad ...(14\cdot 97)$$

or $$-\frac{\mathbf{I}_{p1}}{\mathbf{I}_{p2}} = 1+\frac{r_p+R_l}{R_k(\mu+1)} \qquad ...(14\cdot 98)$$

Thus we see that $\mathbf{I}_{p1}$ is greater than $\mathbf{I}_{p2}$. Typically $-\frac{\mathbf{I}_{p1}}{I_{p2}}=1\cdot 1$. The output voltages $\mathbf{E}_o$ and $\mathbf{E}_o'$ bear the same ratio. The difference between $\mathbf{E}_o$ and $\mathbf{E}_o'$ may, however, be kept small, by keeping the ratio $-\frac{\mathbf{I}_{p1}}{\mathbf{I}_{p2}}$ not much larger than unity. If $\frac{I_1}{I_2}$ is not to exceed 1·1, then

$$R_k \nless 10\,.\,\frac{r_p+R_l}{\mu+1} \qquad ...(14\cdot 99)$$

If I_{p2} is to keep very close to I_{p1}, R_k must be made large.

From the a.c. equivalent circuit,

$$\frac{\mu}{\mu+1}\ \mathbf{E}_i+(\mathbf{I}_{p1}-\mathbf{I}_{p2})\ \frac{r_p+R_l}{\mu+1} = 0 \qquad ...(14\cdot 100)$$

Hence $$\mathbf{I}_{p1}-\mathbf{I}_{p2} = \frac{\mu\mathbf{E}_i}{r_p+R_l} \qquad ...(14\cdot 101)$$

$\mathbf{I}_{p1}$ is approximately equal to $-\mathbf{I}_{p2}$ and hence

$$\mathbf{I}_{p1} \approx \mathbf{I}_{p2} \approx \frac{\mu\mathbf{E}_i}{2(r_p+R_l)} \qquad ...(14\cdot 102)$$

Hence $$\mathbf{E}_o \approx -\mathbf{E}_o' \approx \frac{\mu\mathbf{E}_i R_l}{2(r_p+R_l)} \qquad ...(14\cdot 103)$$

Class AB pushpull audio frequency power amplifier.

In class *AB* operation, grid bias and grid signal voltage are so adjusted that plate current flows for appreciably greater than half the a.c. cycle but less than the complete cycle. The plate current

waveform then departs considerably from the input voltage waveform resulting in harmonic distortion. Consequently single tube class AB amplifiers are never used for audio frequency power amplification. Pushpull operation eliminates even harmonics and thus the harmonics are reduced to be within the permissible limits.

The circuit diagram of pushpull class AB audio frequency power amplifier is the same as that for pushpull class A audio

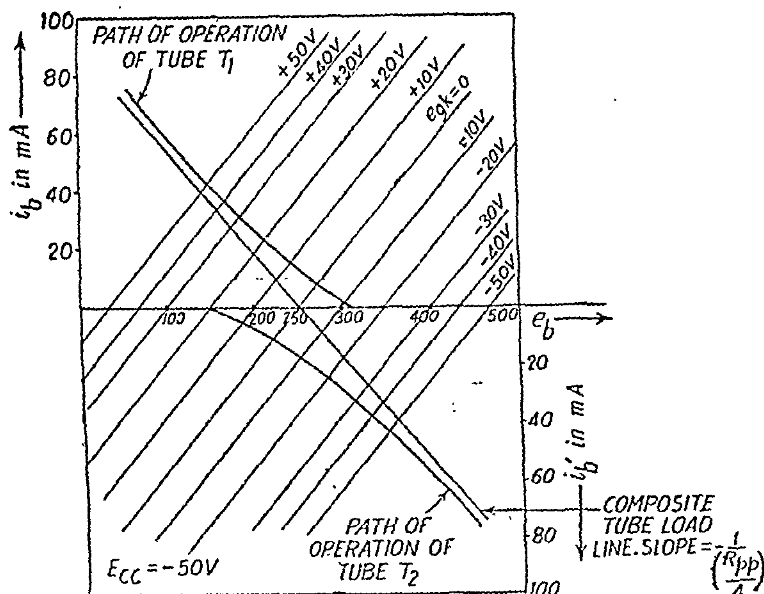


Fig. 14·32. Composite tube characteristics and load lines in class AB pushpull amplifier.

frequency power amplifier. The only difference lies in the operating conditions. As the operation of pushpull amplifier is changed from class A to class AB, it is interesting to examine the changes in (*i*) the shape of the composite tube static characteristic and (*ii*) load lines or paths of operation of individual tubes when the plate supply voltage E_{bb} is specified. Fig. 14·32 shows the composite tube static characteristics, composite tube load line and load lines of individual tubes for a class AB pushpull untuned power amplifier with grid bias of −50 volts and plate supply voltage of 250 volts. A study of the paths of operation of individual tubes indicates that for a grid signal voltage amplitude E_{gm} equal to E_{cc}, the instantaneous plate currents i_b and i_b' become zero and remain so for an appreciable portion of the a.c. cycle. The operation is, therefore, class AB_1. The operation remains class A_1 for only small amplitudes of grid signal. If E_{gm} exceeds E_{cc}, operation becomes class AB_2.

As the grid bias increases, the slope of the composite characteristics changes but the composite static characteristics remain practically straight, parallel and equispaced for equal increments of the grid voltage. The composite dynamic transfer characteristic is still linear although dynamic transfer characteristics of individual

Now consider class AB_1 amplifier operating under limiting condition of maximum grid signal *i.e.* with $E_{gm} \doteq E_{cc}$. If now the signal amplitude E_{gm} is increased still further without changing the grid bias E_{cc}, the operation becomes class AB_2. This gives greater a. c. output power without increasing the plate supply voltage. But the grid current flow results in additional distortion of plate current waveform unless care is taken to use a driver stage with low internal impedance and step-down output transformer.

Usually cathode bias circuit is used. With class AB operation, average plate current I_{bs} increases with grid signal E_{gk} resulting in an increased grid bias. This increased bias may increase the harmonic distortion. To avoid this trouble cathode resistor R_k must be so chosen as to result in grid bias voltage causing limiting class A_1 operation with small grid signal so that even when grid signal voltage has increased sufficiently, grid bias does not become excessively large.

Class AB operation may be considered to be one intermediate between class A and class B operations both of which are well-defined. It is not readily possible, therefore, analytically to determine the condition of maximum power output and efficiency and to calculate the harmonic contents. It is preferred to use either graphical method or direct electrical measurement.

Class B pushpull audio frequency power amplifiers.

The circuit diagram is the same as for class A pushpull

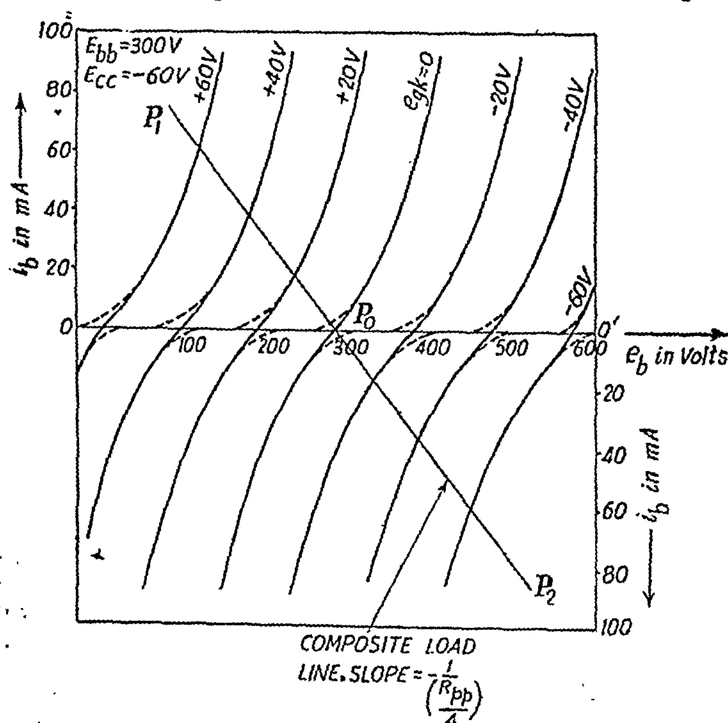


Fig. 14·34. Composite static characteristics of class B pushpull amplifier using triodes.

amplifier. *The only difference lies in the* operating conditions, In class B operation, grid bias is kept equal to cuttoff value so that plate current flows for half the a.c. cycle only. The composite static characteristics follow the corresponding individual tube static plate characteristics over almost their complete range as shown in Fig. 14·34. In the low curent region, therefore, composite tube static characteristics are appreciably curved. The average slop of these composite static characteristics is approximately one-half of the corresponding characteristics for class A operation. Average effective plate resistance of the composite tube for class B operation is, therefore, approximately twice that for class A operation.

[illegible] ight line P_1P_0 for [illegible] age axis P_0O' for [illegible] for tube T_2 path [illegible] itive value of grid [illegible] r negative values of e'_{gk}. Thus at a time only one tube is operating *i.e.* when tube T_1 is conducting tube T_2 is cut off and *vice versa*. For class B operation composite tube static characteristics are, thus, seen to be no longer linear and hence composite dynamic transfer characteristic is also non-linear. Hence even with pushpull circuit for class B operation, harmonic distortion cannot be entirely avoided. Harmonic distortion is maximum for small grid-signal amplitudes since the composite static characteristics have greatest curvature in this region. An accurate determination of harmonic contents may be made by Fourier analysis of the composite tube plate current waveform obtained by applying the input grid signal to the composite dynamic transfer characteristic. Approximate method is not adequate since high order harmonics are present in appreciable magnitude.

Approximate analysis of class B pushpull amplifier. *Following assumptions are made :*

(*i*) input signal voltage is sinusoidal.

(*ii*) output transformer is ideal.

(*iii*) static plate characteristics of tubes T_1 and T_2 are identical and are linear, parallel and equi-spaced for equal increments of grid voltage.

Then the composite tube static characteristics are also linear similar to those shown in Fig. 14·20. Then the composite dynamic transfer characteristic is also linear and so also are individual tube dynamic transfer characteristics as shown in Fig. 14·35.

Individual tube currents flow for alternate half cycles while composite tube current flows for the entire cycle. Obviously for sinusoidal input voltage, maximum values of plate currents of individual tubes are equal and each is equal to maximum value of *composite tube current. Let this maxium plate current be indicated* by I_{pm}. Then from the waveform of i_c, a. c. output power of two tubes is,

$$P_{ac} = \left(\frac{I_{pm}}{\sqrt{2}}\right)\left(\frac{E_{pm}}{\sqrt{2}}\right) = \frac{1}{2} I_{pm} E_{pm} \quad ...(14·105)$$

Now consider class AB_1 amplifier operating under limiting condition of maximum grid signal *i.e.* with $E_{gm} \doteq E_{cc}$. If now the signal amplitude E_{gm} is increased still further without changing the grid bias E_{cc}, the operation becomes class AB_2. This gives greater a. c. output power without increasing the plate supply voltage. But the grid current flow results in additional distortion of plate current waveform unless care is taken to use a driver stage with low internal impedance and step-down output transformer.

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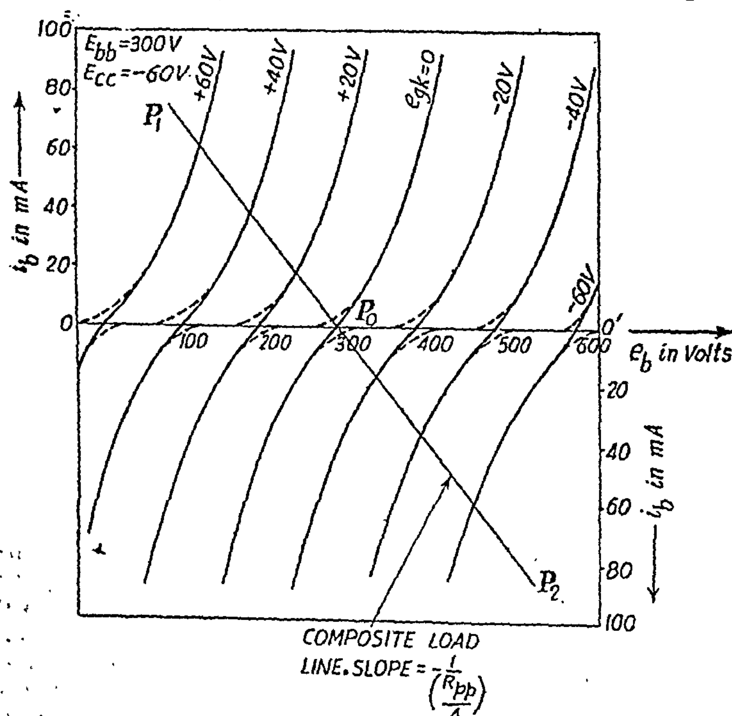


Fig. 14·34. Composite static characteristics of class B pushpull amplifier using triodes.

amplifier. The only difference lies in the operating conditions. In class B operation, grid bias is kept equal to cuttoff value so that plate current flows for half the a.c. cycle only. The composite static characteristics follow the corresponding individual tube static plate characteristics over almost their complete range as shown in Fig. 14·34. In the low curent region, therefore, composite tube static characteristics are appreciably curved. The average slop of these composite static characteristics is approximately one-half of the corresponding characteristics for class A operation. Average effective plate resistance of the composite tube for class B operation is, therefore, approximately twice that for class A operation.

Path of operation of tube T_1 is along the straight line P_1P_0 for positive values of grid signal e_{gt} and is along voltage axis P_0O' for negative value of grid signal voltage e_{gt}. Similarly for tube T_2 path of operation is along the straight line P_2P_0 for positive value of grid signal voltage e'_{gt} and is along voltage axis P_0O for negative values of e'_{gt}. Thus at a time only one tube is operating *i.e.* when tube T_1 is conducting tube T_2 is cut off and *vice versa*. For class B operation composite tube static characteristics are, thus, seen to be no longer linear and hence composite dynamic transfer characteristic is also non-linear. Hence even with pushpull circuit for class B operation, harmonic distortion cannot be entirely avoided Harmonic distortion is maximum for small grid-signal amplitudes since the composite static characteristics have greatest curvature in this region. An accurate determination of harmonic contents may be made by Fourier analysis of the composite tube plate current waveform obtained by applying the input grid signal to the composite dynamic transfer characteristic. Approximate method is not adequate since high order harmonics are present in appreciable magnitude.

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$$P_{ac} = \left(\frac{I_{pm}}{\sqrt{2}}\right)\left(\frac{E_{pm}}{\sqrt{2}}\right) = \frac{1}{2} I_{pm} E_{pm} \quad ...(14\cdot105)$$

where E_{pm} is the amplitude of a.c. component of output volage of composite tube and is also equal to maximum value of output voltage of each tube.

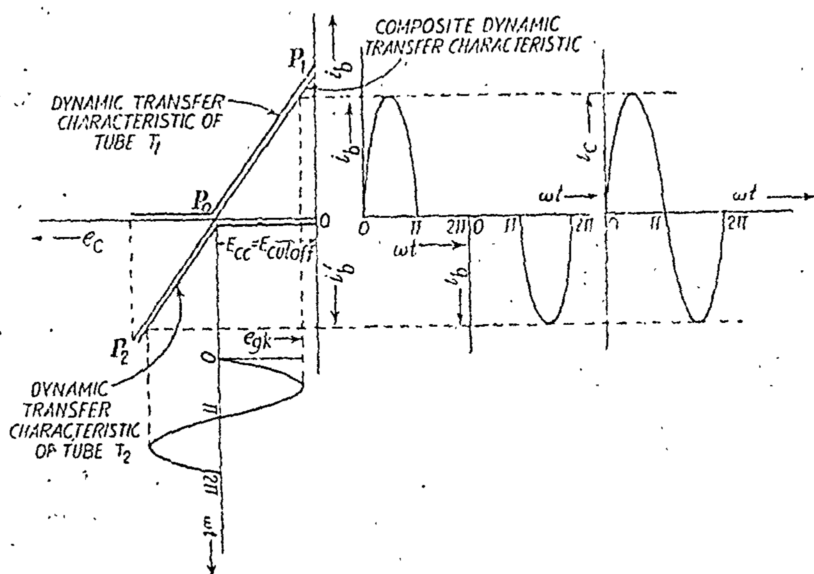


Fig. 14·35. Dynamic transfer characteristics and plate current waveforms of composite tube and individual tubes.

Average value of plate current of each tube is given by,

$$I_{bs} = \frac{1}{2\pi}\int_0^{2\pi} i_b d(\omega t) = \frac{1}{2\pi}\int_0^{\pi} I_{pm} \sin \omega t \, d(\omega t) \qquad \ldots(14\cdot106)$$

or

$$I_{bs} = \frac{I_{pm}}{\pi} \qquad \ldots(14\cdot107)$$

Power supplied from the plate supply source is given by,

$$P_{bb} = 2E_{bb}.\, I_{bs} = \frac{2}{\pi}.\, E_{bb}.I_{pm} \qquad \ldots(14\cdot108)$$

Hence plate circuit efficiency of the class B pushpull amplifier is given by,

$$\eta_p = \frac{P_{ac}}{P_{bb}} = \frac{\frac{1}{2}E_{pm}\, I_{pm}}{\frac{2}{\pi}\, E_{bb}\, I_{pm}} = \frac{E_{pm}}{E_{bb}} \times \frac{\pi}{4} = \frac{E_{pm}}{E_{bb}} \times 78\cdot5\% \quad (14\cdot109)$$

From Eq. (14·109) the plate circuit efficiency increases with increase of E_{pm} *i. e.* with the increase of grid signal voltage. Amplitude E_{pm} may have maximum value equal to E_{bb}. Hence maximum plate circuit efficiency of class B amplifier is 78·5%. E_{pm} may be made to approach E_{bb} by one of the following two means : (*i*) path of operating point approaches the plate voltage axis. This means that load resistance R_l be made very high in which case the a.c. output power becomes very low. (*ii*) grid voltage of each tube may be made high positive during part of the a.c. cycle. This requires

a large amount of power from the driver stage. Since neither of the above two conditions is practically feasible, the plate circuit efficiency of class B pushpull amplifiers is never made to exceed about 60%.

In the class B audio frequency power amplifier, in accordance with Eq. 14·107, the d.c. component of current through the plate supply source increases with increasing grid-signal *voltage and hence* [illegible] is not practical. A fixed grid bias [illegible] ttery is required. *Again* since I_{bt} [illegible] late supply sources used must have [illegible] rder to prevent variation of plate supply voltage with variation of grid-signal voltage.

Pushpull class B amplifiers may use pentodes instead of triodes. The composite static characteristics then also have shapes similar to individual tube characteristics and have *knees* as well. But the load impedance may be so selected as to restrict the operation to linear region of composite static characteristics. In that case the composite dynamic transfer characteristic is also almost linear and *the composite tube plate current waveform is essentially sinusoidal* for a sinusoidal grid-signal voltage. No attempt is made to match the load impedance with the effective plate resistance of the composite tube.

When triodes are used in class B pushpull power amplifier, for maximum power output, resistance $\frac{R_{pp}}{4}$ corresponding to the slope of the composite tube load line, should be made equal to dynamic plate resistance r_p of a single tube. However, the plate-to-plate resistance $R_{pp} = 4 \left(\frac{N_1}{N_2}\right)^2 R_l$ should be selected considering such factor as harmonic generation, a.c. output power, plate dissipation, grid-[illegible]

low internal impedance and by using a step-down output transformer [illegible]

EXERCISES

1. A linear A_1 triode amplifier uses load resistance of 5000 ohms, plate supply voltage $E_{bb} = 350$ volts, grid bias $= -30$ volts, grid signal $e_{cg} = 10 \sin 2\pi\, 50t$. At zero-signal operating point, plate voltage $E_b = 220$ volts, amplification factor $\mu = 8$, and dynamic path resistance $r_p = 3000$ ohms. Calculate (*i*) magnitude of voltage gain (*ii*) a.c. output voltage (*iii*) power supplied from plate power source (*iv*) zero signal plate dissipation (*v*) plate dissipation with signal applied (*vi*) a.c. power output (*vii*) plate circuit efficiency.

2. A linear A_1 triode amplifier uses bias $= -20$ volts, and plate supply voltage of 360 volts. Zero signal plate voltage $E_b = 240$ volts and zero signal plate current $= 20$ mA. μ and r_p of tube are respectively 10 and 8000 ohms. Safe dissipation of value and

load resistors are 4 watts each. Find the maximum and minimum grid signal voltage that may be applied without exceeding the dissipation limits.

3. Calculate the maximum a.c. output power obtainable from A_1 triode power amplifier having $\mu = 20$, $r_p = 800$ ohms and grid signal voltage $= 10$ volts r.m.s.

4. A class A triode power amplifier uses triode having amplification factor $\mu = 10$ and anode slope resistance $r_p = 4000$ ohms. What signal voltage at the grid should be applied to produce available power output of 1 watt. With this signal voltage applied, find the a.c. power output if the load impedance is (*a*) 6000 ohms resistive (*b*) $4000 + j4000$ ohms.

5. In a class A_1 triode power amplifier load impedance consists of inductor of reactance 4000 ohms in series with a variable resistance of 3000 ohms. Calculate the value of load resistance R_l which will give maximum output power if signal voltage has amplitude of 15 volts.

6. A class A_1 transformer-coupled power amplifier uses triode having amplification factor $\mu = 6$ and dynamic plate resistance $r_p' = 1800$ ohms in the operating region. Load is a loudspeaker of resistance 2Ω in series with an inductive reactance of 5Ω. Turns ratio is $1 : 25$. Calculate the power-fed to the loudspeaker if the input signal is a voltage of 10 volts r.m.s.

7. A class A_1 power amplifier is required to amplify without distortion by not allowing operation below a minimum plate current. Tube has amplification factor $\mu = 12$ and dynamic plate resistance $r_p = 6000$ ohms. Zero signal plate voltage is prescribed at 200 volts. Load resistance of R_l and grid bias E_{cc} are chosen to get maximum power output while maintaining A_1 operation. E_{cc} so used is -15 volts. Calculate the value of load resistance, maximum output power and maximum power sensitivity.

8. Prove that for maximum output power with undistorted amplification in a triode, with a prescribed zero-signal plate voltage, load resistance R_l must be equal to $2\,r_p$. Hence prove that maximum power sensitivity of amplifier then is $\frac{2}{9}\,\mu_{gm}$.

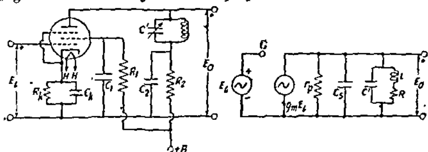
CHAPTER XV

TUNED VOLTAGE AMPLIFIERS

A parallel tuned ciruit or anti-resonant circuit has a high impedance at the resonant frequency and the impedance falls off sharply at frequencies on either side. If such a circuit is used as the plate load in an amplifier, the voltage gain of the amplifier also varies in a manner similar to the tuned circuit impedance *i. e.* the voltage gain is maximum at the resonant frequency and it drops off sharply as the frequency is either lowered or raised. If the amplifier [illegible] very large so that [illegible] is the tuned circ[illegible] pentodes, the voltage gain curve is nothing but the impedance curve of the tuned circuit multiplied by transconductance g_m. Such tuned amplifiers can, therefore, be conveniently used for amplification of a narrow band of frequencies. An important example is the radio frequency amplifier which amplifies either a single radio frequency signal or a [illegible]

that its resonant frequency may be adjusted to be at the centre of the band of frequencies to be amplified. Throughout this narrow band of frequencies concerned, the impedance of the tuned circuit and hence the voltage gain remains more or less constant. In tuned voltage amplifiers the nonlinear distortion is very small because (*i*) the operation is class A_1 and (*ii*) the tuned plate circuit impedance and the voltage gain becomes negligibly small at harmonic frequencies which may be generated due to nonlinearity.

Basic Tuned Voltage Amplifier. Fig 15 1 (*a*) shows the circuit of basic tuned voltage amplifier using a pentode and cathode-bias. R_1 and R_2 are screen grid and plate dropping resistors. C_1 and C_2 are bypass condensers. These combinations of dropping resistors and bypass condensers are usually referred to as "decoupling circuits" and they serve two purposes (*i*) they provide proper



(b) A.C. equivalent circuit

rodes, *i. e.*

prevent variations in the current to one electrode from influencing the voltages of other electrodes in the same or other stages of a cascade amplifier. This coupling occurs through the common power supply. Decoupling or bypass condensers must then be made large enough to provide almost zero impedance path to common earth connection. Then the capacitance between plate and all electrodes is effectively in shunt with the tuning condenser. This is indicated as C_s in the a. c. equivalent circuit of Fig. 15·1 (*b*).

The voltage gain of this amplifier is given by,

$$\mathbf{A} = -g_m \mathbf{Z} \qquad \ldots(15{\cdot}1)$$

where $\mathbf{Z}$ is the total impedance including r_p and is given by,

$$\frac{1}{\mathbf{Z}} = \frac{1}{r_p} + \frac{1}{\mathbf{Z}_t} \qquad \ldots(15{\cdot}2)$$

where $\mathbf{Z}_t$ is the impedance of anti-resonant circuit consisting of inductance L and total shunt capacitance $C = C' + C_s$.

The effective anti-resonant circuit is shown in Fig. 15·2. R is the small resistance of inductor L. The impedance $\mathbf{Z}_t$ of this circuit is then given by,

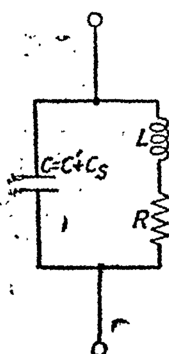


Fig. 15·2. Anti-resonant circuit

$$\mathbf{Z}_t = \frac{(R + j\omega L)\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{\dfrac{L}{CR} - \dfrac{j}{\omega C}}{1 + j\dfrac{\omega L}{R}\left(1 - \dfrac{1}{\omega^2 LC}\right)} \qquad \ldots(15{\cdot}3)$$

Resonant angular frequency ω_0 is given by,

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \ldots(15{\cdot}4)$$

Then the magnification factor or figure of merit Q_0 of the circuit at resonance is given by,

$$Q_0 = \frac{\omega_0 L}{R} \qquad \ldots(15{\cdot}5)$$

$$\text{Hence } \mathbf{Z}_t = \frac{\dfrac{L}{CR} - \dfrac{j}{\omega C}}{1 + jQ_0\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)} \qquad \ldots(15{\cdot}6)$$

$$= \frac{\dfrac{L}{CR}\left(1 - j\dfrac{1}{Q_0} \cdot \dfrac{\omega_0}{\omega}\right)}{1 + jQ_0\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)} \qquad \ldots(15{\cdot}7)$$

Let δ indicate the drift in frequency expressed as a fraction of the resonant frequency, then

$$\delta = \frac{\omega - \omega_0}{\omega_0} = \frac{\omega}{\omega_0} - 1 \qquad \ldots(15{\cdot}8)$$

or $$\frac{\omega}{\omega_0}=1+\delta \qquad ...(15\cdot9)$$

Hence $$\mathbf{Z}_t=\frac{\frac{L}{CR}\left[1-j\frac{1}{Q_0}\times\frac{1}{1+\delta}\right]}{1+j2Q_0\delta\left[\frac{1+\delta/2}{1+\delta}\right]} \qquad ...(15\cdot10)$$

At resonance $\omega=\omega_0$ and $\delta=0$

Hence at resonance,

$$\mathbf{Z}_t=R_t=\frac{L}{CR}\left(1-j\frac{1}{Q_0}\right) \qquad ...(15\cdot11)$$

If $Q_o>10$ as is usually the case, then

$$R_t\approx\frac{L}{CR}=Q_0^2R=Q_0\sqrt{\frac{L}{C}}$$

$$=\omega_0LQ_0 \qquad ...(15\cdot12)$$

Q_0 may be put in the alternative forms,

$$Q_0=\frac{R_t}{\omega_0L}=R_t\,\omega_0C \qquad ...(15\cdot13)$$

Eqn. (15·12) shows that the shunt impedance R_t of the anti-resonant circuit at resonance with $Q_0>10$ is essentially resistive.

Voltage gain at resonance is given by,

$$\mathbf{A}_{res}=-g_m\mathbf{Z}=\frac{-g_m}{\frac{1}{r_p}+\frac{1}{\mathbf{Z}_t}}=\frac{-g_m}{\frac{1}{r_p}+\omega_0LQ_0} \qquad ...(15\cdot14)$$

or $$\mathbf{A}_{res}=\frac{-g_m\omega_0LQ_0}{1+\frac{\omega_0LQ_0}{r_p}} \qquad ...(15\cdot14a)$$

or $$\mathbf{A}_{res}=-g_m\,\omega_0LQ_e \qquad ...(15\cdot15)$$

where $$Q_e=\frac{Q_0}{1+\frac{\omega_0LQ_0}{r_p}} \qquad ...(15\cdot16)$$

Altrenatively $$Q_e=\frac{R_t/X_e}{1+\frac{R_t}{r_p}}=\frac{R_{tp}}{X_e} \qquad ...(15\cdot16a)$$

where R_{tp} is resistance of parallel combination of R_t and r_p.

Thus the effective Q_e of the tuned amplifier circuit is modified by the shunting resistance r_p.

At any frequency ω near the resonant frequency ω_0, the impedance $\mathbf{Z}_t$ of the tuned circuit is given by,

$$\mathbf{Z}_t=\frac{R_t\left[1-j\frac{1}{Q_0(1+\delta)}\right]}{1+j2Q_0\delta\left(1+\frac{\delta/2}{1+\delta}\right)} \qquad ...(15\cdot17)$$

But $\delta \ll 1$, so that

$$\mathbf{Z}_t \approx R_t \frac{1 - j\dfrac{1}{Q_0}}{1+j2Q_0\delta} \qquad \text{...(15·18)}$$

If $Q_o \gg 1$, $\mathbf{Z}_t \approx \dfrac{R_t}{1+j2Q_0\delta}$ ...(15·18a)

The corresponding expression for voltage gain is, then,

$$\mathbf{A} = \frac{-g_m}{\dfrac{1}{r_p} + \dfrac{1+2jQ_0\delta}{\omega_0 LQ_0}} = \frac{-g_m\,\omega_0 LQ_0}{(1+2jQ_0\delta) + \dfrac{\omega_0 LQ_0}{r_p}}$$

$$= \frac{-g_m\,\omega_0 LQ_0 \Big/ \left(1 + \dfrac{\omega_0 LQ_0}{r_p}\right)}{1+j2Q_0\delta \Big/ \left(1 + \dfrac{\omega_0 LQ_0}{r_p}\right)}$$

$$= \frac{-g_m\,\omega_0 LQ_e}{1+j2\delta Q_e} \qquad \text{...(15·19)}$$

Alternatively $\mathbf{A} = \dfrac{-g_m\,R_{tp}}{1+j2\delta Q_e}$ ...(15·19a)

Hence the voltage gain ratio $\dfrac{\mathbf{A}}{\mathbf{A}_{res}}$ is given by,

$$\frac{\mathbf{A}}{\mathbf{A}_{res}} = \frac{1}{1+j2\delta Q_e} \qquad \text{...(15·20)}$$

Magnitude $\dfrac{A}{A_{res}} = \dfrac{1}{\sqrt{1+(2\delta Q_e)^2}}$ ...(15·20a)

and phase angle of $\dfrac{\mathbf{A}}{\mathbf{A}_{res}}$ is given by,

$$\varphi = -\tan^{-1}(2\delta Q_e) \qquad \text{...(15·20b)}$$

If r_p is large as compared with Z_t as is usually the case with pentode, then

$$\mathbf{A}_{res} = -g_m\,R_t = -g_m \omega_0 LQ_0 \qquad \text{...(15·21)}$$

and $\mathbf{A} = \dfrac{-g_m\,\omega_0 LQ_0}{1+j2Q_0\delta}$ ...(15·21a)

Hence $\dfrac{\mathbf{A}}{\mathbf{A}_{res}} = \dfrac{1}{1+j2Q_0\delta}$ ...(15·22)

Magnitude $\dfrac{A}{A_{res}} = \dfrac{1}{\sqrt{1+(2Q_0\delta)^2}}$ ...(15·22a)

and phase angle of $\frac{\mathbf{A}}{\mathbf{A}_{res}}$ is given by

$$\phi = -\tan^{-1}(2Q_0\delta) \qquad ...(15\cdot22b)$$

Equation (15·22) for pentode is similar to Eqn. (15·20) for a triode. Fig. 15·2 shows a plot of Eqn (15·20 a) or (15·22 a). Relative response $\frac{A}{A_{res}}$ is plotted against frequency deviation δ for different values of magnification factor Q.

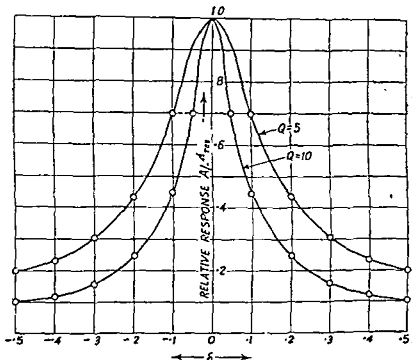


Fig. 15·2. Relative response of single tuned amplifier.

It may be seen from the response curves of Fig. 15·2 that the relative response falls more sharply as the magnification factor Q increases.

Fig. 15·3 shows universal curves for magnitude and phase angle of the relative response $\mathbf{A}/\mathbf{A}_{res}$ of a single tuned amplifier. The magnitude and phase angle of $\frac{\mathbf{A}}{\mathbf{A}_{res}}$ are plotted against δQ. The curves serve universally for all parallel resonant circuits within the range of magnification factor Q indicated. At resonance magnitude of relative response is maximum and it falls off sharply on either side. The phase angle is zero at resonance and increases on either side. The phase angle is inductive on frequencies lower than the resonant frequency and capacitive on frequencies higher than the

resonant frequency. As the frequency deviates more and more from the resonant frequency, the phase angle approaches plus and minus 90 degrees.

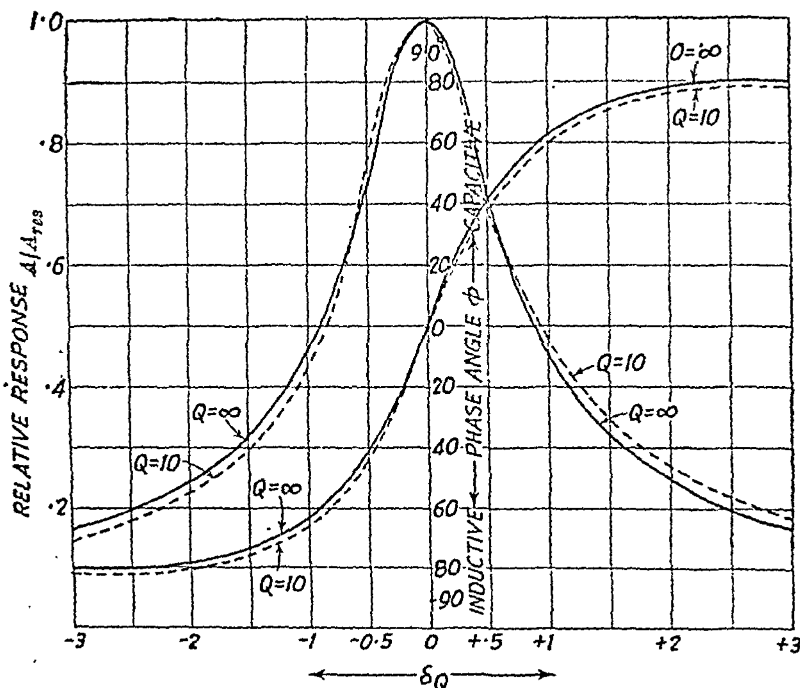


Fig. 15·3. Universal curves of the magnitude and phase angle of relative response $\frac{\mathbf{A}}{\mathbf{A}_{res}}$ of a single tuned amplifier.

When frequency is so adjusted that $\delta = \pm \frac{1}{2Q_e}$, the denominator of Eqn. (15·20) becomes $1 \pm j\,1$, then the relative response $\frac{\mathbf{A}}{\mathbf{A}_{res}}$ assumes a magnitude $\frac{1}{\sqrt{2}} = 0{\cdot}707$. If a constant current generator feeds the resonant circuit, the power dissipated in the tuned circuit at such frequencies is one-half of its value at the resonant frequency. These frequencies are, therefore, referred to as "half power frequencies" and are given by

$$\delta = \pm \frac{1}{2Q_e} \qquad \ldots(15{\cdot}23)$$

The bandwidth of a circuit is the frequency internal between the two half power frequencies. Let ω_1 and ω_2 be the two half power angular frequencies. The bandwidth $\Delta\omega$ is given by,

$$\Delta\omega = \omega_2 - \omega_1 = \frac{[(\omega_2 - \omega_0) + (\omega_0 - \omega_1)] \times \omega_0}{\omega_0}$$

$$= (\delta + \delta)\omega_0 = 2\delta\omega_0 \qquad \ldots(15{\cdot}24)$$

But from Eqn 15·23 for half power frequencies

$$2\delta=\frac{1}{Q_s} \text{ so that } \Delta\omega=\frac{\omega_0}{Q_s} \qquad ...(15\cdot25)$$

Also

$$\Delta\omega=\frac{\omega_0}{R_{tp}\,\omega_0 C}=\frac{1}{R_{tp}\,C} \text{ radians/sec.} \qquad ...(15\cdot26)$$

The response of tuned amplifier may be considered to be almost constant *with the 3-db* frequencies. *In accordance with* Eqn. *(15·25)*, this 3-*db* bandwidth is quite small for large Q circuits. Typical value of Q_s is ten Further from Eqn. (15·26,) we conclude that 3-*db* bandwidth depends only on resonant impedance R_{tp} and the tuning capacitance C *and is independent of resonant frequency.*

TUNED CASCADE AMPLIFIERS

Usually it is required to use a number of tuned voltage amplifiers *is cascade in order to obtain large overall voltage gain.* These cascade tuned amplifiers may be put into the following three categories :

(*A*) Single tuned amplifiers.

(*B*) Double tuned amplifiers.

(*C*) Stagger tuned amplifiers

Single tuned amplifiers may be of two types —(*i*) capacitance coupled and (*ii*) transformer coupled

(*A*) Single Tuned Capacitance Coupled Amplifier

Fig. 15·4 shows the circuit diagram. It is similar to R.C. coupled amplifier except that it uses a tuned circuit instead of a resistance as the plate load. There exists a tendency of oscillation due to feedback through the plate to control grid capacitance C_{gp} and this tendency increases with the increase of frequency. The tendency

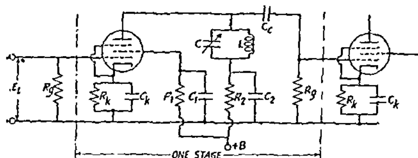


Fig 15·4. Circuit diagram of single tuned capacitance coupled amplifier.

of oscillation is strong because of the tuned plate-load impedan
This tendency is suppressed by the following two means : (*i*) u
of screen grid tubes *i.e.* tetrodes or pentodes having extremely l
C_{gp} *and (ii) by shielding the plate and grid circuits.*

Usually the same d.c. voltage source is used for providing the plate and screen grid voltages to all the tubes. The screen grid and plate voltages E_{c2} and E_b may be adjusted by proper selection of the dropping resistances R_1 and R_2.

With proper decoupling circuits, impedance between cathode, suppressor grid, screen-grid and lower terminal of the tuned circuit and the common earth connection is almost zero. Hence the capacitance between the plate and these other electrodes in one stage caused due to interelectrode capacitances or wiring capacitances, is effectively in shunt with the tuning condenser. Accordingly Fig. 15·5 shows the a. c. equivalent circuit of the single tuned capacitance coupled amplifier of Fig. 15·4. C_g represents the input shunt capacitance of the second stage. R_g is the grid leak resistance of the second stage and may be considered to include the input shunt resistance of the second stage as well. Coupling condenser C_c is usually large enough as to have reactance at the operating frequency negligibly small compared with the impedance of R_g-C_g combination. The capacitances C_s and C_g may be grouped with the tuning capacitance C' to constitute the total effective tuning capacitance C.

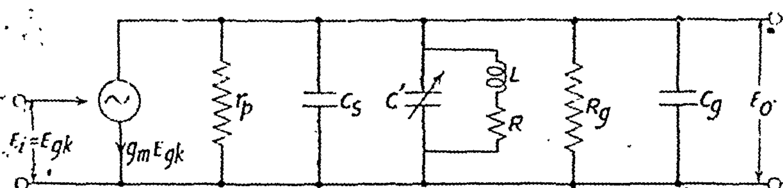


Fig. 15·5. A.C. equivalent circuit of single tuned capacitance coupled amplifier of Fig. 15·4.

From the a.c. equivalent circuit of Fig. 15·5, the voltage gain of the amplifier is given by,

$$\mathbf{A} = -g_m \mathbf{Z} \qquad \ldots(15·27)$$

where $\mathbf{Z}$ is the total impedance of r_p, R_g and $\mathbf{Z}_t$ in parallel and is given by the relation,

$$\frac{1}{\mathbf{Z}} = \frac{1}{r_p} + \frac{1}{R_g} + \frac{1}{\mathbf{Z}_t} \qquad \ldots(15·28)$$

where $\mathbf{Z}_t$ is the impedance of the anti-resonant circuit consisting of R, L and C.

$$C = C_s + C' + C_g \qquad \ldots(15·29)$$

Following the analysis of single stage tuned amplifier, we may prove that $\mathbf{Z}_t$ is given by,

$$\mathbf{Z}_t \approx \frac{R_t}{1+j2Q_0\delta} \qquad \ldots(15·30)$$

where impedance at resonance $R_t = \dfrac{L}{CR} = \omega_0 L Q_0$.

Voltage gain at resonance is given by,

$$\mathbf{A}_{res} = \frac{-g_m}{\frac{1}{r_p} + \frac{1}{R_g} + \frac{1}{R_t}} = -g_m R_{ptg} \qquad \text{...(15·31)}$$

where R_{ptg} is the impedance of parallel combination of r_p, R_g and R_t.

$$\text{Alternatively } \mathbf{A}_{res} = \frac{-g_m R_t}{1 + \frac{R_t}{r_p} + \frac{R_t}{R_g}} = \frac{-g_m \omega_0 LQ_0}{1 + \frac{\omega_0 LQ_0}{r_p} + \frac{\omega_0 LQ_0}{R_g}}$$

$$= -g_m \omega_0 LQ_e \qquad \text{...(15·32)}$$

where $$Q_e = \frac{Q_0}{1 + \frac{\omega_0 LQ_0}{r_p} + \frac{\omega_0 LQ_0}{R_g}} \qquad \text{...(15·33)}$$

$$\text{Alternatively } Q_e = \frac{R_{ptg}}{X_e} = R_{ptg}\,\omega_0 C \qquad \text{...(15·33a)}$$

Voltage gain at any frequency ω is given by

$$\mathbf{A} = \frac{-g_m}{\frac{1}{r_p} + \frac{1}{R_g} + \frac{1 + j2Q_0\delta}{\omega_0 LQ_0}} \qquad \text{...(15·34)}$$

or $$\mathbf{A} = \frac{-g_m \omega_0 LQ_0}{1 + j2Q_0\delta + \frac{\omega_0 LQ_0}{r_p} + \frac{\omega_0 LQ_0}{R_g}}$$

$$= \frac{-g_m \omega_0 LQ_e}{1 + j2\delta Q_e} \qquad \text{...(15·34a)}$$

where Q_e is given by Eqn. (15·33)

Hence $$\frac{\mathbf{A}}{\mathbf{A}_{res}} = \frac{1}{1 + j2\delta Q_e} \qquad \text{...(15·35)}$$

$$\text{Magnitude } \frac{A}{A_{res}} = \frac{1}{\sqrt{1 + (2\delta Q_e)^2}} \qquad \text{...(15·35a)}$$

and phase angle of $\frac{\mathbf{A}}{\mathbf{A}_{res}}$ is given by,

$$\phi = -\tan^{-1} 2\delta Q_e \qquad \text{...(15·36)}$$

Half power bandwidth is given by,

$$\Delta\omega = \frac{\omega_0}{Q_e} = \frac{1}{R_{ptg} C} \qquad \text{...(}$$

Gain bandwidth product of the amplifier is given by,

$$A_{res} \times \Delta\omega = g_m \omega_0 LQ_e \times \frac{\omega_0}{Q_e}$$

$$= \frac{g_m L}{LC} = \frac{g_m}{C} \text{ radians/sec.} \qquad \ldots(15\cdot38)$$

$$\text{or } A_{res} \times \Delta f = \frac{g_m}{2\pi C} \text{ cycles/sec.} \qquad \ldots(15\cdot38a)$$

Thus we see that the gain bandwidth product of the amplifier is independent of frequency and depends on g_m of the tube and total shunt capacitance. Thus this gain-bandwidth product may be maximised by reducing the shunt capacitance C. In the limiting case no tuning capacitance may be used and the inductance L may resonate with the existing interelectrode capacitances and stray capacitance. This gain bandwidth product is then a figure of merit of the tube. A tube with high g_m and low interelectrode capacitances should be used. Universal curves of Fig. 15·3 apply to this amplifier as well provided Q_e as calculated from Eqn. (15·33) or (15·33a) is used.

Single Tuned Transformer Coupled Amplifier. Fig. 15·6 shows the basic circuit diagram of single tuned transformer coupled amplifier. An air cored transformer is generally used for coupling the output of one stage to the input of the next. The secondary circuit is usually tuned and the primary is untuned as shown in Fig. 15·6.

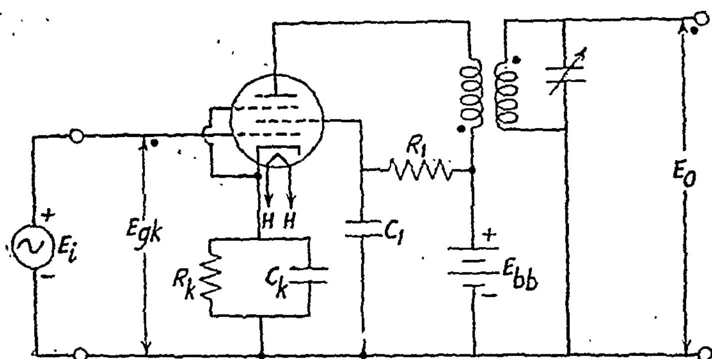


Fig. 15·6. Circuit of single-tuned transformer coupled amplifier.

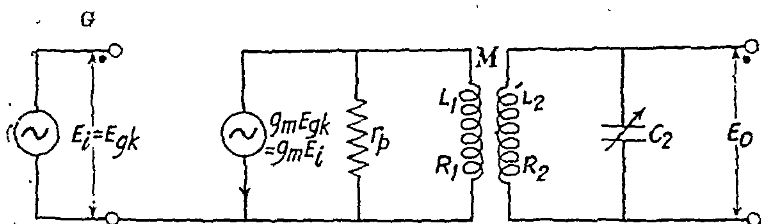


Fig. 15·7. A.C. equivalent circuit of single-tuned transformer coupled amplifier of Fig. 15·6.

A.c. equivalent circuit of the amplifier is shown in Fig. 15·7. It neglects all stray and inter-electrode capacitances. The assump-

tion is permissible provided that the primary inductance is low and the coupling between the primary and secondary is tight.

L_1 and R_1 are respectively primary inductance and resistance

L_2 and R_2 are respectively secondary inductance and resistance.

M is the mutual inductance.

C_2 is the total shunt capacitance on the secondary including the input shunt capacitance of the next stage.

The plate resistance r_p is usually large compared with R_1 and reactance ωL_1 so that the voltage induced in the secondary winding of the transformer is given by the relation,

$$\mathbf{E}_{indd} \approx j\omega M (g_m \mathbf{E}_i) \quad ...(15\cdot39)$$

The output voltage $\mathbf{E}_o$ is then that part of the induced voltage $\mathbf{E}_{indd}$ which is developed across the condenser C_2. Hence $\mathbf{E}_o$ is given by,

$$\mathbf{E}_o = \frac{j\omega M (g_m \mathbf{E}_i) \times \frac{1}{j\omega C_2}}{R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \frac{\omega^2 M^2}{r_p}} \quad ..(15\cdot40)$$

In Eqn. (15·40), $\frac{\omega^2 M^2}{r_p}$ is the impedance of the primary reflected into the secondary circuit. The voltage gain of the amplifier is then given by,

$$\mathbf{A} = \frac{\mathbf{E}_o}{\mathbf{E}_i} = \frac{\mu M / C_2}{r_p\left[R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right)\right] + \omega^2 M^2} \quad ...(15\cdot41)$$

At resonance $\quad \omega_0 = \frac{1}{\sqrt{L_2 C_2}} \quad .(15\cdot42)$

The voltage gain is maximum at resonance and is given by,

$$\mathbf{A}_{res} = \mu \frac{M / C_2}{r_p R_2 + \omega_0^2 M^2} \quad .(15\cdot43)$$

But magnification factor of tuned circuit at resonance is given by,

$$Q_0 = \frac{\omega_0 L_2}{R_2} = \frac{1}{\omega_0 C_2 R_2} \quad .(15\cdot44)$$

Hence Eqn. (15·43) may be put as,

$$\mathbf{A}_{res} = g_m \frac{\omega_0 M Q_0}{1 + \frac{\omega_0^2 M^2}{r_p R_2}} \quad ...(15\cdot45)$$

Eqn. (15·45) may be put in the alternative form,

$$\mathbf{A}_{res} = g_m \omega_0 M Q_e \quad (15\cdot46)$$

where Q_e is the effective Q of the tuned circuit and is given by,

$$Q_e = \frac{Q_0}{1 + \frac{\omega_0^2 M^2}{r_p R_2}} \quad \ldots(15{\cdot}47)$$

Eqn. (15·47) for transformer coupled amplifier is similar to Eqn. (15·33) for capacitance coupled tuned amplifier except that it has M instead of L. In the case of transformer coupled tuned amplifier however, the voltage gain of the amplifier may be increased within reasonable limits by increasing the value of mutual inductance M without reducing the Q of the tuned circuit considerably. But the voltage gain cannot be increased excessively by this means since M also appears in the denominator of Eqn. (15·43) as well. It may thus be seen that there occurs an optimum value of M for which voltage gain A_{res} is maximum. This optimum value of M may be found by equating $\frac{\partial \mathbf{A}_{res}}{\partial M}$ to zero.

Thus from Eqn. (15·43),

$$\frac{\partial \mathbf{A}_{res}}{\partial M} = \frac{\frac{\mu}{C_2}\left[r_p R_2 + \omega_0^2 M^2\right] - \frac{\mu M}{C_2}\left(2\omega_0^2 M\right)}{(r_p R_2 + \omega_0^2 M^2)^2} = 0 \quad \ldots(15{\cdot}48)$$

Hence optimum value of M is given by,

$$M_{opt} = \frac{\sqrt{r_p R_2}}{\omega_0} = \sqrt{r_p R_2 L_2 C_2} \quad \ldots(15{\cdot}49)$$

Substituting the value of M_{opt} from Eqn. (15·49) into Eqn. (15·43), we get,

$$\mathbf{A}_{res})_{opt} = \mu \frac{\frac{\sqrt{r_p R_2}}{\omega_0 C_2}}{r_p R_2 + r_p R_2} = g_m \frac{\sqrt{r_p R_2}\, Q_0}{2} \quad \ldots(15{\cdot}50)$$

$$= \mu \frac{Q_0}{2} \sqrt{\frac{R_2}{r_p}} \quad \ldots(15{\cdot}51)$$

Eqn. (15·50) shows that for optimum voltage gain at resonance to be large, g_m and r_p of the tube must be large. Also the parallel impedance of the tuned circuit must be large.

Expression for bandwidth of the amplifier may be obtained from the general expression (15·41) for the voltage gain at any frequency.

For any frequency ω close to resonant frequency ω_0,

let us put $$\delta = \frac{\omega}{\omega_0} - 1 \quad \ldots(15{\cdot}52)$$

Also $$Q_o = \frac{\omega_0 L_2}{R_2} = \frac{1}{\omega_0 C_2 R_2} = \frac{1}{R_2}\sqrt{\frac{L_2}{C_2}} \quad \ldots(15{\cdot}53)$$

Hence $\omega L_2 - \frac{1}{\omega C_2} = \sqrt{\frac{L_2}{C_2}}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$

$$\approx 2\delta\sqrt{\frac{L_2}{C_2}} \approx 2\delta R_2 Q_0 \qquad \ldots(15.54)$$

Eqn. (15·41) may then be put as,

$$\mathbf{A} = \frac{\frac{\mu M}{C_2}}{r_p R_2\left[1 + j\frac{1}{R_2}\,2\delta R_2 Q_0\right] + \omega^2 M^2}$$

$$= \frac{g_m\,\omega_0 M Q_0}{1 + \frac{\omega_0^2 M^2}{r_p R_2} + j2\delta Q_o} = \frac{g_m\,\omega_0 M \cdot \frac{Q_o}{1 + \frac{\omega_0^2 M^2}{r_p R_2}}}{1 + \frac{j\,2\delta Q_o}{1 + \frac{Q_0^2 M^2}{r_p R_2}}}$$

$$\approx \frac{g_m\,\omega_o\,M Q_e}{1 + j2\delta Q_e} \qquad \ldots(15.55)$$

Where Q_e is given by Eqn (15 47).

$$\text{Hence } \frac{\mathbf{A}}{\mathbf{A}_{res}} = \frac{1}{1 + j2\delta Q_e} \qquad \ldots(15.56)$$

It may be noted that the Eqn. (15·56) for transformer coupled [illegible] tuned [illegible] band-

$$\Delta\omega = \frac{\omega_0}{Q_e} \qquad \ldots(15.56a)$$

In present-day amplifiers using tetrodes or pentodes, the value of r_p is very high so that it becomes difficult in practice to get optimum value of M as given by Eqn. (15 49) To obtain these values of M, the distributed capacitances of the windings may become extremely large and the self resonant frequency may be low enough to make the coil useless. Hence with pentodes, the value of mutual inductance M is chosen far below the optimum value as given by Eqn. (15·49).

In pentodes and tetrodes, plate resistance r_p is generally large compared with $\frac{\omega_0^2 M^2}{R_2}$ so that Eqn. (15 45) for voltage gain at resonance reduces to the approximate form,

$$\mathbf{A}_{res} \approx g_m\,\omega_0 M Q_0 \qquad \ldots(15.57)$$

(B) Double Tuned Voltage Amplifiers.

Double tuned amplifiers are used extensively as I. F. (Intermediate Frequency) amplifiers in radio receivers of both amplitude modulation and frequency modulation types. The advantage obtained by the use of double tuned circuit is that an almost constant voltage gain may be obtained over a band of frequencies and the voltage gain falls rapidly outside this constant gain band. Such a result cannot be achieved in a single tuned amplifier.

Fig. 15·8 shows the basic circuit diagram of a double tuned voltage amplifier.

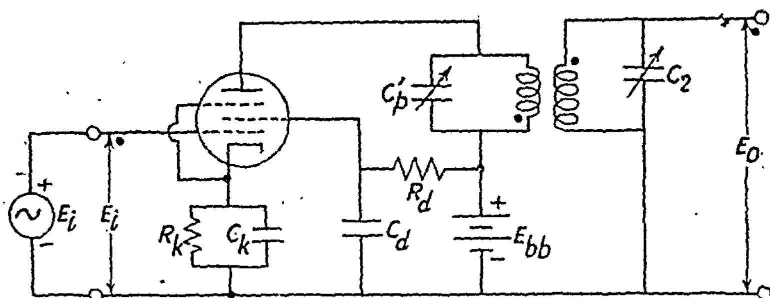


Fig. 15·8. Circuit diagram of double tuned voltage amplifier.

Fig. 15·9 shows the a. c. equivalent circuit of the amplifier. C_p is the primary stray and wiring capacitance. This comes in parallel with the primary tuning capacitance C'_p to constitute total primary shunt capacitance C_1.

Thus $$C_1 = C_p + C'_p.$$

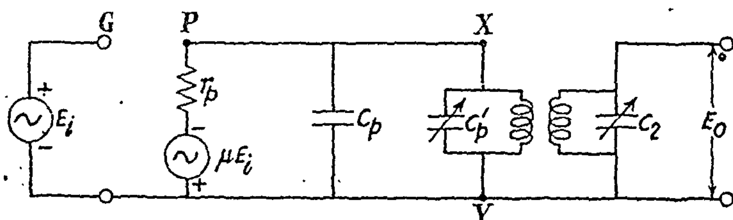


Fig. 15·9. A. C. equivalent circuit of double tuned amplifier.

To simplify the equivalent circuit of Fig. 15·9, Thevenin's theorem is applied to the portion of the circuit to the left of the points $X-Y$. The equivalent generator has the open-circuit voltage of,

$$\mathbf{E} = \frac{\mu \mathbf{E}i \left(\frac{1}{j\omega C_1} \right)}{r_p + j\omega C_1} \qquad \ldots(15{\cdot}58)$$

But for pentodes, $r_p > \frac{1}{\omega C_1}$ so that

$$\mathbf{E} \approx \frac{\mu \mathbf{E}i}{j\omega C_1 r_p} \approx \frac{g_m \mathbf{E}i}{j\omega C_1} \qquad \ldots(15{\cdot}59)$$

The internal source impedance of the equivalent voltage generator is given by,

$$\mathbf{Z}_i = \frac{r_p\left(\frac{1}{j\omega C_1}\right)}{r_p + \frac{1}{j\omega C_1}} \approx \frac{1}{j\omega C_1} \qquad ...(15\cdot60)$$

Hence on application of Thevenin's theorem, the equivalent circuit of Fig. 15·9 reduces to the form shown in Fig. 15·10.

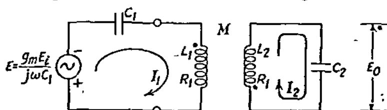


Fig. 15·10. Modified a.c. equivalent circuit of double tuned amplifier.

The equivalent circuit of Fig. 15·10 may be analysed by the usual method of network analysis. Thus we get,

$$-\mathbf{E} = r_{11}\,\mathbf{I}_1 + r_{12}\,\mathbf{I}_2 \qquad ...(15\cdot61)$$

$$O = r_{12}\,\mathbf{I}_1 + r_{22}\,\mathbf{I}_2 \qquad ...(15\cdot62)$$

Substituting the value of $\mathbf{I}_1$ from Eqn. (15·62) into Eqn. (15·61), we get,

$$-\mathbf{E} = r_{11}\left(\frac{-r_{22}\,\mathbf{I}_2}{r_{12}}\right) + r_{12}\cdot\mathbf{I}_2$$

or $$\mathbf{I}_2 = \frac{+\mathbf{E}\,r_{12}}{r_{11}\,r_{22} - r^2_{12}} \qquad ...(15\cdot63)$$

From the equivalent circuit of Fig. 15·10,

$$r_{11} = R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \qquad ...(15\cdot64)$$

$$r_{12} = j\omega M \qquad ...(15\cdot65)$$

$$r_{22} = R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right) \qquad ...(15\cdot66)$$

Hence the voltage gain of the amplifier is given by,

$$\mathbf{A} = \frac{\mathbf{E}_o}{\mathbf{E}_i} = \frac{\mathbf{I}_2\frac{1}{j\omega C_2}}{\mathbf{E}_i} = \frac{\frac{\mathbf{E}.r_{12}}{r_{11}\,r_{22} - r^2_{12}}\cdot\frac{1}{j\omega C_2}}{\mathbf{E}_i}$$

$$= \frac{1}{\mathbf{E}_i}\left[\frac{g_m\,\mathbf{E}_i}{j\omega C_1}\cdot\frac{r_{12}}{r_{11}\,r_{22} - r^2_{12}}\cdot\frac{1}{j\omega C_2}\right]$$

$$= \frac{\left(\frac{g_m}{j\omega C_1}\right)\left(\frac{1}{j\omega C_2}\right)j\omega M}{r_{11}\,r_{22} - r^2_{12}} \qquad ...(15\cdot67)$$

Since both the primary and secondary tuned circuits are tuned to the same frequency ω_0,

$$\omega_0=\frac{1}{\sqrt{L_1C_1}}=\frac{1}{\sqrt{L_2C_2}} \qquad ...(15\cdot68)$$

Also let $$Q_1=\frac{\omega_0L_1}{R_1}=\frac{1}{\omega_0C_1R_1} \qquad ...(15\cdot69)$$

and $$Q_2=\frac{\omega_0L_2}{R_2}=\frac{1}{\omega_0C_2R_2} \qquad ...(15\cdot70)$$

Coefficient of coupling is defined as,

$$K=\frac{X_{mr}}{\sqrt{X_{1r}.X_{2r}}}=\frac{\omega_0M}{\sqrt{\omega_0L_1.\omega_0L_2}}=\frac{M}{\sqrt{L_1L_2}} \qquad ...(15\cdot71)$$

where X_{mr} is the coupling reactance at resonance and X_{1r} and X_{2r} are resonant reactances of the primary and secondary tuned circuits.

A coupling dissipation factor p is defined as

$$p=k^2\,Q_1\,Q_2 \qquad ...(15\cdot72)$$

Hence p may also be put as,

$$p=\frac{(\omega_0M)^2}{R_1\,R_2} \qquad ...(15\cdot72a)$$

Then r_{11}, r_{12} and r_{22} may be put as below:—

$$r_{11}=R_1+j\left(\omega L_1-\frac{1}{\omega C_1}\right)=R_1(1+j2\delta Q_1) \qquad ...(15\cdot73)$$

$$r_{22}=R_2+j\left(\omega L_2-\frac{1}{\omega C_2}\right)=R_2(1+j2\delta Q_2) \qquad ...(15\cdot74)$$

and $$r_{12}=j\omega M \qquad ...(15\cdot75)$$

Substituting these values of r_{11}, r_{22} and r_{12} in Eqn. (15·67) we get,

$$\mathbf{A}=\frac{-j\,g_m\left(\frac{M}{\omega_0C_1C_2}\right)}{R_1R_2(1+j2\delta Q_1)(1+j2\delta Q_2)+\omega_0^2M^2}$$

$$=\frac{-jg_m\frac{\omega_0M}{\sqrt{R_1R_2}}\cdot\frac{1}{\omega_0C_1R_1}\cdot\frac{1}{\omega_0C_2R_2}\cdot\sqrt{R_1R_2}}{(1+j2\delta Q_1)\,(1+j2\delta Q_2)+\frac{\omega_0^2M^2}{R_1R_2}}$$

$$=\frac{-j\,g_m\sqrt{p}\,Q_1Q_2\sqrt{R_1R_2}}{1+p+2j\delta(Q_1+Q_2)-4\,\delta^2Q_1Q_2} \qquad ...(15\cdot76)$$

At resonance $\delta=0$ so that voltage gain at resonance is given by,

$$\mathbf{A}_{res}=\frac{-jg_m\sqrt{p}\,Q_1Q_2\sqrt{R_1R_2}}{1+p} \qquad ...(15\cdot77)$$

Hence the ratio of voltage gain A at any frequency ω close to ω to gain at resonance is given by

$$\frac{A}{A_{res}}=\frac{1+p}{1+p-4\delta^2 Q_1Q_2+j2\delta(Q_1+Q_2)}$$

$$=\frac{1}{\left(1-\frac{4\delta^2 Q_1Q_2}{1+p}\right)+j\frac{2\delta(Q_1+Q_2)}{1+p}} \quad ...(15{\cdot}78)$$

Eqn. (15·78) determines the shape of the response curve of the double-tuned amplifier for a given value of parameter p, or coefficient of coupling k.

The value of parameter p for maximum voltage gain at resonance may be determined by equating to zero the differential $\frac{d\,A_{res}}{d\,p}$. This gives the value of p equal to unity for maximum gain at resonance. This determines the value of critical coupling k_c. Fig. 15·10 shows the response curves of a double tuned amplifier with $Q_1=Q_2$

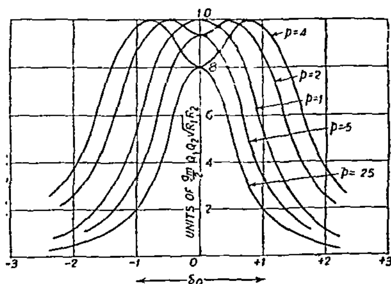


Fig. 15·10. Response curves of a double tuned amplifier with $Q_1=Q_2$.

It is seen from Fig 15·10 that for $p=1$, the response curve has maximum response at resonance frequency ω_0 and has maximum single-peak flatness at resonance. For values of p smaller than unity, the response curve is rounded at the top and the gain at resonance may be less than that with $p=1$. If p is greater than unity, double peaked response curve results as shown in Fig. 15·10.

Maximum and minimum voltage gains of overcoupled double tuned amplifier :—For overcoupled circuits i.e. with $p>1$, the positions of the peaks of the response curve may be found by differentiat-

ing the square of the absolute value of A and equating it to zero. The square of the absolute value of A is taken since the gain is maximum without regard to the phase. From Eqn. (15·76),

$$|A|^2=(g_m\ Q_1Q_2\sqrt{R_1R_2})^2\ \frac{p}{(1+p-4\delta^2\ Q_1Q_2)^2+[2\delta(Q_1+Q_2)]^2} \quad ...(15·79).$$

Equating $\frac{d\,|A|^2}{d\delta}$ to zero, we get

$$\frac{d}{d\delta}\left[(1+p-4\delta^2\ Q_1Q_2)^2+[2\delta(Q_1+Q_2)]^2\right]=0$$

or $\quad 2[1+p-4\delta^2\ Q_1Q_2][-4Q_1Q_2\times 2\delta]+2[2\delta(Q_1+Q_2)]$
$[2(Q_1+Q_2)]^2=0$

or $\quad 1+p-4\delta^2Q_1Q_2=\frac{(Q_1+Q_2)^2}{2Q_1Q_2}$ ...(15·80).

Hence $\quad \delta=\pm\frac{1}{2}\sqrt{\frac{1+p}{Q_1Q_2}-\frac{(Q_1+Q_2)^2}{2(Q_1Q_2)^2}}$

$$=\pm\frac{1}{2}\sqrt{\frac{1}{Q_1Q_2}+k^2-\frac{1}{2}\left(\frac{Q_1+Q_2}{Q_1Q_2}\right)^2} \quad ...(15·81)$$

Usually Q_1 is kept equal to Q_2. Otherwise also Q_1 is kept close to Q_2 so that $\sqrt{Q_1Q_2}\approx\frac{Q_1+Q_2}{2}$

Hence $\quad \delta=\pm\frac{1}{2}\sqrt{k^2-\frac{1}{Q_1Q_2}}\,.$ ...(15·82)

or $\quad \delta\sqrt{Q_1Q_2}=\pm\frac{1}{2}\sqrt{k^2Q_1Q_2-1}$
$=\pm\frac{1}{2}\sqrt{p-1}$ ...(15·83)

Substituting the value of δ from Eqn. (15·83) into Eqn. (15·76), we get the maximum value of voltage gain A as given by,

$$\mathbf{A}_{max}=-j\ (g_m\ Q_1Q_2\sqrt{R_1R_2})\ \frac{\sqrt{p}}{2(1+j\sqrt{p-1}} \quad ...(15·84)$$

Numerical value of $\mathbf{A}_{max}$ is given by,

$$\mathbf{A}_{max}=\tfrac{1}{2}(g_m\ Q_1Q_2\sqrt{R_1R_2}) \quad ...(15·85)$$

Thus we see that maximum voltage gain for overcoupled case is the same as the maximum gain for critical coupling k_c.

The minimum voltage gain of an overcoupled double tuned amplifier is obtained at resonant frequency and is given by Eqn. (15·77). Thus

$$\mathbf{A}_{min}=\frac{-j\sqrt{p}\ g_m\ Q_1Q_2\ \sqrt{R_1R_2}}{1+p} \quad ...(15·86)$$

Magnitude of this gain is given by,

$$A_{min}=g_m\ Q_1Q_2\ \sqrt{R_1R_2}\ \frac{\sqrt{p}}{1+p} \quad ...(15·86a)$$

Primary winding current in double-tuned amplifier :—The primary winding current may be obtained by solving Eqns. (15·61) and (15·62) for I_1. Fig. 15·11 shows the nature of variation of magnitude of I_1 with frequency deviation.

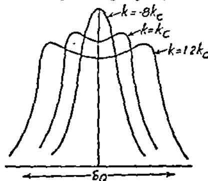


Fig. 15·11. Variation of primary winding current I_1 of a double-tuned amplifier with frequency deviation.

When the coefficient of coupling k is appreciably smaller than the critical coupling coefficient k_c, the primary winding current I_1 is almost unaffected by the secondary circuit and hence waveform of I_1 has a single peak as for a single-tuned circuit. For $k=k_c$, at resonant frequency the primary winding current I_1 decreases since the resistive impedance of the secondary at resonance is reflected to the primary side. At frequencies below resonance, reflected impedance is inductive and at some frequency [illegible] impedance just cancels the primary capacitive [illegible] ularly at frequencies above [illegible] and at some frequency, the reflected capacitive reactance just cancels the primary inductive reactance resulting in another current peak

When $k>k_c$, the double peaks get more separated and are more pronounced.

Response curves with unequal Q'.—With unequal Q's it may be proved that the response curve of a double-tuned amplifier is flattest when,

$$p=\frac{1}{2}\left(\frac{Q_1}{Q_2}+\frac{Q_2}{Q_1}\right) \qquad \text{...(15·87)}$$

With this condition satisfied, we say that the tuned circuits are "transitionally" *coupled and the* corresponding transitional *coupling coefficient k_t is given by*,

$$k_t=\sqrt{\frac{1}{2}\left(\frac{1}{Q_1^2}+\frac{1}{Q_2^2}\right)} \qquad \text{...(15·88)}$$

If coupling coefficient is kept greater than transitional coupling coefficient k_t, the response curve becomes double-humped whereas for values of k less than k_t, the response curve has a single peak.

Frequency interval between response peaks:—

With unequal Q's, with $L_1=L_2=L$ say, and with $k>k_t$, it may be shown that the frequency interval between peaks is given by,

$$\Delta\omega=\frac{1}{L}\sqrt{(\omega_0 M)^2-\frac{R_1^2+R_2^2}{2}} \qquad \text{...(15·89)}$$

Further if $R_1=R_2=R$ say, Eqn. (15·89) reduces to,

$$\Delta\omega=\frac{1}{L}\sqrt{(\omega_0 M)^2-R^2} \quad ...(15·90)$$

so that

$$\frac{\Delta\omega}{\omega_0}=\frac{1}{\omega_0 L}\sqrt{(\omega_0 M)^2-R^2}=\sqrt{\left(\frac{\omega_0 M}{\omega_0 L}\right)^2-\left(\frac{R}{\omega_0 L}\right)^2}$$

$$=\sqrt{k^2-k_c^2} \quad ...(15·91)$$

It may be proved that the frequency bandwidth over which the response is greater than that at resonant frequency is $\sqrt{2}\,\Delta\omega$ where $\Delta\omega$ is the frequency interval between resonance peaks as given by Eqn. (15·89) or Eqn. (15·91).

Although critical coupling provides optimum response characteristics, it is usually preferred to use under-coupled double-tuned circuits wherever narrow band is permissible. The response is then single peaked and permits easy frequency alignment of tuned circuits. The alignment procedure then consists in simply tuning each circuit for maximum response at resonance frequency. With double peaked response of overcoupled circuits, alignment is very critical. Similarly difficulty arises with circuits having critical coupling.

Half-power bandwith of double-tuned Amplifiers with critical coupling and $Q_1=Q_2$.

Let $Q_1=Q_2=Q$. Then the relative voltage gain is given by,

$$\frac{\mathbf{A}}{\mathbf{A}_{res}}=\frac{1}{(1-2\delta^2 Q^2)+j\,2\delta Q} \quad ...(15·92)$$

$$\text{Magnitude } \frac{A}{A_{res}}=\frac{1}{\sqrt{1+4\delta^4 Q^4}} \quad ...(15·93)$$

At half-power frequencies,

$$4\delta_0^4 Q^4=1$$

Hence 3-*db* or half-power bandwith is given by,

$$B=2\delta_0 f_0=2f_0\frac{1}{\sqrt{2}\,Q}=\frac{\sqrt{2}\,f_0}{Q} \quad ...(15·94)$$

Thus we see that 3-*db* bandwith of the double-tuned amplifier is $\sqrt{2}$ times the 3-*db* bandwidth of corresponding single tuned amplifier.

Gain Bandwidth Product of Tuned Amplifiers. We may now summarise the performance of tuned amplifiers in terms of the gain bandwidth product. The voltage gain at resonance is given by the following expressions which have already been discussed :

Single Tuned Capacitance Coupled Amplifier : $\mathbf{A}_{res}=-g_m\omega_0 LQ_e$

Single Tuned Transformer Coupled Amplifier : $\mathbf{A}_{res}=g_m\omega_0 MQ_e$

Double Tuned Amplifier : $\mathbf{A}_{res}=-j\,\frac{1}{2}g_m\,Q_1Q_2\sqrt{R_1R_2}$
$(p=1)$

The 3-*db* bandwidth of these amplifiers have also been found and are given below :

Single tuned capacitance coupled Amplifier : $B=\frac{f_0}{Q_e}$

Single tuned transformer coupled Amplifier : $B=\frac{f_0}{Q_e}$

Double tuned amplifier : $B=\frac{\sqrt{2}\,f_0}{\sqrt{Q_1Q_2}}$

The gain bandwidth product of these tuned amplifiers is calculated as below :

(*i*) Single tuned capacitance coupled Amplifier :

$$A_{res}\,B=g_m\omega_0L\,Q_e\,\frac{f_0}{Q_e}=\frac{g_m f_0}{\omega_0 C}=\frac{g_m}{2\pi C} \qquad ...(15\cdot95)$$

To increase the gain-bandwidth product, C must be reduced. In the limit, the capacitance C may be due only to interelectrode capacitances.

Then
$$\left(A_{res}\,B\right)_{max}=\frac{g_m}{2\pi\,(C_i+C_o)} \qquad ...(15\cdot96)$$

where C_0 is the output capacitance of amplifier tube and C_i is the input capacitance of the next stage. This Eqn. (15·96) is the same as for the R.C. coupled amplifier.

(*ii*) Single tuned transformer coupled Amplifier .

$$A_{res}.B=g_m\,\omega_0MQ_e\,\frac{f_0}{Q_e}=g_m\omega_0M\,f_0$$

$$=\frac{g_m f_0}{\omega_0C_2}\cdot\frac{M}{L_2}=\frac{g_m}{2\pi C_2}\cdot\frac{M}{L_2} \qquad ...(15\cdot97)$$

To increase the gain bandwidth product C_2 must be reduced. In the process of reducing C_2, a limit is reached when the tuning capacitance has been reduced to zero leaving only the input capacitance C_i of the next stage. Then

$$\left(A_{res}\,.\,B\right)_{max}=\frac{g_m}{2\pi C_i}\cdot\frac{M}{L_2} \qquad ...(15\cdot98)$$

(*iii*) Double Tuned Amplifier :

$$A_{res}\,.\,B=\frac{g_m}{2}\;Q_1Q_2\,\sqrt{R_1R_2}\,.\,\sqrt{2}\;\frac{f_0}{\sqrt{Q_1Q_2}}$$

$$=\frac{g_m}{\sqrt{2}}\sqrt{Q_1Q_2\,R_1R_2}\;.\;\frac{\omega_0}{2\pi}$$

$$=\frac{g_m}{2\pi\sqrt{2}}\;\frac{1}{\sqrt{C_1C_2}} \qquad ...(15\cdot99)$$

In order to increase the gain-bandwidth product C_1 and C_2 must be reduced. In the process of reducing C_1 and C_2, a limit is reached when tuning capacitances have become zero leaving only the output capacitance C_o of the amplifier tube and input capacitance C_i of the next amplifier stage. Then,

$$\left(A_{res}' B\right)_{max}=\frac{g_m}{2\pi}\cdot\frac{1}{\sqrt{2C_oC_i}} \quad ...(15\cdot100)$$

Comparing Eqn. (15·100) with Eqn. (15·96) we observe that if $C_i=C_o$, the gain-bandwidth product of the double tuned amplifier is $\sqrt{2}$ as great as that for single tuned capacitance coupled amplifier.

Voltage Gain and Bandwidth of Multi-stage Tuned Amplifier-

Usually a tuned amplifier consists of a number of stages. Obviously then the overall voltage gain is equal to the multiplication of the voltage gains of individual stages but simultaneously the overall bandwidth is reduced. We here derive expressions for overall voltage gain and bandwidth of multi-stage tuned amplifiers of single tuned, double tuned and stagger tuned types.

(*A*) *Single Tuned Multi-stage Amplifier* :—Let there be n identical tuned stages connected in cascade. From Eqn. (15·35), the relative voltage gain of overall cascade amplifier is given by,

$$\left(\frac{A}{A_{res}}\right)_{cascade}=\frac{(A)^n}{(A_{res})^n}=\frac{1}{[1+(2\delta Q_e)]^{\frac{n}{2}}} \quad ...(15\cdot101)$$

At half power frequencies,

$$[1+(2\delta_0 Q_e)^2]^{\frac{n}{2}}=\sqrt{2}$$

or $$1+(2\delta_0 Q_e)^2=2^{\frac{1}{n}}$$

or $$2\delta_0 Q_e=\sqrt{2^{\frac{1}{n}}-1} \quad ...(15\cdot102)$$

Hence 3-*db* bandwidth of n-stage cascade tuned amplifier is,

$$B_n=2\delta_0 f_0\frac{\sqrt{2^{\frac{1}{n}}-1}}{Q_e/f_0} \quad ...(15\cdot103)$$

But 3-*db* bandwidth of one stage is given by,

$$B=\frac{f_0}{Q_e} \quad ...(15\cdot104)$$

Hence $$B_n=B\sqrt{2^{\frac{1}{n}}-1} \quad ...(15\cdot105)$$

From Eqn. (15·105), it is seen that on cascading n stages, the 3-db bandwidth gets multiplied by the factor $\sqrt{2^{\frac{1}{n}}-1}$. Value of factor $\sqrt{2^{\frac{1}{n}}-1}$ is equal to 0·643 for $n=2$, and 0·510 for $n=3$ so that 3-db bandwidth is reduced to 64·3 and 51 per cent of the single-stage bandwidth in the case of 2 and 3 stage amplifiers respectively. For a given resonance frequency, in order to maintain a prescribed 3-db bandwidth, it is necessary to reduce the Q of the tuned circuits continuously as the number of stages in cascade is increased.

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in this case, the 3-db bandwidth reduces rapidly with the increase in the number of stages used in cascade. Thus the 3-db bandwidth of a 4-stage single tuned amplifier is only 0·44 of the 3-db bandwidth of each stage. To get an overall bandwidth of say 4 Mc/s at 40 Mc/s (typical of I.F. amplifier in Television), the bandwidth of each stage must be $\frac{4}{\cdot 44}=9\cdot 1$ Mc/s. Such a wide bandwidth requirement has two implications. (i) the gain per stage is very small and (ii) Q is small. In this case $Q=\frac{40}{9\cdot 1}=4\cdot 4$. Small Q results in a very flat *response curve i.e.,* it extends into adjoining channels with sufficient strength. This is undesirable. Hence for high gain wideband amplification, single tuned amplifiers are, in general, not suitable.

(*B*) *Double-tuned Multi-stage Cascade Amplifier* :—Let there be n identical stages in cascade and let Q_e be the effective Q of the tuned circuits in each stage. Let the factor p be equal to unity. Then the relative voltage gain of this n-stage cascade double-tuned amplifier may be obtained from Eqn. (15·93) and is given by,

$$\left(\frac{A}{A_{res}}\right)_{cascade}=\frac{(A)^n}{(A_{res})^n}=\frac{1}{\left(1+4\delta^4Q^4\right)^{\frac{n}{2}}} \quad ...(15\cdot 106)$$

Hence

$$\delta_o Q=\sqrt[4]{\frac{\sqrt{2^{\frac{1}{n}}-1}}{4}} \quad ...(15\cdot 107)$$

The 3-db bandwidth of n-stage double-tuned amplifier is than given by,

$$B_n=2\,\delta_o f_o=\sqrt{2}\sqrt[4]{\left(2^{\frac{1}{n}}-1\right)}\times\frac{f_o}{Q_o} \quad ...(15\cdot 108)$$

But 3-db bandwidth of one double-tuned amplifier is given by,

$$B=\sqrt{2}\,\frac{f_o}{Q_o}$$

Hence. $B_n = B\sqrt[4]{\left(2^{\frac{1}{n}}-1\right)}$ ...(15·109)

From Eqn. (15·109) we conclude that by cascading the 3-*db* bandwidth is reduced by the factor $\sqrt[4]{2^{\frac{1}{n}}-1}$. Thus for a 2-stage double tuned amplifier, the 3-*db* bandwidth reduces to 80·2% of the value for one stage. This reduction in bandwidth is much smaller than the corresponding reduction in single tuned amplifiers. This results because the response curve of a double-tuned amplifier has steeper sides compared with that of a single tuned amplifier. From this consideration, for high gain wideband amplification double-tuned cascade amplifiers are preferred to single tuned cascade amplifiers. For a specified gain and bandwidth fewer tubes are required in this case. The passband characteristic is also superior since it falls more rapidly outside the passband.

Double-tuned circuits are, however, more difficult to align. For good results, alignment has to be done with reference to oscilloscopic pattern. Further double-tuned cascade amplifiers are more sensitive to variations in tube capacitances and coil inductances than is the case with single tuned circuits. Hence inspite of their superior response curve, from the above practical consideration, double-tuned cascade amplifiers are not used commonly for high gain wideband amplification. They are however very commonly used as I.F. (Intermediate Frequency) amplifiers in Medium-Wave and Short Wave radio receivers having only 2 or 3 stages and small bandwidth requirements. Here, since the number of stages is small, alignment is not very difficult and since the bandwidth required is small, high gain may be achieved along with sharp cutoff beyond the passband.

(C) Stagger Tuned Amplifiers

It has been observed that for high gain wideband amplification, single-tuned and double-tuned cascade amplifiers are not entirely satisfactory. The most satisfactory amplifier for such high gain wideband operation is the so-called "stagger tuned" amplifier. It consists of a number of single-tuned amplifier stages, each tuned to slightly different resonant frequencies and each having a bandwidth equal to or less than the overall bandwidth. Thus the consecutive stages are "staggered" in frequency and hence the name "stagger tuned amplifier." The overall response of several stages can be made to have both the requisite degree of flatness over the passband and adequate steepness of slope at the band limits. The gain bandwidth product of overall amplifier is considerably greater than that obtainable with all the stages tuned to the same frequency.

In stagger tuned amplifiers, several stages are considered as a group. A group of two stages is called a "pair," three stages a "triple", four stages a "quadruple" etc. The improvement in performance over single frequency operation increases with the number of stages in a group. For most of the high gain wideband amplifiers staggered pairs or triples are generally used.

Fig. 15·12 shows the response curves of two tuned circuits res-

$$\frac{A}{A_{res}}=\frac{1}{1+j2\delta Q_e} \qquad ...(15\cdot110)$$

At 3-*db* frequencies, $2\delta_0 Q_e=1$ and hence 3-*db* bandwidth B is given by

$$B=2\ \delta_0 f_0=\frac{f_0}{Q_e}$$

Since stage No. 1 is tuned to a frequency $\delta_0 f_0$ below f_0, the relative voltage gain of this stage is given by,

$$\left(\frac{A}{A_{res}}\right)_1=\frac{1}{1+j2(\delta Q_e+\delta_0 Q_e)}=\frac{1}{1+j(2\delta Q_e+1)} \qquad ...(15\cdot111)$$

where δ refers to the band centre frequency.

The stage No. 2 is tuned to a frequency $\delta_0 f_0$ above f_0, and hence the relative voltage gain of this stage is given by,

$$\left(\frac{A}{A_{res}}\right)_2=\frac{1}{1+j2(\delta Q_e-\delta_0 Q_e)}=\frac{1}{1+j(2\delta Q_e-1)} \qquad ...(15\cdot112)$$

Hence the overall voltage gain of the two stages is given by,

$$\left(\frac{A}{A_{res}}\right)_{pair}=\left(\frac{A}{A_{res}}\right)_1\left(\frac{A}{A_{res}}\right)_2$$

$$=\frac{1}{[1+j(2\delta Q_e+1)][1+j(2\delta Q_e-1)]}$$

$$=\frac{1}{2-(2\delta Q_e)^2+2j(2\delta Q_e)} \qquad ...(15\cdot113)$$

Magnitude of $=\left(\frac{A}{A_{res}}\right)_{pair}$ is given by,

$$\left(\frac{A}{A_{res}}\right)_{pair}=\frac{1}{\sqrt{4+(2\delta Q_e)^4}}=\frac{0.5}{\sqrt{1+4\delta^4 Q_e^4}} \qquad ...(15\cdot114)$$

Eqn. (15·114) for stagger tuned pair is similar in nature to Eqn. (15·93) for a critically coupled double-tuned amplifier. Consequently staggered pair has a 3-*db* bandwidth $\sqrt{2}$ times as great as that of each of the individual single tuned circuits constituting the pair.

From Eqn. (15·114) the voltage gain at band centre frequency is given by,

$$A_0=\frac{(A_{res})^2}{2} \qquad ...(15\cdot115)$$

Hence gain bandwidth product is given by,

$$A_0 B_{pair}=\frac{(A_{res})^2}{2}+\sqrt{2}\ B_1=\sqrt{2}(A_{res})^2 B_1 \qquad ...$$

where B_1 is the 3-*db* bandwidth of single tuned amplifier. This is larger than the corresponding figure of $0{\cdot}643\,(A_{res})^2\,B_1$ for a

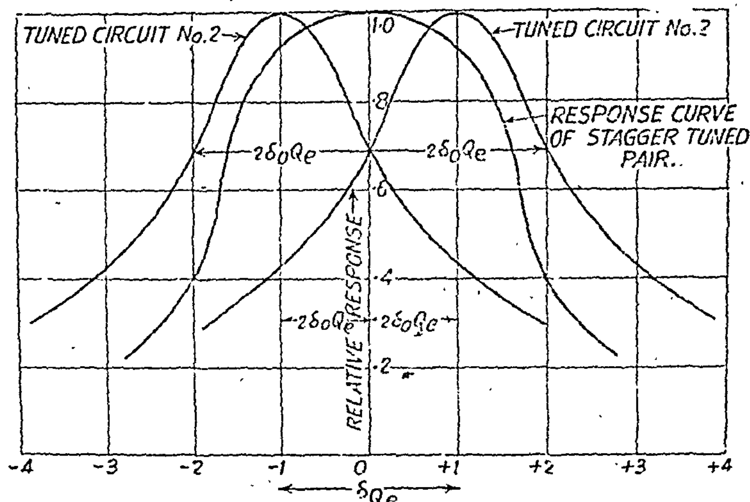


Fig. 15·12. Response curves of stagger-tuned pair and those of constituen single-tuned stages

two-stage single-tuned amplifier with each stage tuned to the same frequency.

The principle of stagger tuning may be extended to triples, quadruples etc. Such an extension is easily possible since each stage is single-tuned and there is no interaction among the tuning elements of different stages. The design of a stagger tuned amplifier depends upon band centre frequency f_0 and overall relative bandwidth $d=\left(\frac{B}{f_0}\right)_{overall}$.

The important advantages of stagger tuned amplifier are as follows :—

(*i*) Stagger tuned amplifiers can be designed for any relative bandwidth upto $d=2$ with relative sharp cutoff at the band limits.

(*ii*) The gain bandwidth product of a stagger tuned stage is considerably greater than that obtained on tuning the different stages of the amplifier to the same frequency. Thus a 6-stage stagger tuned amplifier consisting of 3 staggered pairs has gain bandwidth product twice that of a 6-stage synchronously tuned single tuned amplifier. The gain bandwidth product of overall stagger tuned amplifier increases as the number of stages in a group increases. Thus if the same 6-stage stagger tuned amplifier, consists of 2 staggered triples, its gain-bandwidth product increases to 2·5 times the gain bandwidth product of 6-stage synchronously tuned single-tuned amplifier.

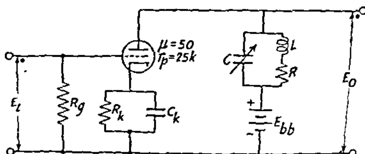
(*iii*) Since the various stages of a stagger tuned amplifier resonate at different frequencies, instability of operation by positive feedback is reduced.

(*iv*) Alignment of stagger tuned stages is extremely simple. All that is necessary in this case is to apply at the input of the first stage, a signal of frequency equal to resonant frequency of say stage one and then to tune this stage for maximum output of the last stage. The procedure is repeated for each of the stagger tuned stages feeding the signal still at the input of the first stage. Oscilloscopic inspection of response curve is not necessary. Thus alignment procedure is much simpler than in the case of multi-stage double-tuned amplifier.

(*v*) The phase response of a stagger tuned amplifier is, in general, superior to *i.e.* more linear over the passband than that of a single tuned amplifier of the same gain and bandwidth.

The main disadvantage of the stagger tuned amplifier is its low input impedance at high frequencies. It may be as low as a few thousand ohms. This excessively loads the tuned circuits preceding the stagger tuned amplifier. Such a loading affects the double tuned circuits to a lesser degree so that for frequencies above about 100 Mc/s, double tuned circuits are preferred.

Example 1. *In the tuned voltage amplifier shown in the diagram, $L=500\ \mu H$, $R=10\ \Omega$, tuning condenser $C=500\ \mu\mu F$. $E_i=\cdot 2$ volt r.m.s. at the resonant frequency. Calculate* (i) *resonant frequency* (ii) *Impedance of tuned circuit at resonance* (iii) *Voltage gain at resonance* (iv) *output voltage at resonance* (v) *Q of tuned circuit at resonance* (vi) *effective Q of tuned circuit including r_p.*



Solution. (i) Resonant frequency $f_0=\dfrac{1}{2\pi\sqrt{LC}}$

$$=\frac{1}{2\pi\sqrt{500\times10^{-6}\times500\times10^{-12}}}\ c/s$$
$$=318\ kc/s.$$

(ii) Impedance of tuned circuit at resonance is given by the approximate relation,

$$R_t=\frac{L}{CR}=\frac{500\times10^{-6}}{500\times10^{-12}\times10}=10^5\ \text{ohms}$$

(*iii*) Voltage gain at resonance is given by,

$$A_{res}=-g_m Z=\frac{-g_m}{\frac{1}{r_p}+\frac{1}{R_t}}=\frac{-\frac{50}{25\times10^3}}{\frac{1}{25\times10^3}+\frac{1}{10^5}}=-40$$

(*iv*) Output voltage at resonance $\mathbf{E}_0=-40\times0{\cdot}2=-8{\cdot}0$ volts.

(*v*) Q of tuned circuit at resonance is given by,

$$Q_0=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{10}\sqrt{\frac{500\times10^{-6}}{500\times10^{-12}}}=100$$

(*vi*) Effective Q of the tuned circuit including r_p is given by,

$$Q_e=\frac{R_{tp}}{X_c}$$

where $X_c=\frac{1}{\omega_0 C}=\sqrt{\frac{L}{C}}=\sqrt{\frac{500\times10^{-6}}{500\times10^{-12}}}=1000$ ohms.

and $R_{tp}=\frac{r_p\,.\,R_t}{r_p+R_t}=\frac{25\times10^3\times100\times10^3}{(25+100)\times10^3}=20\times10^3$ ohms.

Hence $Q_e=\frac{R_{tp}}{X_c}=\frac{20\times10^3}{1000}=20.$

Example 2. *A tuned voltage amplifier uses pentode having $g_m=1$ m℧ and $r_p=1$ MΩ. The tuned circuit consists of a coil of inductance 200 μH and resistance 12 Ω in shunt with a tuning condenser adjusted to 450 μμF. Calculate (i) resonant frequency (ii) Q of the tuned circuit at resonance (iii) impedance of the tuned circuit at resonance (iv) effective Q of the tuned circuit including r_p. (v) Voltage gain at resonance. Neglect the shunt capacitances. If the frequency of the applied signal is increased to 10 kc/s above resonance, calculate the magnitude and phase angle of voltage gain.*

Solution.

(*i*) $$f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{200\times10^{-6}\times450\times10^{-12}}}\text{ c/s}=532\text{ kc/s.}$$

(*ii*) $$Q_0=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{12}\sqrt{\frac{200\times10^{-6}}{450\times10^{-12}}}=55{\cdot}5$$

(*iii*) Impedance of tuned circuit at resonance is given by

$$R_t=\omega_0 L Q_0=\frac{L}{CR}=Q_0^2 R=(55{\cdot}5)^2\times12=37\times10^3\text{ ohms.}$$

(*iv*) Impedance of R_t and r_p is parallel is given by

$$R_{tp}=\frac{r_p\,.\,R_t}{r_p+R_t}=\frac{10^6\times37\times10^3}{10^3(1000+37)}=35{\cdot}8\times10^3\text{ omhs.}$$

Hence voltage gain at resonance is given by,

$$A_{res}=-g_m\,R_{tp}=-1000\times10^{-6}\times35{\cdot}8\times10^3=-35{\cdot}8$$

Effective Q of the plate circuit is given by,

$$Q_e = \frac{R_{tp}}{X_c} = R_{tp}\sqrt{\frac{L}{C}} = 35{\cdot}8\times10^3\sqrt{\frac{450\times10^{-12}}{200\times10^{-6}}}$$

$$=54{\cdot}6$$

At a frequency 10 kc/s above resonance,

$$\delta = \frac{f-f_0}{f_0} = \frac{10\times10^3}{532\times10^3} = {\cdot}0188$$

Relative voltage gain is given by,

$$\frac{\mathbf{A}}{\mathbf{A}_{res}} = \frac{1}{1+j2\delta Q_e} = \frac{1}{1+j2\times{\cdot}0188\times54{\cdot}6} = \frac{1}{1+j2{\cdot}05}$$

$$\text{or} \quad \mathbf{A} = \frac{-35{\cdot}8}{1+j2{\cdot}05}$$

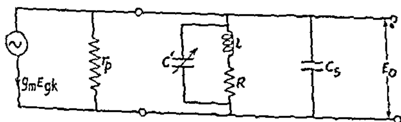
$$\text{Magnitude } A = \frac{35{\cdot}8}{\sqrt{1+(2{\cdot}05)^2}} = 15{\cdot}7$$

$$\text{Phase angle of } \mathbf{A} = \varphi = 180^\circ - \tan^{-1}(2{\cdot}05)$$

$$= 180 - 64^\circ = 116^\circ$$

Example 3. *A tuned voltage amplifier uses a triode having $\mu=50$ and $r_p=20$ kΩ. The tuned circuit consists of coil of impedance $10+j2\pi f\times300\times10^{-6}$ in shunt with a tuning capacitance variable between the limits 30–500 μμF. The total shunt capacitance caused by wiring and interelectrode capacitance is 20 μμF. Calculate the maximum and minimum frequencies of resonance and the capacitance of tuning condenser to obtain resonance at 0·5 Mc/s. With tuning condenser adjusted to produce resonance at 0·5 Mc/s, calculate (i) Q_0 of tuned circuit at resonance (ii) impedance of tuned circuit at resonance (iii) effective Q of the plate circuit at resonance including r_p. (iv) Voltage gain at resonance (v) magnitude and phase angle of voltage gain at frequency 20 kc/s above resonance (vi) half power bandwidth (vii) additional shunt resonance required to produce a half power bandwidth of 50 kc/s.*

Solution. A.c. equivalent circuit of this amplifier is given below.



Total tuning capacitance $C = C' + C_s$

Maximum value of capacitance $C_{max} = 30+20 = 50$ μμF.

Minimum value of capacitance $C_{min} = 500+20 = 520$ μμF

$$f_{max}=\frac{1}{2\pi\sqrt{LC_{min}}}$$

$$=\frac{1}{2\pi\sqrt{300\times10^{-6}\times50\times10^{-12}}}\text{ c/s}=1\cdot3\times10^{6}\text{ c/s.}$$

$$f_{min}=\frac{1}{2\pi\sqrt{LC_{max}}}$$

$$=\frac{1}{2\pi\sqrt{300\times10^{-6}\times520\times10^{-12}}}\text{ c/s}$$

$$=0\cdot403\times10^{6}\text{ c/s.}$$

Value of C for resonance at 0·5 Mc/s is given by,

$$0\cdot5\times10^{6}=\frac{1}{2\pi\sqrt{300\times10^{-6}\times C}}$$

Hence $C=338\times10^{-12}$ Farad.

Required tuning condenser Capacity $C'=(338-20)=318$ $\mu\mu$F.

At 0·5 Mc/s

(*i*) $$Q_0=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{10}\sqrt{\frac{300\times10^{-6}}{338\times10^{-12}}}=94$$

(*ii*) Impedance of the tuned circuit at resonance

$$R_t=\frac{L}{CR}=Q^2R=(94)^2\times100=88\cdot36\times10^{3}\text{ ohms.}$$

(*iii*) Effective Q of the tuned circuit is given by,

$$Q_e=\frac{Q_e}{1+\frac{\omega_0LQ_0}{r_p}}=\frac{Q_0}{1+\frac{R_t}{r_p}}=\frac{94}{1+\frac{88\cdot36}{20}}=17\cdot35$$

(*iv*) Voltage gain at resonance is given by,

$$\mathbf{A}_{res}=-g_m\frac{\omega_0LQ_0}{1+\frac{\omega_0LQ_0}{r_p}}=\frac{-g_mR_t}{1+\frac{R_t}{r_p}}$$

$$g_m=\frac{50}{20000}=2\cdot5\times10^{-3}\text{ mho.}$$

Hence $$\mathbf{A}_{res}=-2\cdot5\times10^{-3}\times\frac{88\cdot36\times10^{3}}{5\cdot418}=-40\cdot7$$

(*v*) At 20 kc/s above resonant frequency,

$$\delta=\frac{20\times10^{3}}{0\cdot5\times10^{6}}=\cdot04$$

Hence $$\mathbf{A}=\frac{\mathbf{A}_{res}}{1+j2\delta Q_e}=\frac{-40\cdot7}{1+j2\times\cdot04\times17\cdot35}=\frac{-40\cdot7}{1+j1388}$$

Magnitude $$\mathbf{A}=\frac{40\cdot7}{\sqrt{1+(1\cdot388)^2}}=23$$

Phase angle of A is given by,

$$\phi = 180^\circ - \tan^{-1} 1{\cdot}388$$
$$= 180^\circ - 54^\circ\, 14' = 125^\circ\, 46'$$

(*vi*) Half power bandwidth is given by,

$$B = \frac{f_o}{Q_e} = \frac{0{\cdot}5 \times 10^6}{17{\cdot}35} = 28{\cdot}8 \times 10^3 \text{ c/s}$$

(*vii*) If B is to be made 50 kc/s,

$$Q_e' \text{ must, } \frac{f_o}{50 \times 10^3} = \frac{0{\cdot}5 \times 10^6}{50 \times 10^3} = 10$$

Let additional resistance in shunt be R_0

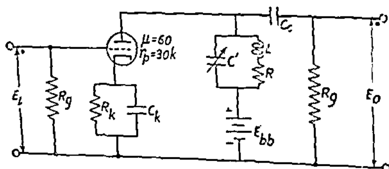
Then $$Q_e' = \frac{Q_e}{1 + \frac{R_{t9}}{R_0}}$$

or $$10 = \frac{17{\cdot}35}{1 + \frac{17{\cdot}6 \times 10^3}{R_0}}$$

Hence $$R_0 = 23{\cdot}9 \times 10^3 \text{ ohms.}$$

Example 4. *In the capacitance coupled tuned voltage amplifier shown in the* [illegible] *as below : $L = 400$ μH, $C' = 380$* [illegible] *d shunt capacitance $C_s = 20$ $\mu\mu F$.* [illegible] *lculate (i) frequency of resonance of plate circuit, (ii) Q_0 of the* [illegible] *circuit at resonance $(C = C' + C_s)$, (iii) effective value of Q of plate circuit including r_p and R_g, (iv) voltage gain at resonance, (v) half power bandwidth (vi) gain bandwidth product (vii) voltage gain at frequency of 20 kc/s below resonance.*

Solution. (i) $C = C' + C_s = 380 + 20 = 400$ $\mu\mu$F.



Frequency of resonance $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{400 \times 10^{-6} \times 400 \times 10^{-12}}}$$

$$= 398 \times 10^3 \text{ c/s.}$$

(*ii*) Q_0 of $L-C-R$ circuit at resonance is given by,

$$Q_0=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{8}\times\sqrt{\frac{400\times10^{-6}}{400\times10^{-12}}}=125.$$

(*iii*) $\omega_0=2\pi f_0=2\pi\times398\times10^3=2{\cdot}5\times10^6$ radians/sec.

$$Q_e=\frac{Q_0}{1+\omega_0 L\, Q_0\left(\frac{1}{r_p}+\frac{1}{R_g}\right)}$$

$$=\frac{125}{1+2{\cdot}5\times10^6\times400^{-6}\times125\left(\frac{1}{30\times10^3}+\frac{1}{500\times10^3}\right)}=23{\cdot}1$$

(*iv*) $\mathbf{A}_{res}=-g_m\,\omega_0 L\, Q_e$

$$=-\frac{60}{30\times10^3}\times25\times10^{-6}\times400\times10^{-6}\times125=-46{\cdot}2$$

(*v*) Half power bandwidth B

$$=\frac{f_0}{Q_e}=\frac{398\times10^3}{23{\cdot}1}\text{ c/s}$$

$$=17{\cdot}25 \text{ kc/s.}$$

(*vi*) Gain bandwidth product $=B\times A_{res}$

$$=(17{\cdot}25\times10^3)\times46{\cdot}2=796\times10^3$$

(*vii*) At 20 kc/s below resonance,

$$\delta=\frac{20\times10^3}{398\times10^3}={\cdot}0502$$

Voltage gain $\quad \mathbf{A}=\frac{\mathbf{A}_{res}}{1+j2\delta Q_e}=\frac{-46{\cdot}2}{1+j\,{\cdot}1004\times23{\cdot}1}$

$$=\frac{-46{\cdot}2}{1+j\,2{\cdot}32}$$

Magnitude $\quad A=\frac{46{\cdot}2}{\sqrt{1+(2{\cdot}32)^2}}=18{\cdot}3.$

Example 5. *A single tuned capacitance coupled voltage amplifier has tuned circuit coil having inductance of 200 μH and resistance of 6Ω. In shunt with this coil is the tuning condenser with capacitance variable from 30 to 500 μμF. The output capacitance C_o of the amplifier tube is 10 μμF and input capacitance C_i of the next stage is 20 μμF. Grid leak resistance R_g is 500 kΩ. The amplifier tube has $\mu=40$ and $r_p=20$ kΩ. Calculate (i) value of tuning condenser C′ for resonance at 500 kc/s (ii) effective Q of the plate circuit including r_p and R_g at resonance (iii) voltage gain at resonance (iv) 3-db bandwidth (v) gain bandwidth product (vi) maximum possible value of gain bandwidth product. If inductance L is kept constant for this maximum gain-bandwidth product, calculate the frequency of resonance.*

Solution. (i) $C=\frac{1}{\omega^2 L}=\frac{1}{(2\pi\times500\times10^3)^2\times200\times10^{-6}}$

$=505\times10^{-12}$ Farad.

$C_s=C_i+C_o=20+10=30\ \mu\mu F$

Hence tuning condenser $C'=C-C_s=505-30=475\ \mu\mu F$.

(ii) Impedance of tuned circuit at resonance is given by,

$$R_t=\frac{L}{CR}=\frac{200\times10^{-6}}{505\times10^{-12}\times8}=49.5\times10^3 \text{ ohms}$$

$$Q_o=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{1}{8}\sqrt{\frac{200\times10^{-16}}{505\times10^{-12}}}=78.5$$

$$Q_e=\frac{Q_o}{1+R_t\left[\frac{1}{r_p}+\frac{1}{R_g}\right]}$$

$$=\frac{78.5}{1+49.5\times10^3\left[\frac{1}{20\times10^3}+\frac{1}{500\times10^3}\right]}=22$$

(iii) $A_{res}=-g_m\,\omega_0 L\,Q_e$

$$=-\frac{40}{20\times10^3}\times(2\pi\times500\times10^3)\times(200\times10^{-6})\times22$$

$=-27.6$

(iv) 3-*db* bandwidth $B=\frac{f_o}{Q_e}=\frac{500\times10^3}{22}$

$=22.8\times10^3$ cycles/sec.

(v) Gain bandwidth product $A_{res}\times B=27.6\times22.8\times10^3$

$=630\times10^3$ c/s.

(vi) Maximum gain bandwidth product is given by,

$$\left(A_{res}\times B\right)_{max}=\frac{g_m}{2\pi[C_i+C_o+C_{min}]}$$

$$=\frac{2\times10^{-3}}{2\pi[20+10+30]\times10^{-12}}$$

$=5.3\times10^6$ c/s

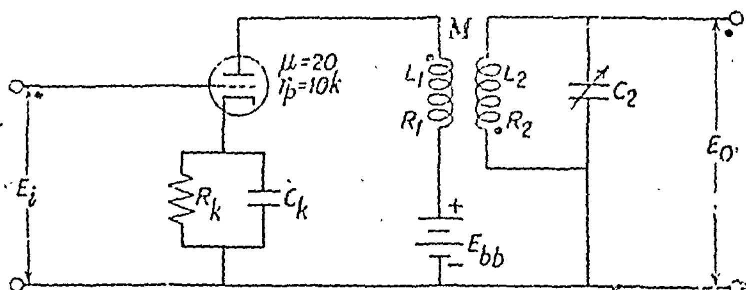
Total tuning capacity is 60 $\mu\mu F$. Hence frequency of resonance for maximum gain bandwidth product condition is given by,

$$f'_o=\frac{1}{2\pi\sqrt{200\times10^{-6}\times60\times10^{-12}}}$$

$=1.45\times10^6$ c/s

Example 6. *In the transformer coupled single tuned voltage amplifier shown, $C_2=35-500\ \mu\mu F$, $L_2=300\ \mu H$, $R_2=10\Omega$, $M=30\ \mu H$, C_2 is adjusted to 300 $\mu\mu F$. Calculate (i) frequency of resonance f_r (ii) Q_o of tuned circuit at resonance (iii) voltage gain at resonance (iv) effective Q of the tuned circuit including r_p reflected into the secondary*

circuit (v) Optimum value of mutual inductance M and the corresponding value of voltage gain at resonance (vi) With value of $M=30\ \mu H$. Calculate the magnitude and phase angle of voltage gain at a frequency 20 kc/s above the resonant frequency (vii) half power bandwidth.



Solution.

(i) $$f_0=\frac{1}{2\pi\sqrt{L_2C_2}}=\frac{1}{2\pi\sqrt{300\times10^{-6}\times300\times10^{-6}}}$$
$$=530\times10^3\ \text{c/s.}$$

(ii) Q_o at resonance is given by,

$$Q_0=\frac{1}{R_2}\sqrt{\frac{L_2}{C_2}}=\frac{1}{10}\sqrt{\frac{300\times10^{-6}}{300\times10^{-12}}}=100.$$

(iii) Voltage gain at resonance is given by,

$$\mathbf{A}_{res}=\mu\frac{M/c_2}{r_pR_2+\omega_0^2M^2}$$

$$=20\frac{(30\times10^{-6})/(300\times10^{-12})}{(10^4\times10)+(2\pi\times530\times10^3)^2\times(30\times10^{-6})^2}$$

$$=\frac{2\times10^6}{10^5+(1{\cdot}06\pi\times30)^2}=18{\cdot}2$$

(iv) $$Q_e=\frac{Q_o}{1+\dfrac{\omega_0^2M^2}{r_pR_2}}=\frac{100}{1+\dfrac{(2\pi\times530\times10^3)^2(30\times10^{-6})^2}{10\times10^4}}=91$$

(v) $$M_{opt}=\frac{\sqrt{r_pR_2}}{\omega_0}=\sqrt{r_pR_2L_2C_2}$$

$$=\sqrt{10^4\times10\times300\times10^{-6}\times300\times10^{-12}}$$
$$=94{\cdot}7\ \mu H$$

$$(\mathbf{A}_{res})_{opt}=\frac{g_m}{2}\sqrt{r_pR_2}\,Q_o$$

$$=\frac{20}{10^4}\times\frac{1}{2}\times\sqrt{10^4\times10}\times100=31{\cdot}6$$

(vi) At frequency 20 kc/s above resonance,

$$\delta=\frac{20}{530}={\cdot}0377$$

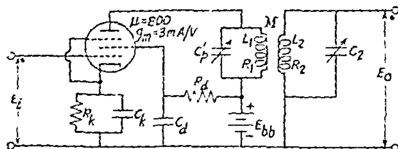
$$A=\frac{A_{res}}{1+j\,2\delta Q_e}=\frac{18.2}{1+j\,2\times .0377\times 91}=\frac{18.2}{1+j\,6.86}$$

Magnitude $A=\dfrac{18.2}{\sqrt{1+(6.86)^2}}=2.62$

Phase angle $\phi=\tan^{-1}6.86=-81°\ 42'$

(*vii*) 3-*db* bandwidth $B=\dfrac{f_o}{Q_e}=\dfrac{530\times 10^3}{91}=5.83\times 10^3$ c/s.

[illegible]



Solution.

(*i*) $$f_o=\frac{1}{2\pi\sqrt{L_2C_2}}=\frac{1}{2\pi\sqrt{200\times 10^{-6}\times 450\times 10^{-12}}}=$$
$$=531\times 10^3\ \text{c/s}.$$

(*ii*) $$Q_2=\frac{\omega_0 L_2}{R_2}=\frac{1}{R_2}\sqrt{\frac{L_2}{C_2}}=\frac{1}{10}\sqrt{\frac{200\times 10^{-6}}{450\times 10^{-12}}}=66.7$$

$Q_1=Q_2=66.7$

$$A_{res}=\frac{-j\sqrt{p}\,g_m\,Q_1Q_2\sqrt{R_1R_2}}{1+p}$$

$$=\frac{-j\sqrt{0.81}\times 3\times 10^{-3}\times 66.7\times 66.7\times 10}{1+0.81}=-j\,67.4$$

Magnitude $A_{res}=67.4$.

(*iv*) At frequency 2 kc/s above resonant frequency,

$$\delta=\frac{2}{531}=3.78\times 10^{-3}$$

Hence voltage gain is given by,

$$A=\frac{-jg_m\sqrt{p}\ Q_1Q_2\ \sqrt{R_1R_2}}{1+p-4\delta^2Q_1Q_2+j\,2\delta(Q_1+Q_2)}$$

$$=\frac{-j\,3\times10^{-3}\sqrt{0\cdot81}\times66\cdot7\times66\cdot7\times10}{1\times0\cdot81-4(3\cdot78\times10^{-3}\times66\cdot7)^2-j\,2\times3\cdot78\times10^{-3}\times133\cdot4}$$

$$=\frac{-j\,120}{1\cdot556+j\,1\cdot01}$$

Magnitude $A=\dfrac{120}{\sqrt{(1\cdot556)^2+(1\cdot01)^2}}=65.$

Example 8. *In a double tuned voltage amplifier, primary and secondary resonant circuits are identical and each is tuned to 2 Mc/s. Both primary and secondary windings have inductance of 150 μH and resistance of 10 Ω, coupling dissipation factor p=1. Mutual conductance g_m of the tube is 4mA/V and amplification factor μ=50. Calculate (i) value of tuning condensers (ii) Q of resonant circuits at resonance (iii) voltage gain at resonance (iv) relative voltage gain $\frac{A}{A_{res}}$ at a frequency 10 kc/s above resonance (v) 3-db bandwidth.*

Solution.

(i) $C=\dfrac{1}{\omega_0^2L}=\dfrac{1}{(2\pi\times2\times10^6)^2\times(150\times10^{-6})}$

$=42\cdot2\times10^{-2}$ Farad

(ii) $Q_1=Q_2=\dfrac{1}{R_2}\sqrt{\dfrac{L_2}{C_2}}=\dfrac{1}{10}\sqrt{\dfrac{150\times10^{-6}}{42\cdot2\times10^{-12}}}\doteq188$

(iii) $\mathbf{A}_{res}=\dfrac{-j\sqrt{p}\;g_mQ_1Q_2\sqrt{R_1R_2}}{1+p}$

$=\dfrac{-j\times4\times10^{-3}\times188\times188\times10}{1+1}=-j\,708$

Magnitude $A_{res}=708.$

(iv) At 10 *kc/s* above resonance,

$$\delta=\frac{10}{2000}=5\times10^{-3}$$

For $p=1$, and $Q_1=Q_2=Q$ say,

$$\frac{\mathbf{A}}{\mathbf{A}_{res}}=\frac{1}{(1-2\,\delta^2Q^2]+j\,2\delta Q}$$

Magnitude $\dfrac{A}{A_{res}}=\dfrac{1}{\sqrt{1+4\delta^4Q^4}}=\dfrac{1}{\sqrt{1+4(5\times10^{-3}\times188)^4}}=0\cdot493$

(v) 3-*db* bandwidth is given by,

$$B=\sqrt{2}\,\frac{f_o}{Q}=\frac{\sqrt{2}\times2\times10^6}{188}=15\cdot05\times10^3\text{ c/s.}$$

Example 9. *A radio frequency amplifier consists of four stages of capacitance coupled single tuned stages each tuned to a frequency of 1 Mc/s. Effective Q of each stage is 20. Mutual conductance g_m of each tube is 3 mA/V and inductance of each tuned circuit is 200 μH.*

Calculate the voltage gain at resonance 3-db and bandwidth of each stage and that of overall amplifier.

Solution. Gain of each stage at resonance is given by,

$$A_{res} = g_m \omega_o L Q_e$$
$$= 3 \times 10^{-3} \times (2\pi \times 10^6) \times 200 \times 10^{-6} \times 20$$
$$= 75.5$$

Gain of 4 stages at resonance $= (75.5)^4 = 32.48 \times 10^6$

3-*db* bandwidth of each stage $= B = \frac{f_o}{Q_e} = \frac{10^6}{20} = 50$ kc/s.

Overall 3-*db* bandwidth $\quad B_n = B\sqrt{2^{\frac{1}{4}} - 1}$

$$= 50 \times 10^3 \times 0.435 = 21.75 \text{ kc/s.}$$

Example 10. *An Intermediate Frequency Amplifier consists of three stages of double tuned amplifier. Each stage is tuned to a frequency of 550 kc/s. Effective Q of each tuned circuit in each stage is 30. The voltage gain of each stage is 40. Calculate the overall gain and overall 3-db bandwidth. Coupling dissipation factor p=1.*

Solution. Overall gain $= (40)^3 = 64000$

3-*db* bandwidth of each stage is $B = \sqrt{2}\frac{f_o}{Q} = \frac{\sqrt{2} \times 550 \times 10^3}{30}$

$$= 25.9 \times 10^3 \text{ c/s.}$$

Overall 3-*db* bandwidth $\quad B_n = B\sqrt[4]{2^{\frac{1}{3}} - 1}$

$$= 25.9 \times 0.713 = 18.5 \text{ kc/s.}$$

EXERCISES

1. A tuned voltage amplifier uses as tuned circuit a coil of inductance 250 μH and resistance 10Ω in parallel with a condenser of 500 $\mu\mu F$. μ of the triode used is 40 and dynamic plate resistance r_p is 20 $k\Omega$. Calculate (*i*) resonant frequency (*ii*) Q_o of the tuned circuit at resonance (*iii*) impedance of the tuned circuit at resonance (*iv*) effective Q_e of the plate circuit including r_p (*v*) voltage gain of the amplifier.

2. A tuned voltage amplifier uses pentode having g_m equal t[illegible] of 800 $k\Omega$. The tuned circui[illegible] and resistance 15 ohms wit[illegible] e 400 $\mu\mu F$. The total shun[illegible] rode capacitances and wirin[illegible] equency of resonance (*ii*) Q_o of the tuned circuit at resonance including the shunt capacitances (*iii*) impedance of the tuned circuit (*iv*) effective Q of the total plate circuit impedance including r_p at resonance (*v*) voltage gain at resonance (*vi*) gain at frequencies 20 and 40 kc/s below resonant frequency.

3. A tuned voltage amplifier uses a triode having $\mu=40$, and $r_p=16\ k\Omega$. The tuned circuit consists of a coil and a tuning condenser in its parallel. Coil has inductance of 400 μH and resistance of 15 Ω. The tun'ng condenser has its capacity variable between 35 to 500 $\mu\mu F$. The additional shunt capacitance due to wiring and inter-electrode capacitance is 25 $\mu\mu F$. Calculate the maximum and minimum frequencies at which this tuned circuit may resonate and the value of tuning condenser capacity at resonant frequency of 0·6 Mc/s. With condenser adjusted for resonance at 0·6 Mc/s, calculate (*i*) Q_0 of the tuned circuit (*ii*) impedance of tuned circuit at resonance (*iii*) total plate circuit impedance at resonance (*iv*) Q_e of total plate circuit at resonance (*v*) voltage gain at resonance (*vi*) magnitude and phase angle of voltage gain at frequencies 10 kc/s and 20 kc/s below resonance (*vii*) 3-*db* bandwidth (*viii*) additional shunt resistance required to increase the bandwidth by 50%.

4. Draw the circuit diagram of a single tuned capacitancecoupled voltage amplifier using a triode. Draw its a.c. equivalent circuit and hence derive expressions for (*i*) frequency of resonance f_o (*ii*) Q of tuned circuit at resonance (*iii*) effective Q of the plate circuit including r_p and R_g at resonance (*iv*) impedance of the tuned circuit alone at resonance (*v*) impedance of plate circuit including r_p and R_g (*vi*) voltage gain at resonance (*vii*) voltage gain **A** at any frequency ω close to resonant frequency ω_0 (*viii*) half power bandwidth (*ix*) gain bandwidth product.

5. A capacitance coupled single tuned voltage amplifier uses a triode having $\mu=50$ and mutual conductance of 2 mA/V. Tuned circuit uses a tuning condenser variable from 30 to 500 $\mu\mu F$. Output shunt capacitance C_o of amplifier tube is 12 $\mu\mu F$ and input shunt capacitance C_i of next stage is 22 $\mu\mu F$. Calculate the maximum gain bandwidth product obtainable from this amplifier. If the coil of the tuned circuit has inductance of 100 μH and resistance of 5 ohms, and grid leak resistance R_g is 1 Meg ohm. Calculate (*i*) the frequency of resonance and (*ii*) the voltage gain of the amplifier under this maximum gain bandwidth product condition.

6. A transformer coupled single tuned voltage amplifier uses a triode having $\mu=40$ and $r_p=15\ k\Omega$. The secondary winding has inductance $L_2=400\ \mu H$ and resistance $R_2=12\Omega$. Mutual inductance $M=40\ \mu H$. The variable tuning condenser across the secondary is adjusted to 200 $\mu\mu F$. Calculate (*i*) frequency of resonance f_o (*ii*) Q_o of the tuned circuit at resonance (*iii*) effective Q of the tuned circuit including r_p reflected to the secondary side (*iv*) voltage gain at resonance (*v*) Optimum value of mutual inductance to get maximum voltage gain at resonance (*vi*) voltage gain at resonance with optimum mutual inductance (*vii*) magnitude and phase angle of voltage gain at frequencies 10 and 20 kc/s below the resonant frequency with $M=40\ \mu H$ (*viii*) 3-*db* bandwith.

7. In a double tuned voltage amplifier, primary and secondary resonant circuits are identical and each is tuned to 1 Mc/s. Both

primary and secondary windings have inductance of 300 μH and resistance of 8Ω. The coupling dissipation factor $p = k^2Q_1Q_2 = 0.64$ g_m of tube is 4 mA/V and amplification factor is 60. Calculate (i) value of tuning condensers (ii) Q of tuned circuits at resonance (iii) voltage gain at resonance (iv) voltage gain at a frequency 4 kc/s below *resonant frequency*.

8. A critically coupled double tuned voltage amplifier has both the primary and secondary tuned to 1·5 Mc/s. The primary and the secondary windings are identical and each has inductance of 250 μH and resistance of 12 ohms. The amplifier tube has mutual conductance of 2 mA/V and amplification factor of 40. Calculate, (i) value of tuning condenser to obtain resonance at 1·5 Mc/s (ii) Q of the tuned circuit at resonance (iii) voltage gain at resonance (iv) relative voltage gain $\frac{A}{A_{res}}$ at a frequency 6 kc/s above resonance and (v) 3-db bandwidth.

9. A radio frequency voltage amplifier uses 5 stages of capacitance coupled single tuned stages each tuned to a frequency of 500 kc/s. Effective Q of plate circuit of each stage is 25. The amplifier tube has mutual conductance of 2 mA/V. The coil in the tuned circuit of each stage has inductance of 100 μH. Calculate the gain at resonance and 3-db bandwidth of each stage and of the overall amplifier.

10. A four stage Intermediate Frequency Amplifier uses double tuned stages each stage tuned to 565 kc/s. Q of each tuned stage is 20. Voltage gain of each stage is 30. All tuned circuits are critically coupled. Calculate the overall voltage gain and overall 3-db bandwidth.

CHAPTER XVI

TUNED POWER AMPLIFIERS

A tuned power amplifier amplifies with good efficiency, radio frequency power either at a single radio frequency or in a narrow band of radio frequencies. It differs from tuned voltage amplifier in that its primary function is to produce large R.F. power. Since the amount of power handled is large, it is essential that the plate circuit efficiency of tuned power amplifier be large. In fact tuned power amplifier converts the d.c. power from the plate supply source into R.F. power. The amplifier must, therefore, be so designed and set that this conversion of d.c. power into R.F. power takes place with maximum efficiency.

The circuit arrangement of tuned power amplifier remains essentially the same as that of the tuned voltage amplifier. Basic circuit diagram of a tuned power amplifier is shown in Fig. 16·1.

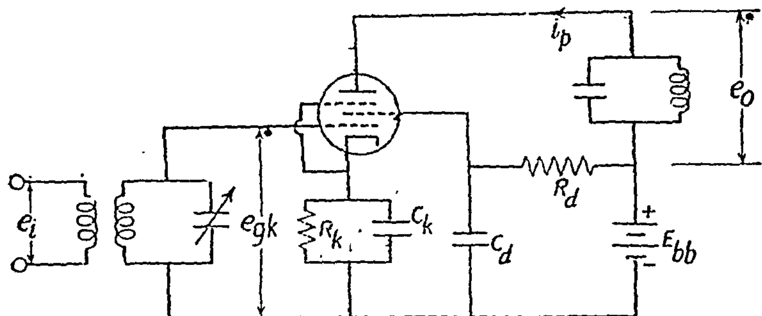


Fig. 16·1. Basic circuit diagram of a tuned power amplifier.

Since power handled in tuned power amplifiers is large, plate circuit efficiency must be large. Hence class *A* operation is hardly ever used. Class *B* and class *C* operation giving high plate circuit efficiency are utilized. In class *B* operation, negative grid bias is adjusted to plate current cutoff so that plate current flows for half the a.c. cycle. In class *C* operation, negative grid bias is adjusted appreciably beyond the cutoff value so that plate current flows in the form of pulses lasting for periods appreciably less than half the a.c. cycle. In either case, since the plate current does not flow for the entire a.c. cycle of the applied input voltage, harmonic components comparable in magnitude with the fundamental, are produced in the plate circuit. But plate load is a tuned circuit of high *Q* which responds to a narrow band of frequencies centred about the resonant frequency. Response at second, third etc. harmonics of fundamental is negligible if *Q* of the tuned circuit is greater than ten. Thus all harmonic terms are easily eliminated. But if *Q* is large, response of tuned amplifier is limited to a narrow band of frequencies. In radio broadcasting and radio communication, using amplitude modulation,

wever, bandwidth requirement is small, usually not exceeding kc/s Hence these tuned power amplifiers having Q of the order ten may be conveniently used. In practice, in amplitude modu- :ion radio transmitters, class C tuned power amplifiers are used to nplify the R.F. carrier before modulation takes place whereas iss B tuned power amplifiers are used to amplify amplitude odulated R F. carrier. Tuned class C power amplifier cannot used for amplification of such an amplitude modulated carrier ice the amplitude may vary over a wide range and plate current nnot reproduce these amplitude variations since it flows for a nall part of the a c. cycle. Class B amplifiers alone may be used r such a purpose.

Response of tank circuit at second harmonic frequency.

In Fig. 16 1 parallel tuned circuit constitutes the plate load. i general, however, the output developed across the tuned circuit coupled to the load impedance. This coupling may be either pacitive or inductive. Inductive coupling is used commonly. irther a tuning condenser C_2 is placed in the load circuit to produce sonance with the coil at the single frequency. Fig 16 2 shows the rcuit arrangement.

Let the amplifier operate at frequency ω_0. Then C_2 is so ljusted as to resonate with L_2 at angular frequency ω_0 Thus

$$\omega_0=\frac{1}{\sqrt{L_2C_2}} \quad ..(16\text{ }1)$$

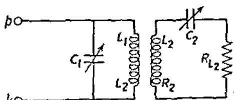
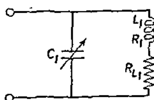
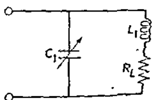


Fig 16·2. Typical method of coupling the load impedance R_{l2} to the parallel tuned circuit.

At resonance the im- edance of the secondary rcuit is (R_{l2} and R_2) sistive. This resistance ansferred to the primary de becomes $R_{l1}=\frac{(\omega M)^2}{(R_{l2}+R_2)}$. he equivalent tuned circuit ien becomes as shown in Fig. 16 3 (a) Resistances R_{l1} and R_1 iay be combined to form resistance R_l resulting in equivalent rcuit of Fig. 16·3 (b)



(a) Equivalent circuit of tuned plate circuit.



(b) Modified form of equivalent circuit.

Fig. 16 3. Equivalent circuit of plate tuned cir

The impedance at resonance of this tuned circuit of Fig. 16·3 (*b*) is resistive and for ideal conditions is given by,

$$Z(\omega_0)= R_t=\frac{L_1}{C_1R_t} \qquad ...(16·2)$$

If the tank circuit is ideal, the impedance $Z(n\omega_0)$ at any of the harmonic frequencies will be zero. But since ideal conditions do not exist in practice, harmonic frequency impedance $Z\ (n\omega_0)$ is never zero although it may be made quite small. Thus for any parallel tuned circuit, impedance at any frequency is given by,

$$Z=\frac{L_1}{C_1R_t}\quad\frac{1+\delta-j\left(\frac{1}{Q}\right)}{1+\delta+jQ\delta\ (2+\delta)} \qquad ...16·3)$$

where $$\delta=\frac{\omega-\omega_0}{\omega_0} \qquad ...(16·4)$$

At resonance $\omega=\omega_0$ and $\delta=0$, so that

$$Z(\omega_0)=R_t\left(1-j\frac{1}{Q_o}\right)$$

$$=R_t\sqrt{1+\frac{1}{Q_o^2}}\underline{|-\tan^{-1} 1/Q_0} \qquad ...(16·5)$$

If Q_o is large, $Z(\omega_o)$ is equal to R_t and phase shift is zreo. Typically $Q_0=10$, at which value,

$$Z(\omega_0)=R_t\times 1·005 - \angle\underline{5·7°} \qquad ...(16·6)$$

Equation (16·6) shows that for $Q_o=10$, impedance of tuned circuit at resonance is almost resistive and equal to R_t. Thus for large values of Q_o,

$$\mathbf{Z}(\omega_0)=R_t=\frac{L_1}{C_1R_t}=\omega_oL_1Q_o$$

$$=R_lQ_o^2=Q_o\sqrt{\frac{L_1}{C_1}} \qquad ...\ (16·7)$$

At Second Harmonic Frequency : $\omega=2\omega_o$ and $\delta=1$, so that Eqn. (16·3) becomes,

$$\mathbf{Z}(2\omega_0)=R_t\frac{1-j\frac{1}{2Q_0}}{1+j1·5Q_0}$$

$$=R_t\frac{0·25-j\left[\frac{1}{2Q_0}+1·5Q_0\right]}{1+2·25Q_o^2} \qquad ...\ (16·8)$$

For Q_o *equal to ten or greater*, Eqn. (16·8) becomes,

$$\mathbf{Z}\ (2\omega_0)=\frac{R_t}{j\ 1·5Q_o}=-j\frac{1}{1·5}\sqrt{\frac{L_1}{C_1}} \qquad ...\ (16·9)$$

Then $$\frac{Z(2\omega_0)}{Z(\omega_0)}=\frac{R_t\left(\frac{1}{j\,1\cdot5Q_o}\right)}{R_t}=\frac{1}{j\,1\cdot5Q_o} \quad \ldots (16\cdot10)$$

Power output at ω_0 is given by,

$$P_t(\omega_0)=I^2_{p1}\,Z(\omega_0)_{real}=I^2_{p1}.R_t=I^2_{p1}\times Q_o^2R_t \quad \ldots(16\cdot11)$$

where I_{p1} is the r.m s. value of fundamental component of plate current and $Z(\omega_0)_{real}$ is the real component of $Z(\omega_0)$.

Similarly power output at $2\omega_0$ is given by,

$$P_t\,(2\omega_0)=I^2_{p2}\,Z(2\omega_0)_{real}=\frac{I^2_{p2}R_t}{4(1+2\cdot25Q_0^2)} \quad \ldots (16\cdot12)$$

where I_{p2} is the r.m s. value of second harmonic component of plate current and $Z(2\omega_0)_{real}$ is the real component of $Z(2\omega_0)$.

Hence relative output power is given by,

$$\frac{P_t(\omega_o)}{P_t(2\omega_o)}=\frac{I^2_{p1}.R_t}{I^2_{p2}.\frac{R_t}{4(1+2\cdot25Q_o^2)}}$$

$$=\left(\frac{I_{p1}}{I_{p2}}\right)^2.\,4(1+2\cdot25Q_o^2) \quad \ldots (16\cdot13)$$

Let us consider the extreme condition when $I_{p1}=I_{p2}$. Then relative output power is given by,

$$\frac{P_t(\omega_o)}{P_t(2\omega_o)}=4(1+2\cdot25Q_o^2) \quad \ldots (16\cdot14)$$

Typically $Q_o=10$, so that,

$$\frac{P_t(\omega_o)}{P_t(2\omega_o)}=4(1+2\cdot25\times10^2)=904.$$

... power at the second harmonic ... $Q_o=10$ and $I_{p1}=I_{p2}$, is neg-

Out of the total power developed in the tank circuit, a part is lost in the resistance R_1 of the coil. If condenser C_1 also has a dissipative component, further loss takes place.

Out of the total power $(I_{p1}Q_o)^2\,R_t$ delivered to the tuned circuit, the part delivered to the secondary is given by,

$$P_{t1}=(I_{p1}Q_o)^2.R_{t1}=(I_{p1}Q_o)^2.\,\frac{\omega_0^2M^2}{R_{t2}+R_2} \quad \ldots (16\cdot15)$$

Then $$\eta_1=\frac{\text{power delivered to the secondary}}{\text{power delivered to the primary}}$$

$$=\frac{P_{t1}}{P_t(\omega_0)}$$

$$=\frac{(I_{p1}Q_o)^2R_{t1}}{(I_{p1}Q_o)^2R_t}=\frac{R_{t1}}{R_{t1}+R_1}$$

Out of this total secondary power P_{l1}, the part P_{l2} is developed across the load resistance R_{l2} and is given by,

$$P_{l2}=P_{l1}\cdot\frac{R_{l2}}{R_{l2}+R_2}=(I_{p1}\cdot Q_o)^2R_{l1}\cdot\frac{R_{l2}}{R_{l2}+R_2}$$

$$=(I_{p1}Q_o)^2\cdot\frac{\omega_0{}^2M^2}{R_{l2}+R_2}\cdot\frac{R_l}{R_{l2}+R_2} \quad ...(16\cdot17)$$

Hence $$\eta_2=\frac{\text{Power delivered to the load}}{\text{Power delivered to the secondary}}$$

$$=\frac{P_{l2}}{P_{l1}}=\frac{R_{l2}}{R_{l2}+R_2} \quad ...(16\cdot18)$$

The ratio of power delivered to the load to total power delivered to the tank circuit is called the "circuit transfer efficiency" and may be indicated by η.

Hence $$\eta=\frac{P_{l2}}{P_l(\omega_0)}=\frac{P_{l2}}{P_{l1}}\cdot\frac{P_{l1}}{P_l(\omega_0)}=\eta_2.\eta_1 \quad ...(16\cdot19)$$

Substituting the values of powers, we get,

$$\eta=\frac{R_{l2}}{R_{l2}+R_2}\cdot\frac{R_{l1}}{R_{l1}+R_1} \quad ...(16\cdot20)$$

Eqn. (16·19) may be put in the alternative form,

$$\eta=\frac{P_{l2}}{P_{l2}+P_t}=\frac{P_l(\omega_0)-P_t}{P_l(\omega_0)} \quad ...(16\cdot21)$$

where P_t is the power lost in the tank circuit and is given by,

$$P_t=(I_{p1}Q_o)^2\left[R_1+R_{l1}\cdot\frac{R_2}{R_2+R_{l2}}\right] \quad ...(16\cdot22)$$

The expressions for η_1 and η_2 may be put in the alternative form. Thus

$$\eta_1=\frac{R_{l1}}{R_1+R_{l1}}=1-\frac{R_1}{R_1+R_{l1}}$$

$$=1-\frac{\omega_0L_1/(R_1+R_{l1})}{\omega_0L_1/R_1}=1-\frac{Q_{01l}}{Q_{01}} \quad ...(16\cdot23)$$

where $Q_{01}=\dfrac{\omega_0L_1}{R_1}=Q$ of the primary at resonance without loading

and $Q_{01l}=\dfrac{\omega_0L_1}{R_1+R_{l1}}=Q$ of the primary circuit at resonance including the resistance of the secondary reflected to the primary side

Similarly we may put,

$$\eta_2=1-\frac{Q_{02l}}{Q_{02}} \quad ...(16\cdot24)$$

where $Q_{02}=\frac{\omega_0 L_2}{R_2}$ and $Q_{02l}=\frac{\omega_0 L_2}{R_2+R_{l2}}$

Hence $$\eta=\eta_1\eta_2=\left(1-\frac{Q_{01l}}{Q_{01}}\right)\left(1-\frac{Q_{02l}}{Q_{02}}\right) \quad ..(16\cdot25)$$

For obtaining high circuit transfer efficiency, circuit elements must be arranged to give high values of unloaded Q values namely Q_{01} and Q_{02} and low values of loaded Q values namely Q_{01l} and Q_{02l}. Typically Q_{01l} and Q_{02l} are kept equal to or greater than ten so that the harmonic terms may be suppressed sufficiently. Unloaded Q's depend on (*i*) power output (*ii*) construction of the coil (*iii*) frequency of operation In broadcast band *i.e.* in the frequency range of 500-1600 Kc/s, unloaded Q's are typically 100-200 for low power coils and 400-800 for high power coils.

Significance of low values of loaded Q's. The following advantages result from the use of low values of Q's of loaded coils :

(*i*) *High circuit transfer efficiency* : This follows directly from Eqn. (16·25).

(*ii*) *Broader bandwidth* . If δ is small, and Q_{0l} is not excessively low, relative response from Eqn (16·4) becomes,

$$\frac{\mathbf{Z}(\omega)}{\mathbf{Z}(\omega_0)}=\frac{1}{1+jQ_{0l}\left[\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right]} \quad .(16\cdot26)$$

... lowered, bandwidth increases. It ... 14) that the second harmo- ... But it has been found that even for values of Q_{0l} as low as ten, second harmonic power is extremely small compared with power at resonant frequency

From Eqn. (16·7) we observe that a low value of Q_{0l} for a prescribed value of R_l may be obtained by using large value of ratio L/c. Large value of L/c is achieved when tube and wiring capacitances alone are used in the tuned circuit. High value of inductance L must then be used in shunt to get resonance at the prescribed frequency To get stable operation, however, a small physical condenser is usually used in the tuned circuit. In practice the value of Q_{0l} is chosen as low as possible, consistent with the requirement of the permissible harmonic content.

CLASS B TUNED POWER AMPLIFIER

In class B operation, the grid bias voltage E_{cc} is adjusted to cutoff value so that with zero signal voltage, no plate current flows while with the application of an a.c. signal, plate current flows during the positive half of applied a.c. voltage cycle. Such class B tuned amplifiers are used in Radio Transmitters for power amplification of modulated carrier.

The analysis of this amplifier may be made using one of the following two methods :

(*A*) **Semi-graphical Analysis :** This method is applicable to class *C* amplifiers as well and is applicable over a wide range of operation. Essentially it is a trial and error method and consists in selecting certain operating conditions which yield eventually the desired values of power output, plate dissipation and plate efficiency.

(*B*) **Approximate quantitative analysis :** According to this method it is possible to derive expressions for the power output and plate circuit efficiency *in terms of known and adjustable quantities.* The results of this analysis may, therefore, by applied quickly. This method of analysis is discussed below. The following steps give the analysis used :

Step 1. *Development of simple approximate analytical expression for the characteristics of the amplifier tube :* A study of the tube parameters makes evident the following relation between the plate current i_b; grid voltage e_c and plate voltage e_b of a triode

$$i_b = g_m \left[e_c + \frac{e_b}{\mu} \right] \text{ for } \left(e_c + \frac{e_b}{\mu} \right) \geqslant 0 \qquad \ldots(16{\cdot}27)$$

where i_b=instantaneous total plate current

e_b=instantaneous total plate voltage

and e_c=instantaneous total grid voltage.

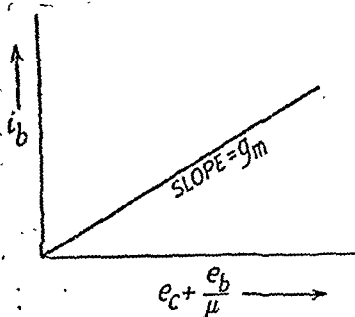


Fig. 16·4. Approximate triode characteristic.

Fig. 16·4 shows a plot of i_b as a function of the control voltage $e_c + \frac{e_b}{\mu}$.

Step. 2. *Estabishment of a relationship between a.c. plate currents and voltages:* Here it is assumed that a high *Q parallel tuned circuit has at* impedance at harmonics of resonant frequency negligible in comparison with the impedance at the resonant frequency.

Let the grid signal voltage be sinusoidal and let the plate circuit be tuned to the applied signal frequency. For class *B* operation, plate current flows for half the periodic time and hence is rich in harmonics. At the resonant frequency and at each harmonic frequency, the voltage developed across the tuned circuit is multiplication of the plate current component and tuned circuit impedance at the pertinent frequency. Since the impedance at harmonics is negligibly small compared with that at resonant frequency particularly in high-*Q* circuits, the output voltage at harmonics is negligible. *Only the signal frequency voltage appears at the output. Further the d.c. resistance of the inductor in the tuned plate load is usually very small and hence the d.c. voltage drop in the tuned circuit is negligibly small.*

Under zero signal condition,

$e_c = E_{cc}$, and $e_b = E_{bb}$, so that

$$e_c + \frac{e_b}{\mu} = E_{cc} + \frac{E_{bb}}{\mu} \quad \ldots(16{\cdot}28)$$

But in class B operation,

$$E_{cc} = E_{cutoff} = -\frac{E_{bb}}{\mu} \quad \ldots(16{\cdot}29)$$

and hence from Equations (16·28) and (16·29), the quantity $\left(e_c + \frac{e_b}{\mu}\right)$ is zero for zero-signal condition. Further from Eqn. (16·27), for zero signal condition, i_b is also zero

With the application of a grid-signal voltage e_{gk} under class B condition,

$$\left(e_c + \frac{e_b}{\mu}\right) = (E_{cc} + e_{gk}) + \left(\frac{E_{bb} + e_p}{\mu}\right)$$

$$= \left(E_{cc} + \frac{E_{bb}}{\mu}\right) + e_{gk} + \frac{e_p}{\mu}$$

$$= e_{gk} + \frac{e_p}{\mu} \quad \ldots(16{\cdot}30)$$

Hence $$i_b = g_m\left[e_{gk} + \frac{e_b}{\mu}\right] \quad \ldots(16{\cdot}31)$$

provided that $\left(e_{gk} + \frac{e_b}{\mu}\right) \geqslant 0$.

Step 3. *Development of a linear equivalent circuit for the fundamental component of plate current and voltage.* The plate voltage e_p is sinusoidal because the tank circuit is tuned to the input sinusoidal signal voltage e_{gk}. Hence the quantity $\left[e_{gk} + \frac{e_p}{\mu}\right]$ is also sinusoidal, irrespective of the phase angles of the two quantities e_{gk} and $\frac{e_p}{\mu}$. Current i_b will then be zero during the ... he positive ... shows the ... at maxi- ... veform of ... nly cosine ... otal plate ... at the origin. Further ... nate at the fundamental ... circuit is a resistance of value R_l. The fundamental frequency ... plate voltage E_{p1} is then given by,

$$\mathbf{E}_{p1} = -\mathbf{I}_{p1} \cdot R_l$$

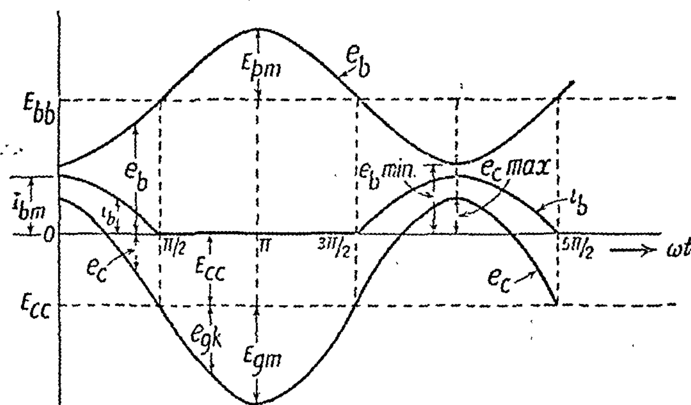


Fig. 16·5. Waveforms of grid voltage e_c, plate current i_b and plate voltage e_b in a class B tuned amplifier.

Harmonic components of plate voltage are negligibly small and hence the subscript 1 may be dropped and $\mathbf{E}_{p1}$ may simply be written as $\mathbf{E}_p$. Thus

$$\mathbf{E}_p = -\mathbf{I}_{p1} \cdot R_l \qquad \text{...(16·32}a\text{)}$$

From Eqn. (16·32a), we see that e_p becomes maximum negative when i_b is maximum positive or when e_{gk} is maximum positive. The e_{gk} is 180 degrees out of phase with the plate voltage e_p. Waveform of e_{gk} is shown in Fig. 16·5.

Let I_{bm} be the maximum value of plate current i_b.

Then $\quad i_b = I_{bm} \cos \omega t$ for positive half cycle of e_{gk} ...(16·33)

and $\quad i_b = 0$ for negative half cycle of e_{gk} ...(16·34)

The average value of plate current i_b is given by,

$$I_{bs} = \frac{1}{2\pi} \int_0^{2\pi} i_b \, d(\omega t)$$

$$= \frac{1}{\pi} \int_0^{2\pi} I_{bm} \cos \omega t \, d(\omega t) \qquad \text{...(16·35)}$$

or

$$I_{bs} = \frac{I_{bm}}{\pi} \qquad \text{...(16·36)}$$

This value of average plate current is the same as that obtained in the case of class B untuned power amplifier.

From Fourier analysis, the value of amplitude of the fundamental component of plate current is given by,

$$I_{p1m} = \frac{1}{\pi} \int_0^{2\pi} i_b \cos \omega t \, d(\omega t)$$

$$= \frac{2}{\pi} \int_0^{\pi/2} I_{bm} \cos^2 \omega t \, d(\omega t) \qquad \text{...(16·37)}$$

or $$I_{p1m} = \frac{I_{bm}}{2} \qquad \text{...(16·38)}$$

The grid signal voltage e_{sk} and a.c. voltage e_p are given by,

$$e_{sk} = E_{sm} \cos \omega t \qquad \text{...(16·39)}$$

$$e_p = -E_{pm} \cos \omega t \qquad \text{...(16·40)}$$

where E_{sm} and E_{pm} are the amplitudes of grid signal voltage and a.c. plate voltage respectively.

Substituting the values of i_b, e_{sk} and e_p from Equations (16·33), (16·39) and (16·40) into Eqn. (16·31) we get,

$$I_{bm} = g_m \left[E_{sm} - \frac{E_{pm}}{\mu} \right]$$

$$= \sqrt{2}\, g_m \left[E_{sk} - \frac{E_p}{\mu} \right] \qquad \text{...(16·41)}$$

But from Eqn. (16·32a),

$$I_{p1} = \frac{E_p}{R_t} \qquad \text{...(16·42)}$$

Substituting the value I_{bm} from Eqn. (16·38) and value of E_p from Eqn. (16·42) into Eqn. (16·41) yields,

$$2I_{p1m} = \sqrt{2}\, g_m \left[E_{sk} - \frac{I_{p1} R_t}{\mu} \right]$$

$$2\sqrt{2}\, I_{p1}\, r_p = \sqrt{2}\, [\mu E_{sk} - I_{p1} R_t]$$

or $$I_{p1} [2r_p + R_t] = \mu E_{sk}$$

or $$I_{p1} = \frac{\mu E_{sk}}{2r_p + R_t} \qquad \text{...(16·43)}$$

Hence the average plate current I_{bs} is given by,

$$I_{bs} = \frac{I_{bm}}{\pi} \text{ from Eqn. (16·36)}$$

$$= \frac{2\, I_{p1m}}{\pi} \text{ from Eqn (16·38)}$$

$$= \frac{2\sqrt{2}\, I_{p1}}{\pi} \text{ since } I_{p1m} = \sqrt{2}\, I_{p1}.$$

From Eqn. (16·43) we get,

$$I_{bs} = \frac{2\sqrt{2}}{\pi} \cdot \frac{\mu E_{sk}}{2r_p + R_t} \qquad \text{...(16·44)}$$

Equations (16·43) and (16·44) express the fundamental [illegible] nent of plate current and the average value of plate current in [illegible] of grid signal voltage E_{sk} and the tube and circuit parameters. [illegible] these currents are proportional to grid-signal voltage [illegible]. Since I_{p1} is proportional to E_{sk} the circuit behaves as a [illegible] and may be represented by the incremental equivalent [illegible] Fig. 16·6. This circuit differs from that of class A [illegible]

in that its internal impedance is $2r_p$ instead of r_p. The equivalent circuit is valid only for fundamental frequency terms.

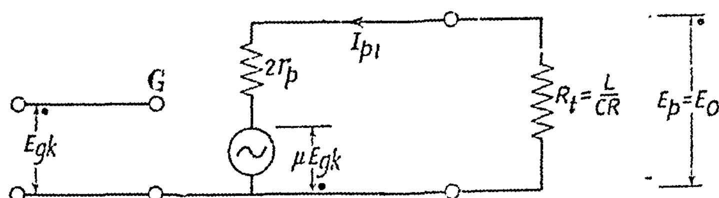


Fig. 16·6. Incremental equivalent circuit of class *B* tuned amplifier.

Step 4. *Use of incremental equivalent circuit to derive expression for voltage gain, power output, plate circuit efficiency and plate dissipation in terms of voltages and circuit parameters.*

From a.c. equivalent circuit of Fig. 16·6,

$$\mathbf{E}_p = \mu \mathbf{E}_{gk} \frac{R_t}{2r_p + R_t} \quad \ldots(16\cdot45)$$

and hence

$$\mathbf{A} = \frac{\mathbf{E}_p}{\mathbf{E}_{gk}} = \frac{\mu R_t}{2r_p + R_t} \quad \ldots(16\cdot46)$$

The d.c. power input to the plate circuit P_{bs} is given by,

$$P_{bs} = E_{bb} \cdot I_{bs} = \frac{2\sqrt{2}}{\pi} E_{bb} I_{p1} \quad \ldots(16\cdot47)$$

The a.c. power output P_{ac} is given by,

$$P_{ac} = I^2_{p1} \cdot R_t \quad \ldots(16\cdot48)$$

The plate dissipation P_p is given by,

$$P_p = P_{bs} - P_{ac} = \frac{2\sqrt{2}}{\pi} E_{bb} \cdot I_{p1} - I^2_{p1} \cdot R_t \ldots(16\cdot49)$$

The plate circuit efficiency η_p is given by,

$$\eta_p = \frac{P_{ac}}{P_{bs}} = \frac{\pi}{2\sqrt{2}} \frac{I_{p1} \cdot R_t}{E_{bb}}$$

$$= \frac{\pi}{2\sqrt{2}} \frac{E_p}{E_{bb}} \quad \ldots(16\cdot50)$$

or

$$\eta_p = \frac{\pi}{4} \cdot \frac{E_{pm}}{E_{bb}} = 78\cdot5 \frac{E_{pm}}{E_{bb}} \% \quad \ldots(16\cdot51)$$

where E_{pm} is the amplitude of fundamental component of the plate voltage.

From Eqn. (16·51), we observe that the class *B* tuned amplifier has the same expression for plate circuit efficiency as class *B* untuned push-pull amplifier. In each case, for a prescribed E_{bb}, the plate circuit efficiency increases linearly with the a.c. plate voltage amplitude E_{pm} at fundamental frequency *i.e.* it increases linearly with grid-signal voltage and in the limit approaches 78·5% as the amplitude E_{pm} approaches the plate supply voltage E_{bb}. In practice, however,

due to various limitations, this full theoretical plate circuit efficiency is not realized in practice.

Step 5. *Determination of operating conditions which utilize the tube best.* As the grid signal voltage amplitude E_{gm} increases, the a c. plate voltage E_p increases and a limiting condition is reached when,

$$e_{c\ max}=e_{b\ min} \quad ...(16\cdot52)$$

If E_{gm} is further increased, $e_{c\ max}$ exceeds $e_{b\ min}$ and grid becomes positive with respect to plate. The equation (16·31) and the *equivalent circuit of Fig 16·6 no longer hold good then* Such a condition is called the overexcited condition and results in grid drawing a larger fraction of cathode current. Further secondary electrons from plate are attracted by the grid. The plate current then reduces with the increase of grid signal voltage. The a.c. power output P_{ac} then no longer increases as the square of E_{gk} and excessive grid driving power is required. With the increase of E_{gk} in this overexcited condition, power amplification tends to decrease and harmonics in the plate current i_b increases in magnitude. Due to these limitations, $e_{c\ max}$ is never made to exceed $e_{b\ min}$. In practice $e_{c\ max}$ is kept less than $\frac{1}{2}\ e_{b\ min}$.

Under the limiting condition of $e_{c\ max}=e_{b\ min}$,

$$E_{cc}+E_{gm}=E_{bb}-E_{pm} \quad ...(16\cdot53)$$

But $\quad E_{cc}=-\dfrac{E_{bb}}{\mu}$ and $E_p=\mu E_{gk}\cdot\dfrac{R_t}{2r_p+R_t}$

from Eqn. (16·45)

Hence from Eqn. (16·53),

$$-\frac{E_{bb}}{\mu}+\sqrt{2}\,E_{gk}=E_{bb}-\sqrt{2}\,\mu E_{gk}\frac{R_t}{2r_p+R_t}$$

or

$$E_{go}=\frac{E_{bb}}{\sqrt{2}}\left(1+\frac{1}{\mu}\right)\frac{2r_p+R_t}{2r_p+(\mu+1)R_t} \quad ...(16\cdot54)$$

where the symbol E_{go} is used instead of E_{gk} to indicate this optimum value of E_{gk}.

For a prescribed plate supply voltage E_{bb} and prescribed value of resonant circuit resistance R_t, the optimum power output is obtained when E_{gk} is adjusted to the value E_{go} given by Eqn. (16·54).

For this optimum value of E_{go}, from Eqn. (16·43) we get,

$$I_{p1}=\frac{1}{\sqrt{2}}\cdot\frac{(\mu+1)E_{bb}}{2r_p+(\mu+1)R_t} \quad ...(16\cdot55)$$

and from Eqn. (16·44), average value of plate current is given by,

$$I_{bs}=\frac{2}{\pi}\cdot\frac{(\mu+1)E_{bb}}{2r_p+(\mu+1)R_t} \quad ...(16\cdot56)$$

Substituting the value of I_{bs} and I_{p1} from Equations (16·56) and (16·55) into Eqns. (16·47) through (16·50) we get for optimum operating condition,

$$P_{bs}=E_{bb}\,.\,I_{bs}=\frac{2}{\pi}\,.\,\frac{(\mu+1)E^2_{bb}}{2r_p+(\mu+1)R_t} \quad ...(16\cdot57)$$

$$P_{ac}=I^2_{p1}\,.\,R_t=\frac{1}{2}\,.\,\frac{(\mu+1)^2E^2_{bb}\,R_t}{[2r_p+(\mu+1)R_t]^2} \quad ...(16\cdot58)$$

and

$$P_p=P_{bs}-P_{ac}=\frac{2}{\pi}\,.\,\frac{(\mu+1)E^2_{bb}}{[2r_p+(\mu+1)R_t]}-\frac{1}{2}\frac{(\mu+1)^2E^2_{bb}R_t}{[2r_p+(\mu+1)R_t]^2} \quad ...(16\cdot59)$$

For [a] given tube, parameters μ, g_m and r_p are fixed. E_{bb} is limited by the maximum value of plate voltage e_b. $e_{b\,max}$ under optimum condition is approximately equal to 2 E_{bb}. $e_{b\,max}$ is limited by the tube considerations such as insulation and cathode bombardment. The value of E_{bb} is then one-half of this limiting value of $e_{b\,max}$. Again plate dissipation P_p is limited by the construction of the tube. Thus the quantities μ, r_p, P_p and E_{bb} are fixed for the given tube. For the optimum conditions, we may, therefore, express R_t in terms of these quantities μ, r_p, P_p and E_{bb}. This expression may be obtained by rearranging Eqn. (16·59). We then get the quadratic equation,

$$R_t^2+\left[\frac{4\,r_p}{\mu+1}-\frac{E^2_{bb}}{P_p}\left(\frac{2}{\pi}-\frac{1}{2}\right)\right]R_t+\left[\frac{4\,r_p^2}{(\mu+1)^2}-\frac{E^2_{bb}}{P_p}\,.\,\frac{4\,r_p}{\pi(\mu+1)}\right]=0 \quad ...\ (16\cdot60)$$

Solution of Eqn. (16·60) gives the value of R_t which under the condition $e_{c\,max}=e_{b\,min}$ provides maximum power output for prescribed values of μ, r_p, E_{bb} and P_p.

Step 6. *Use of optimum value of resonant circuit resistance R_t to obtain expressions for I_{p1}, I_{bs}, P_{ac}, P_{bs}, P_p and plate circuit efficiency η_p.* Optimum value of R_t as determined from Eqn. (16·60) may be substituted in Eqn. (16·55) to yield the corresponding value of I_{p1}. This value of I_{p1} may then be substituted in Eqns. (16·47) through (16·50) to get values of P_{bs}, P_{ac}, P_p and η_p. Thus the substitution of the value of I_{p1} from Eqn. (16·55) into Eqn. (16·50) gives the value of plate circuit efficiency as,

$$\eta_p=\frac{\pi}{2\sqrt{2}}\,.\,\frac{R_t}{E_{bb}}\,.\,\frac{1}{\sqrt{2}}\,.\,\frac{(\mu+1)\,E_{bb}}{2r_p+(\mu+1)\,R_t}$$

$$=\frac{\pi}{4}\frac{R_t}{R_t+\dfrac{2r_p}{\mu+1}}$$

$$=78\cdot5\,\frac{R_t}{R_t+\dfrac{2r_p}{\mu+1}}\,\% \quad ...\ (16\cdot61)$$

From Eqn. (16 61) we conclude that for optimum operating condition of $e_{c\ max}=e_{b\ min}$ the plate circuit efficiency depends only on R_l and tube parameters. The plate circuit efficiency may be made to approach the theoretical maximum value of 78.5% by making $R_l \gg \frac{2r_p}{\mu+1}$. But then the output power is small. Hence in practice for best utilization of tube, R_l is selected in accordance with Eqn. (16 61) rather than for maximum plate circuit efficiency. The plate circuit efficiency then is usually much lower than the maximum value of 78.5 per cent.

TUNED CLASS C AMPLIFIERS

A class C amplifier is an amplifier operated with negative grid bias far beyond the cutoff value so that with zero signal, plate current is zero. With the application of grid signal voltage of reasonable magnitude, plate current for a period appreciably less than half the a.c. cycle. Since the plate current flows in the form of pulses, harmonic terms of appreciable magnitude are present in the plate current. To eliminate these harmonic terms from the plate voltage, the plate load is invariably a tuned circuit. Such class *C amplifiers have relatively high plate circuit efficiency and are used* primarily for the production of large amount of power at a single radio frequency as is necessary in radio transmitters. Fig. 16·7 shows the waveforms of grid voltage, plate voltage and plate current in a tuned class C amplifier assuming sinusoidal grid signal voltage.

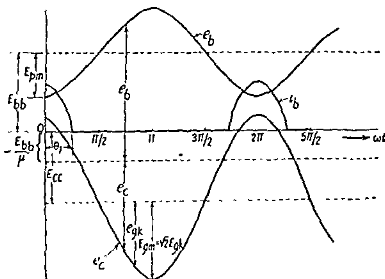


Fig. 16·7 Waveforms of grid voltage, plate voltage and plate current in a class C amplifier

At the resonant frequency, the tuned circuit offers a pure resistance load $R_l = \frac{L}{CR}$ and hence the a.c. plate voltage E_p is 180 degrees out of phase with the grid voltage.

voltage is approximately equal to $\frac{E_{bb}}{\mu}$. For any grid bias voltage greater than $\frac{E_{bb}}{\mu}$, the operation is class C. The plate current then flows in the form of pulses for intervals appreciably less than half the cycle. The plate current i_b contains all harmonics of the fundamental but since the high-Q resonant circuit is tuned to the fundamental, the response of the tuned circuit at all harmonics is negligibly small. The a.c. plate voltage at harmonics of the resonant frequency is, therefore, negligibly small. Such an amplifier has an extremely large plate circuit efficiency since the d.c. plate current is small compared with the a.c. plate current at the fundamental frequency. In practice efficiency of the order of 70–80 % is obtainable with suitable design. But such class C amplifiers cannot be used for amplification of amplitude modulated signal since in an amplitude modulated voltage, the amplitude may vary considerably and may become very small during the modulation minimum. Such a small amplitude may be insufficient to drive current through the class C amplifier tube. The output voltage waveform then differs from the input signal waveform. Hence in R.F. transmitters, class C amplifiers with their high plate circuit efficiency are used only for the purpose of power amplification of the R.F. carrier voltage before modulation has been done. The amplitude modulated carrier is then amplified in R.F. linear amplifiers which are R.F. class B amplifiers biased to what is called the "projected cutoff bias".

ANALYSIS OF CLASS C AMPLIFIERS

A few approximate methods of analysis are available and are used whenever the data of the tube characteristics are incomplete or when the results are desired with the least possible computations. However for better results, analysis is made in accordance with one of the following two methods : (A) Analysis assuming linear tube characteristics and (B) semigraphical analysis.

(A) **Analysis assuming linear tube characteristics.** This method is an extension of the method given in the last article for class B amplifier. This method, however, becomes quite involved since E_{cc} is no longer a single value which provides zero plate current for zero excitation. It becomes one of the circuit parameters. Further, the assumption of linear tube characteristics is not fully justified in class C amplifiers. Again it is required to make use of design charts. Thus this method has the disadvantages of being (i) complicated and (ii) approximate but has the advantage that it gives explicit solution for the optimum operating conditions and hence may be used in design directly. Detailed study of only the semi-graphical method is taken up here.

(B) **Semigraphical Analysis applicable to class B and C amplifiers.** This analysis makes use of the actual static charac-

teristics of the tube. The ... waveform of the plate ... the average component ... component of plate current I_{p1}. These values are subsequently used to calculate power output, plate circuit efficiency etc.

In the case of tuned class C amplifiers, the grid signal voltage E_{gL} and plate voltage E_p are both sinusoidal while the plate current i_b is not sinusoidal. Hence the path of operating point is not linear on either the mutual characteristics (i_b-e_c characteristics) or plate characteristics (i_b-e_c characteristics). But since both the instantaneous plate voltage e_p and the instantaneous grid voltage e_{gk} are sinusoidal, have the same frequency and have phase difference of 180 degrees, one plotted against the other on a rectangular coordinate graph gives a straight line. The path of operation on the e_c-e_b or constant current characteristics is then a straight line.

One-half of the linear path of operation in the constant-current curves is shown in Fig. 16·8. Grid voltage waveform is shown as

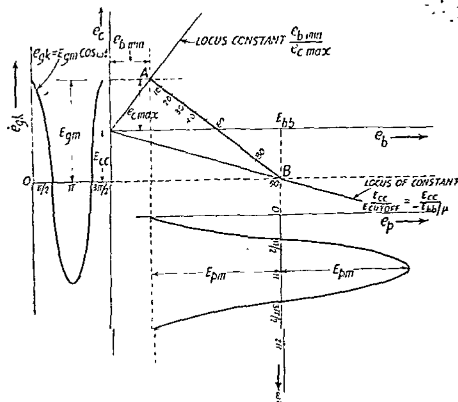


Fig. 16·8. Path of operation on the e_c-e_b characteristic curves in class B and C amplifiers.

sinusoidal variation about the steady value E_{cc}. Similarly plate voltage curve is another sinusoidal variation about the steady ...

E_{bb}. At ωt equal to zero, grid voltage is maximum and plate voltage e_b is minimum. These voltages $e_{c\ max}$ and $e_{b\ min}$ determine the point A on the path of operation as shown in Fig. 16·8. At angle $\omega t=\frac{\pi}{2}$ radians, $e_c=E_{cc}$ and $e_b=E_{bb}$ and hence voltages E_{cc} and E_{bb} determine the point B on the linear path of operation. A straight line is drawn passing through the origin and point B. This line is the locus of constant ratio $E_{cc}/E_{cutoff}=\frac{E_{cc}}{-E_{bb}/\mu}$. Another straight line is drawn passing through the origin and point A. This line is the locus of constant ratio $e_{b\ min}/e_{c\ max}$. The following eight variables present themselves : E_{bb}, E_{cc}, E_p, E_{gk}, $e_{b\ min}/e_{c\ max}$, E_{cc}/E_{cutoff}, $e_{c\ max}$ and $e_{b\ min}$. A study of the geometry of the figure reveals that the coordinates of points A and B are determined by any two pairs of independent variables out of the total of eight variables listed above.

The path of operation AB pertaining to the first and fourth quarters of the applied grid voltage is shown in Fig. 16·8. During the second and third quarter cycles, the path of operation is nothing but further projection of AB through B and is of length equal to AB. But during these second and third quarters, e_{gk} is negative and the plate current i_b is zero for class B and class C operations. This portion of the path of operation is, therefore, of little consequence and omitted. At any instant, the distance along the linear path of operation between the operating P and point B varies as the cosine of the phase angle of the applied signal voltage.

Fig. 16·9 shows the constant-current characteristics for a tube. The path of operation of Fig. 16·8 is also shown superposed on these constant current curves. Then the plate current and grid current at different instants in the applied grid voltage cycle may be found by interpolation between these curves. Obviously maximum plate current $i_{b\ max}$ is obtained at the point P_1 corresponding to ωt equal to zero. The value of $i_{b\ max}$, therefore, constitutes another coordinate for determining the terminus A on the path of operation. This quantity $i_{b\ max}$ may, therefore, be added to the previous list of eight variables.

The points A and B in Fig. 16·9 are selected from the following sets of values :

$$A : i_{b\ max}=250\ mA,\quad \frac{e_{bmin}}{e_{cmax}}=1$$

$$B : \quad E_{bb}=400 \text{ volts},\quad \frac{E_{cc}}{E_{cutoff}}=2$$

Table 16–I shows systematically the amplifier performance as obtained from Fig. 16·9. The nine quantities given at the top of the Table are determined by the geometry of the figure when any four of the nine variables are specified. Next, the length l of the line AB is measured and recorded in Table 16-I. The half cycle

of operation is divided into n equal parts. Let θ_k be the v
of $\frac{\pi k}{n}$ expressed in degrees, k being an integer. θ_k is entere

Fig. 16·9. Path of operation superposed on the constant current characteristics.

the second row. $\cos\theta_k$ is entered in the third row. Typically n is chosen to be 18 so that $\theta_k = 10k$ degrees. As the value of n is increased, say to 24 or 36, the accuracy is correspondingly increased. Fourth row gives $l\cos\theta_k$. These distances $l\cos\theta_k$ are marked along the line AB from terminus B. This process determines the points on the line AB that correspond to the particular angle θ_k in the first quarter cycle. The values of plate current $i_b(\theta_k)$ and the grid current $i_c(\theta_k)$ are obtained by interpolation between the corresponding constant-current curves. These values are entered in rows 5 and 6 of the Table. All the necessary informations from the diagram in Fig. 16·9 have now been completely obtained.

The values of plate current $i_b(\theta_k)$ and grid current $i_c(\theta_k)$ as obtained from Table 16-I are plotted against the angle θ_k as known in Fig. 16·10. The plate current i_b becomes zero when θ_k equals the cutoff angle θ_1 and is shown in Fig. 16·7 and Fig 16 10. This angle θ_1 is entered in the last column in Table 16-I. Let cutoff bias line cross the path of operation AB in point C. Then $\cos\theta_1 = \frac{BC}{BA}$ and hence cutoff angle θ_1 may be obtained by filling in the entries in the last column corresponding to rows 4, 3 and 2 in sequence. Obviously from Fig. 16·7, angle θ_1 is equal to one-half of the total angle of conduction in a cycle of the applied signal voltage. The waveform of grid current i_c may be obtained in a similar manner.

TABLE 16–I

Analysis of class B and class C amplifiers.

(i) E_{bb}...... (ii) E_{cc}...... (iii) E_p......

(iv) E_{gk}...... (v) $\dfrac{e_{b\ min}}{e_{c\ max}}$...... (vi) $e_{c\ max}$......

(vii) $e_{b\ min}$...... (viii) $\dfrac{E_{cc}}{E_{cutoff}}$...... (ix) $i_{b\ max}$......

l=length of line AB=10 cm. (say), $\eta=18$,
k=an integer, θ_1=angle of cutoff.

1	k	0	1	2	3	4	5	6	7	8	9	
2	θ_k	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	θ_1
3	$\cos\theta_k$	1·0	0·985	0·940	0·866	0·766	0·643	0·500	0·342	0·174	0·000	
4	$l\cos\theta_k$ in cm.	1·0	9·85	9·40	8·66	7·66	6·43	5·00	3·42	1·74	0·000	
5	$i_b\ (\theta_k)$											
6	$i_c\ (\theta_k)$											
7	$i_b\ (\theta_k).\cos\theta_k$											
8	$e_g\ (\theta_k)$											
9	$e_c\ (\theta_k)=E_{cc}+e_g(\theta_k)$											
10	$i_c\ (\theta_k).e_c\ (\theta_k)$											

The necessity of plotting the plate and grid currents may be avoided by employing the alternative method of analysis consisting in finding their Fourier components. This method of analysis is given hereunder. Thus the plate current i_b may be represented by the following Fourier series :—

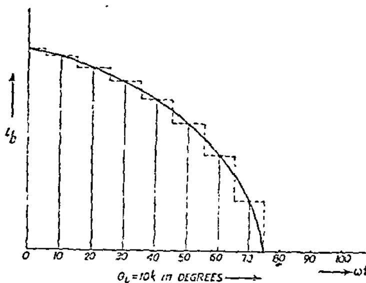


Fig. 16·10. Waveform of plate current i_b obtained from Fig. 16·9 (shown by solid line) and its approximate representation (shown by dotted line)

$$i_b = I_{bs} + [I_{p1m} \cos \omega t - I_{p2m} \cos 2\omega t - \quad ..\ ] \quad . \quad (16\cdot62)$$

This series contains only the cosine terms since the waveform of i_b is an even function. The average value I_{bs} of the plate current i_b is given by,

$$I_{bs} = \frac{1}{2\pi} \int_0^{2\pi} i_b \, d(\omega t) \quad . \quad (16\cdot63)$$

But the plate current wave is symmetrical about the i_b-axis at the origin, so that, we may write,

$$I_{bs} = \frac{1}{\pi} \int_0^{\pi} i_b \, d(\omega t) \quad (16\cdot64)$$

The integral in Eqn (16·64) is nothing but the area under the curve of Fig. 16·10. As a first approximation, the area under the curve may be equated to the area under the dotted step-function curve, which may be obtained by summing up the areas under the individual steps.

The height of the first step is $i_b(0)$ and its width is $\frac{1}{2}\left(\frac{\pi}{n}\right)$ radians so that its area is $\frac{\pi}{2n} i_b(0)$. Similarly the height of second, third etc. steps are $i_b(\theta_k)$ and the width of each step except the last is $\frac{\pi}{n}$ radians, so that their areas are $\frac{\pi}{n} i_b(\theta_k)$ where k is a positive integer. The width of the last step may vary but not much error

is caused by approximating it to $\frac{\pi}{n}$ since the area of this step is quite small.

Hence I_{bs} is approximately given by,

$$I_{bs}=\frac{1}{\pi}\left[\frac{\pi}{2n}i_b(0)+\sum_{k=1,2,3...}\frac{\pi}{n}i_b(\theta_k)\right] \qquad ...(16\cdot65)$$

or

$$I_{bs}=\frac{1}{n}\left[\frac{i_b(0)}{2}+\sum_{k=1,2,3...}i_b(\theta_k)\right] \qquad ...(16\cdot66)$$

Values of $i_b(0)$ and $i_b(\theta_k)$ may be found from row 5 of Table 16-I and hence average values I_{bs} of plate current may be calculated.

The average value of grid current may similarly be found from the data in row 6 of Table 16-1.

The amplitude of the fundamental component is given by,

$$I_{p1m}=\frac{1}{\pi}\int_0^{2\pi}i_b\,(\omega t)\cos\omega t\;d\,(\omega t) \qquad ...(16\cdot67)$$

But $i_b\,(\omega t)$ as well as $\cos\,(\omega t)$ are symmetrical about the current axis at the origin and hence Eqn. (16·67) may be put as,

$$I_{p1m}=\frac{2}{\pi}\int_0^{\pi}i_b\,(\omega t)\cos\,(\omega t)\;d(\omega t) \qquad ...(16\cdot68)$$

Approximate method of summation may be applied to the integral in Eqn. (16·68) as well, so that,

$$I_{p1m}=\frac{2}{\pi}\left[\frac{\pi}{2n}i_b\,(0)\cos\,(0)+\sum_{k=1,2,3...}\frac{\pi}{n}i_b\,(\theta_k)\cos\,(\theta_k)\right] \qquad ...(16\cdot69)$$

or

$$I_{p1m}=\frac{2}{n}\left[\frac{1}{2}i_b\,(0)\cos\,(0)+\sum_{k=1,2,3...}i_b\,(\theta_k)\cos\,(\theta_k)\right] \qquad ...(15\cdot70)$$

The terms of Eqn. (16·70) may be obtained from row 7 of Table 16-I. Row 7 gives the product of $i_b(\theta_k)$ of row 3 and $\cos\,(\theta_k)$ of row 5.

The average grid driving power is given by,

$$P_{cav}=\frac{1}{2\pi}\int_0^{2\pi}i_c(\omega t)\;e_c(\omega t)\;d(\omega t) \qquad ...(16\cdot71)$$

Since both $i_c(\omega t)$ and $e_c(\omega t)$ are symmetrical about the ordinate axis at origin, P_{cav} may be put as,

$$P_{cav}=\frac{1}{\pi}\int_0^{\pi} i_c(\omega t)\, e_c(\omega t)\, d(\omega t) \qquad ...(16\cdot72)$$

The values of $i_c(\omega t)$ are obtained from row 6 in the Table. The values of $e_c(\omega t)$ may be found from row 9 of the table. Alternatively $e_c(\omega t)$ may be obtained by adding to the bias E_{cc} to the voltage $e_g(\omega t)$ obtained from row 8. The values of voltage $e_g(\omega t)$ in row 8 are obtained by multiplying the value of cos (ωt) in row 3 to the amplitude of grid signal voltage. The grid-driving power is obtained by the usual summation and may be expressed as :

$$P_{cav}=\frac{1}{\pi}\left[\frac{\pi}{n}\frac{i_c(0)\, e_c(0)}{2} + \sum_{k=1,2,3}\frac{\pi}{n} i_c(0_k)\, e_c(0_k)\right] \qquad ...(16\cdot73)$$

or

$$P_{cav}=\frac{1}{n}\left[\frac{i_c(0)\, e_c(0)}{2} + \sum_{k=1,2,3} i_c(0_k)\, e_c(0_k)\right] \qquad ...(16\cdot74)$$

The row 10 in the Table gives the product of the quantities in row 6 and row 9 and summation may be performed to evaluate P_{cav} from Eqn. (16·74).

The a.c. power output to the tuned load is given by,

$$P_{ac}=E_p . I_{p1} \qquad ...(16\cdot75)$$

The ratio of E_p to I_{p1} is the input resistance of the tuned load and is selected to satisfy the assumed operating conditions.

The d.c. power input to the plate circuit from the plate supply source is given by,

$$P_{bs}=E_{bb} . I_{bs} \qquad (16\cdot76)$$

Hence the plate circuit efficiency of the amplifier is obtained from the expression,

$$\eta_p=\frac{P_{ac}}{P_{bs}}=\frac{E_p . I_{p1}}{E_{bb} . I_{bs}} \qquad (16\cdot77)$$

The plate dissipation P_p is the difference of the d.c. power from the plate supply source and the a.c. power output and hence is given by,

$$P_p=P_{bs}-P_{ac}=E_{bb}\, I_{bs}-E_p . I_{p1} \qquad ..(1($$

CHAPTER XVII

OSCILLATORS

Oscillator may be defined as an electronic device for producing independently alternating voltage or current from a d.c. voltage source.

In so far as the function of converting d.c. power into alternating power is concerned, oscillator function similar to a vacuum tube amplifier. In either case, d.c. power is supplied from the plate supply source and there results an alternating power across the output terminals. In amplifiers, however, the frequency, waveform and magnitude of the generated alternating power is governed by the controlling a.c. voltage from an external source applied to the control grid of the amplifier tube. In oscillators on the other hand, frequency, waveshape and magnitude of the a.c. voltage generated depends only on the tube and associated circuit and no external controlling voltage is required.

Thus an oscillator essentially converts d.c. power into a.c. power. This function is inverse of the function of a rectifier which converts a.c. power into d.c. power. Because of this function of converting d.c. power into a.c. power, an oscillator may rightly be referred to as an inverter.

The function of generating alternating voltage or current may be performed by a number of other devices. A device commonly used for such a purpose is an alternator which is an electromechanical device. But an oscillator has a number of advantages over alternators. These are :—

(*i*) Generation of alternating voltages or currents over a very wide frequency range.

(*ii*) Freedom from harmonics in one type of oscillators called the sinusoidal oscillators and richness of harmonics in other type of oscillators called the elaxation oscillator.

(*iii*) Constancy of frequency with time.

(*iv*) Simple way in which the frequency of oscillation may be varied.

(*v*) Portability and low cost.

On account of these advantages, oscillators are used in all cases except where large amount of power at a low and constant frequency is required such as a.c. mains power. Oscillators using vacuum tubes are used for a variety of purposes such as (*i*) in Radio Transmitter and Receivers (*ii*) in Television (*iii*) in Radar (*iv*) for measurement purposes (*v*) for high frequency induction heating etc.

Requirements of an Oscillator

Every oscillator consists of the following constituent parts : (*i*) Internal amplifier (*ii*) Feedback network or Negative resistance property and (*iii*) Amplitude limiting device.

(*i*) Internal amplifier. An oscillator is essentially an amplifier with infinite voltage gain. An amplifier forms, therefore, an essential part of any amplifier. The voltage gain of this amplifier is made infinite by proper feedback or by inclusion of a suitable negative resistance device

(*ii*) Feedback device or negative resistance property. For understanding the negative resistance property clearly, let us consider an L-R-C series circuit. Let a current i flow through this circuit. Then,

$$L\frac{di}{dt}+Ri+\frac{1}{C}\int i\,dt=0 \quad \ldots (17\cdot1)$$

or

$$L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{c}=0 \quad \ldots (17\cdot2)$$

or

$$i=\varepsilon^{-\frac{Rt}{2L}} \quad \ldots (17\cdot3)$$

where A is a constant and ε is the base of natural logarithm.

If the components L, R and C pertain to the complete oscillator, then for oscillations to be sustained, *i.e.* for the device to generate power the total resistance R in the circuit should be zero or negative. A positive resistance is a lossy element, *i.e.* results in dissipation of power. A negative resistance must therefore result in generation of power. In Eqn (17·3), current i will therefore, build up with time if R is negative and will die down with time if R is positive. If R is zero, amplitude of current i will remain constant

In a physical circuit, R is a positive quantity and hence a negative resistance is required to neutralize it. An oscillator in which a negative resistance is produced by the amplifier tube or by other means, to neutralize the positive resistance of the oscillating circuit, is known as a negative resistance oscillator

Let us next consider an oscillator in which positive feedback has been provided. The voltage gain $\mathbf{A}_\beta$ of this amplifier is given by,

$$\mathbf{A}_\beta=\frac{\mathbf{A}}{1-\mathbf{A}\beta} \quad \ldots (17\cdot4)$$

where $\mathbf{A}$ is the complex voltage gain of the internal amplifier and β is the complex feedback ratio.

In order that this amplifier may work as an oscillator, voltage gain must be infinite, *i.e.*

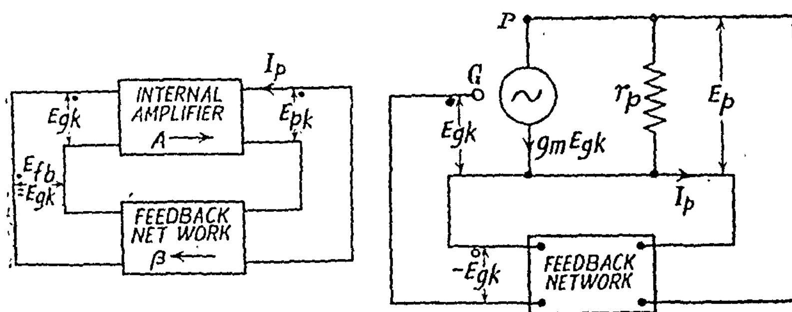
$$1-\mathbf{A}\beta=0$$

or

$$\mathbf{A}\beta=1+j0$$

Such an oscillator is called a "Feedback Oscillator."

Fig. 17·1 (a) shows such a feedback oscillator in Block diagram form while Fig. 17·1 (b) shows the corresponding equivalent circuit.



(a) Block diagram (b) Incremental Equivalent Circuit

Fig. 17.1. Block diagram and incremental equivalent circuit of a feedback oscillator.

If we look from the plate and cathode of the amplifier tube towards the β network, then β network appears as a load. Let its impedance be **Z**.

Then
$$\mathbf{A}=\frac{-\mu\mathbf{Z}}{r_p+\mathbf{Z}} \quad \ldots (17{\cdot}7)$$

and hence from Eqn. (17·6),

$$\beta=\frac{1}{\mathbf{A}}=-\frac{r_p+\mathbf{Z}}{\mu\mathbf{Z}} \quad \ldots (17{\cdot}8)$$

or
$$\beta=-\left[\frac{1}{\mu}+\frac{1}{g_m\mathbf{Z}}\right] \quad \ldots (17{\cdot}9)$$

Eqn. (17·9) is called the Barkhausen criterion for the maintenance of oscillations. Eqn. (17·9) involves four quantities namely β, μ, g_m and **Z**. Out of these, g_m and μ depend upon the tube and its condition of operation. Quantities β and **Z** depend upon the β circuit only. Hence in order to obtain oscillations, these four quantities are so matched that Eqn. (17·9) is satisfied.

An alternative expression for the condition for sustained oscillations involving the transfer impedance of the coupling network is possible. The transfer impedance denoted by $\mathbf{Z}_T$ is defined as the ratio of the output potential to the input current. With reference to Fig. 17·1 (b),

$$\mathbf{Z}_T=\frac{-\mathbf{E}_{gk}}{\mathbf{I}_p}$$

For oscillations to take place,

$$\left(g_m\,\mathbf{E}_{gk}+\frac{\mathbf{E}_p}{r_p}\right)\mathbf{Z}_T=-\mathbf{E}_{gk} \quad \ldots (17{\cdot}10)$$

or
$$g_m\,\mathbf{Z}_T\left(\mathbf{E}g_m+\frac{\mathbf{E}_p}{\mu}\right)=-\mathbf{E}_{gk}$$

But $E_o = AE_{i1}$ where A is the voltage gain of the internal amplifier.

Hence $\quad g_m Z_T E_{i1}\left[1+\frac{A}{\mu}\right]=-E_{i1}$

or $\quad -g_m Z_T\left(1+\frac{A}{\mu}\right)=1 \qquad$ ...(17·10a)

or $\quad g_m=\frac{-1}{Z_T\left(1+\frac{A}{\mu}\right)} \qquad$ ...(17·10b)

Equations (17·10a) and (17·10b) are the alternative forms of Barkhausen criterion for sustained oscillations.

Since β and A are both in general complex quantities, Eqn. (17·8) may be split up into two parts :

Thus $\quad |\beta| \angle\theta=\frac{1}{|A| \angle\phi}=\frac{1}{|A|}\angle-\phi \qquad$ ...(17·11)

Hence $\quad |\beta|=\frac{1}{|A|} \qquad$ ...(17·12a)

and $\quad \angle\theta=\angle-\phi \qquad$ ...(17·12b)

Eqn. (17·12a) denotes the magnitude equilibrium while Eqn. (17·12b) denotes the phase angle equilibrium.

In most of the sine wave oscillators, feedback network is a tuned circuit. Its impedance Z at the frequency of oscillation is a pure resistance equal to R say. Then β also becomes a real number and phase difference is simply π radians. The tuned circuit then serves two functions : (i) providing the feedback network and load impedance and (ii) providing the frequency determining network. The frequency of resonance is determined by the inductive and capacitive elements of the tuned circuit.

Amplitude Limiting Device. As soon as the oscillator is switched on, oscillations build up gradually and the amplitude increases. During this entire process overall resistance of the oscillator is negative. When the amplitude of oscillation has built up considerably, the transconductance g_m and hence the voltage gain A of the internal amplifier get reduced so that the $A\beta$ becomes exactly unity. In terms of negative resistance property of oscillators, we may say that the amplitude of oscillation builds up until such time that the net resistance of the oscillator circuit becomes zero i.e. the negative resistance of the tube has reduced to a value as to just equal the positive resistance of the associated load circuit. This forms the automatic amplitude limiting device. That the gain A or transconductance g_m of an electron tube reduces with the increase of amplitude of the signal is evident from the study of the mutual characteristic of an electron tube.

Classification of Oscillators

Oscillators may be classified in two ways :

(I) Classification depending upon the method of producing Oscillations. According to this method of classification, Oscillators may be classified as (*i*) negative resistance oscillators and (*ii*) feedback oscillators.

A negative resistance oscillator is one in which a negative resistance is inserted by the amplifier tube or otherwise, to neutralise the positive resistance of the oscillating circuit. A typical example is the dynatron oscillator which uses a tetrode operated over the region of plate characteristic offering negative plate resistance, *i.e.* producing reduction in plate current with increase of plate voltage.

In a feedback oscillator, on the other hand, a portion of the output voltage is fedback to the input of internal amplifier to provide positive feedback. The constants are so chosen that the Barkhausen criterion of $A\beta = 1$ is satisfied. The gain of the amplifier is then infinite *i.e.* the amplifier produces output without any input. The amplifier thus becomes an oscillator.

on the portion of the tube characteristic used. Factors **Z** and β are external to the tube and are independent of the operating conditions of the tube.

A study of the mutual characteristics reveals that transconductance g_m *i.e.* the slope of i_b-e_c curve is not constant over wide range of grid voltage variations. This variable nature of g_m appears to be a very undesirable property but in fact this very variable nature of g_m is essential for successful operation of an oscillator. In any oscillator, for given values of μ, **Z** and β, oscillations build up until g_m reaches such a value as to satisfy Eqn. (17·9) under which condition constant amplitude oscillations result. If g_m does not have a ... tions will ... given by ... r sustain- ... up.

Fig. 17·2 shows a transfer characteristic which is considerably nonlinear. A cycle of feedback voltage is also shown. P_1 and P_2

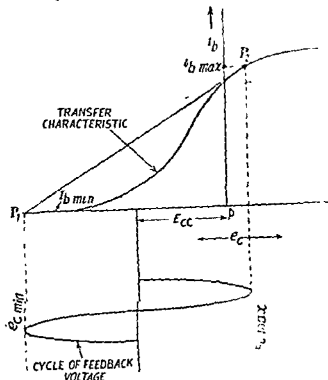


Fig. 17·2. Determination of average transconductance $\bar{g}_m$.

are the extreme points on the transfer characteristic pertaining to the input voltage at the grid. Let $\bar{g}_m$ indicate the slope of the straight line joining points P_1 and P_2. Then $\bar{g}_m$ is the average transconductance over the region of operation.

Then $$\bar{g}_m = \frac{i_{b\ max} - i_{b\ min}}{e_{c\ max} - e_{c\ min}} \qquad \ldots(17\cdot13)$$

Then the modified form of the Barkhausen criterion becomes,

$$\left.\begin{array}{l} -\dfrac{1}{\beta}\left[\dfrac{1}{\bar{g}_m \mathbf{Z}} + \dfrac{1}{\mu}\right] \\ -\ \bar{g}_m \mathbf{Z}_T \left[1 + \dfrac{\mathbf{A}}{\mu}\right] \end{array}\right\} \begin{array}{l} <1 \text{ for decaying oscillations} \\ =1 \text{ for sustained oscillations} \\ >1 \text{ for growing oscillations.} \end{array} \qquad \ldots(17\cdot14)$$

It is possible to obtain an approximate relation between the transconductance g_m and the average transconductance $\bar{g}_m$ as defined above. Assumption made is that the transfer characteristic is linear as shown in Fig. 17·3. Class C condition of operation is illustrated.

Then $$\bar{g}_m = \frac{i_{b\ max}}{2\ E_{gm}} \qquad \ldots(17\cdot15)$$

whereas $$g_m = \frac{i_{b\ max}}{e_{c\ max} - E_{co}} \qquad \ldots(17\cdot16)$$

where E_{co} is the cutoff grid voltage.

Hence $$\frac{\bar{g}_m}{g_m} = \frac{e_{c\ max} - E_{co}}{2\ E_{gm}} \qquad \ldots(17\cdot17)$$

Fig. 17·3. Idealized linear transfer characteristic of oscillator tube used for determination of relation between g_m and $\bar{g}_m$.

Let θ_c be the angle of plate current flow. Then expression (17·17) may be written in terms of θ_c. To do so we first write expression for plate current i_b as below,

$$i_b = g_m\,[-E_{co}+e_{c\,max}-E_{co}+E_{gm}\cos\omega t] \text{ for } i_b \leqslant 0 \quad ...(17{\cdot}18)$$

But i_b becomes zero at

$$\omega t = \frac{\theta_c}{2}.$$

Hence, $$-E_{co}+e_{c\,max}=E_{gm}\left[1-\cos\frac{\theta_c}{2}\right] \quad ...(17{\cdot}19)$$

or $$\frac{-E_{co}+e_{c\,max}}{2\,E_{gm}}=\sin^2\frac{\theta_c}{4} \quad ...(17{\cdot}20)$$

Combining Eqn. (17·20) and (17·17) we get,

$$\frac{\bar{g}_m}{g_m}=\sin^2\frac{\theta_c}{4} \quad ...(17{\cdot}21)$$

Thus $\bar{g}_m$ is proportional to g_m.

Amplitude and Frequency Relations. In Eqn. (17·9) quantities β and **Z** are both complex quantities. For oscillations to be sustained, real and imaginary parts on the left hand side of Eqn. (17·9) must equal respectively real and imaginary parts on the right hand side. Thus Eqn. (17·9) gets spitted into two equations. These two equations then give, for any particular oscillator circuit, requirements of amplitude and frequency.

Fixed-bias operation of an oscillator. Let us assume that heavy fixed bias beyond the cutoff value is applied as shown in Fig.

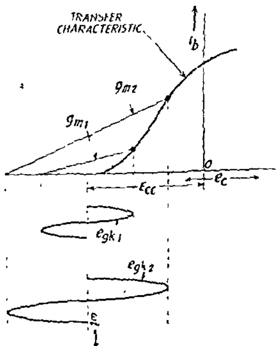
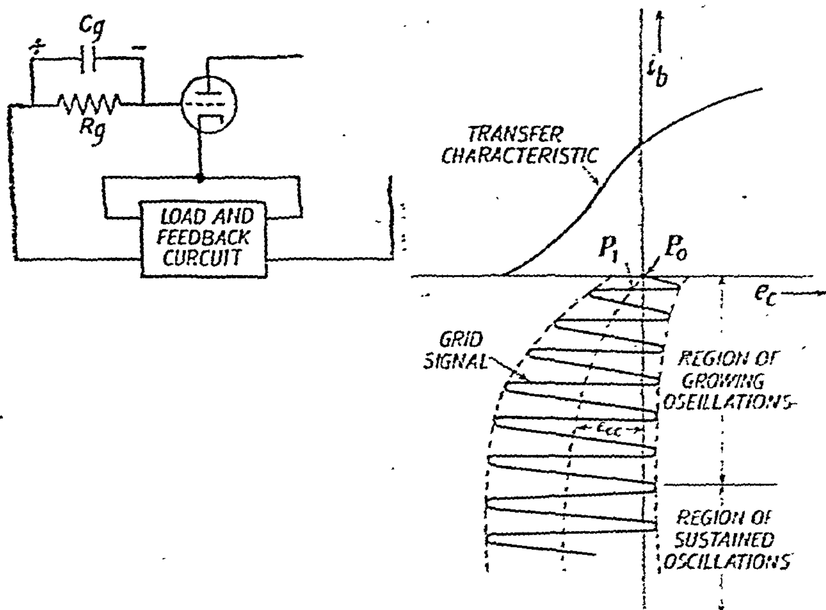


Fig. 17·4. Fixed bias operation of f

17·4. Let there arise a small voltage e_{gk1} due to a transient phenomenon. The value of $\bar{g}_m$ is smaller than that required in Eqn. (17·9) for sustained oscillations. The result is that the oscillations die out. Next if the signal is increased to e_{gk2}, the corresponding value of average transconductance $\bar{g}_{m2}$ may be large enough to result in growing oscillations. The amplitude of oscillation will then increase until the condition of sustained oscillation has reached. Evidently such a circuit is not self-oscillating since in the infant stage of oscillations $\bar{g}_m$ is so small as to cause decay of oscillations. In fact if the oscillator is biased near to or beyond cutoff, the circuit is never self-starting. However, once the oscillations are started by injecting a heavy signal at grid, they are sustained and their amplitude is relatively large. The efficiency is then considerably increased and such an operation is extremely useful in high power oscillators.

Self-starting oscillation with grid resistor-grid capacitor biasing circuit. Fig. 17·5 shows the use of grid resistor-grid condenser combination of biasing the oscillator tube. This method of biasing results in self-starting oscillations. As soon as the circuit is switched on, the grid-bias is zero and the operating point is at P_o. The value of $\bar{g}_m$ is large resulting in growing oscillations. During the first positive half of the generated voltage, the grid runs positive and the resulting grid current charges the condenser in the polarity shown. During the negative half cycle of grid voltage, grid current does not flow and the condenser discharges through R_g. The time constant R_g–C_g is so chosen that an almost steady bias is maintained.



(a) R_g-C_g biased oscillator. (b) Growth of oscillations.

Fig. 17·5. Self-starting oscillations in an oscillator with R_g-C_g biasing circuit.

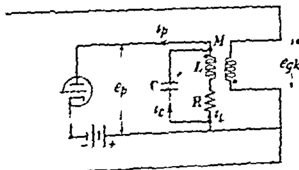
This negative bias displaces the point of operation to the left of the point P_1. The amplitude of oscillation increases continuously with each half cycle of oscillation. Also the bias increases gradually until an equilibrium condition is reached between the amplitude of oscillation and resulting grid E_{cc}. Under this equilibrium condition, the charge placed across the condenser by the grid current just equals the charge removed from the condenser to R_g during the interval of non-flow of grid current. The bias and amplitude of oscillation then become steady. The value of g_m under this condition is then just equal to that required for sustained oscillations. The bias obtained in this manner is at or beyond cutoff

The time constant $R_g \cdot C_g$ must be chosen carefully. If the time- ... After a ... igh. The ... Thus the ... as. After the oscillations have stopped, condenser slowly but continuously, discharges through R_g and bias reduces. When condenser has discharged sufficiently oscillation alternates with the period of rest. The phenomenon is termed as "motor-boating". The period of motor-boating depends upon the period of non-oscillation and hence depends upon the time constant $R_g C_g$. The repetition frequency is of the order of $\frac{1}{R_g C_g}$. Hence to eliminate motor-boating, the time constant $R_g C_g$ should be suitably reduced

SINE WAVE OSCILLATORS

(i) Tuned plate oscillators
(ii) Tuned grid oscillators
(iii) Tuned grid-tuned plate oscillators
(iv) Hartley oscillators
(v) Colpitts oscillators.

Tuned Plate Oscillator



Circuit diagram of tuned plate oscillator

Fig. 17·6 shows the circuit diagram of the tuned plate oscillator. In this oscillator, a parallel tuned circuit or anti-resonant circuit is connected directly in the plate circuit. Another coil inductively coupled to the coil of the tuned circuit provides the grid excitation. Basically the circuit is a class C amplifier with arrangement for providing the grid excitation from the tuned circuit itself. Due to this similarity with tuned class C amplifier, the analysis for class C amplifier applies in this case also.

Expressions for frequency of oscillation and $\bar{g}_m$ for sustained oscillations. There are two methods by which we may derive the expressions for frequency of oscillation and average transconductance $\bar{g}_m$ under condition of sustained oscillation. These are (A) using Barkhausen criterion and (B) using Vander Bijl solution.

(A) Method using Barkhausen Criterion :—The assumptions made are that (i) grid current is negligible and (ii) condenser is loss free.

Then the voltage induced in the secondary is given by,

$$e_{gk}=j\omega M.\, i_l=\mathbf{Z}_m\, i_l \quad \dots (17{\cdot}22)$$

where $\mathbf{Z}_m=j\omega M$

The transfer impedance is given by,

$$\mathbf{Z}_T=\frac{-E_{gk}}{i_p}=\frac{-\mathbf{Z}_m\, i_l}{i_l+i_c} \quad \dots (17{\cdot}23)$$

But $\quad i_c.\, \mathbf{Z}_c=i_l.\, \mathbf{Z}_l \quad \dots (17{\cdot}24)$

where $\quad \mathbf{Z}_c=\frac{1}{j\omega C}$ and $\mathbf{Z}_l=j\omega L.$

Hence

$$\mathbf{Z}_T=\frac{-\mathbf{Z}_m\, i_l}{i+i_l\left(\frac{\mathbf{Z}_l}{\mathbf{Z}_c}\right)}=\frac{-\mathbf{Z}_m\, \mathbf{Z}_c}{\mathbf{Z}_l+\mathbf{Z}_c} \quad \dots (17{\cdot}25)$$

Voltage gain of the internal amplifier is given by,

$$\mathbf{A}=\frac{-\mu\mathbf{Z}}{r_p+\mathbf{Z}} \quad \dots (17{\cdot}26)$$

where

$$\mathbf{Z}=\frac{\mathbf{Z}_l\, \mathbf{Z}_c}{\mathbf{Z}_l+\mathbf{Z}_c} \quad \dots (17{\cdot}27)$$

Substituting the values of $\mathbf{A}$ and $\mathbf{Z}_T$ in Eqn. (17·14) which gives the modified version of Barkhausen criterion for sustained oscillations,

$$-\bar{g}_m\left[-\frac{\mathbf{Z}_m\, \mathbf{Z}_c}{\mathbf{Z}_l+\mathbf{Z}_c}\right]\left[1+\frac{-\mathbf{Z}}{r_p+\mathbf{Z}}\right]=1$$

or

$$\bar{g}_m\frac{\mathbf{Z}_m\, \mathbf{Z}_c}{\mathbf{Z}_l+\mathbf{Z}_c}\left[1-\frac{\mathbf{Z}_l\, \mathbf{Z}_c}{\mathbf{Z}_l\, \mathbf{Z}_c+r_p(\mathbf{Z}_l+\mathbf{Z}_c)}\right]=1$$

or $$\bar{g}_m \mathbf{Z}_m \mathbf{Z}_c \frac{r_p}{r_p(\mathbf{Z}_l+\mathbf{Z}_c)+\mathbf{Z}_l \mathbf{Z}_c}=1$$

or $$\bar{g}_m \mathbf{Z}_m \mathbf{Z}_c=(\mathbf{Z}_l+\mathbf{Z}_c)+\frac{\mathbf{Z}_l \mathbf{Z}_c}{r_p} \quad \text{... (17·28)}$$

or $$\bar{g}_m \frac{M}{C}=R+j\left(\omega L-\frac{1}{\omega C}\right)+\frac{L}{C\, r_p}-j\frac{R}{\omega C\, r_p} \quad \text{... (17·29)}$$

In Eqn. (17·29) equating real part *on the left hand side* to the real part on the right hand side,

$$\bar{g}_m \frac{M}{C}=R+\frac{L}{C.\, r_p} \quad \text{... (17·30)}$$

or $$\bar{g}_m=\frac{\mu RC}{\mu M-L} \quad \text{... (17·31)}$$

Equation (17·31) gives the value of $\bar{g}_m$ for sustained oscillations in a tuned plate oscillator If $\bar{g}_m$ is less than this value, oscillations die out and if $\bar{g}_m$ is more than this value, the oscillations continue to build up and *then with* R_g-C_g bias the point of operation shifts until such a position is reached when $\bar{g}_m$ has the desired value.

Again equating the imaginary parts in Eqn (17·29),

$$\omega L-\frac{1}{\omega C}-\frac{R}{\omega C\, r_p}=0 \quad \text{... (17·32)}$$

or $$\omega^2=\frac{1}{LC}\left(1+\frac{R}{r_p}\right) \quad \text{... (17·33)}$$

Resonant frequency is given by,

$$\omega_0=\frac{1}{\sqrt{LC}}$$

Hence $$\omega=\omega_0\sqrt{1+\frac{R}{r_p}} \quad \text{... (17·34)}$$

or $$\omega\approx\omega_0\left[1+\frac{1}{2}\cdot\frac{R}{r_p}\right] \quad \text{... (17·35)}$$

If $r_p \gg R$, then $\omega\approx\omega_0$.

Eqn (17·34) shows that the frequency of oscillation ω is slightly greater than the resonant frequency ω_0. A change in the value of R will cause a change in the frequency of oscillation. This change in the frequency of oscillation is not desirable and in order to avo this, the load is not put directly on the oscillator. An amplifi stage *is* placed between the oscillator and the load. This amplifi is called the buffer amplifier and does not draw power from t the oscillator. It may further be seen that the tube plays insignificar role in determining the frequency of oscillation. The extern circuit components almost completely control the frequency.

(B) Vander Bijl Solution :—With reference to gram of Fig. 17·6,

$$e_{gk}=M\frac{di_l}{dt} \qquad \ldots (17\cdot36)$$

$$i_l+i_c=I_p \qquad \ldots (17\cdot37)$$

and
$$R.\,i_l+L\frac{di_l}{dt}=\frac{1}{C}\int i_c\,dt \qquad \ldots (17\cdot38)$$

The a.c. equivalent circuit of the plate circuit of the oscillator is drawn in Fig. 17·7.

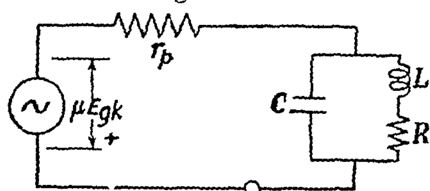


Fig. 17·7. A.C. equivalent circuit of tuned plate oscillator of Fig. 17·6

From Fig. 17·7,

$$\mu e_{gk}=L\frac{di_l}{dt}+R.\,i_l+r_p.\,i_p \qquad \ldots (17\cdot39)$$

From Eqns. (17·36), (17·37) and (17·39), we get,

$$\mu\,M\frac{di_l}{dt}=L\frac{di_l}{dt}+Ri_l+r_p(i_c+i_l) \ldots (17\cdot40)$$

From Eqn. (17·38) by differentiation we get,

$$\frac{i_c}{C}=R\frac{di_l}{dt}+L\frac{d^2i_l}{dt^2}$$

or
$$i_c=RC\frac{di_l}{dt}+LC\frac{d^2i_l}{dt^2} \qquad \ldots (17\cdot41)$$

Substituting the value of i_c from Eqn. (17·41) into Eqn. (17·40) we get,

$$LC\,r_p\frac{d^2\,i_l}{dt^2}+\left[L+CR\,r_p-\mu\,M\right]\frac{di_l}{dt}+(R+r_p)\,i_l=0 \qquad \ldots (17\cdot42)$$

For oscillations to be sustained, the coefficient of $\frac{d_{il}}{d_t}$ in Eqn. (17·42) must be zero.

Hence $L+CR\;r_p=\mu\;M$

or
$$g_m=\frac{\mu CR}{\mu M-L} \qquad \ldots (17\cdot43)$$

Since the oscillator operates with a bias beyond cutoff, average value of g_m must be considered. Hence in Eqn. (17·43) we replace g_m by $\bar{g}_m$.

Thus
$$\bar{g}_m=\frac{\mu CR}{\mu M-L} \qquad \ldots (17\cdot44)$$

Eqn. (17·44) is the same as Eqn. (17·31) obtained by application of Barkhausen criterion.

Putting the coefficient of $\frac{di_l}{dt}$ in Eqn. (17·42) to be zero, we get,

$$\frac{d^2i_l}{dt^2}+\frac{R+r_p}{LCr_p}\,i_l=0 \qquad \ldots (17\cdot45)$$

Solving Eqn. (17·45) we get,

$$\omega=\sqrt{\frac{R+r_p}{LCr_p}}=\omega_0\sqrt{1+\frac{R}{r_p}} \qquad \ldots(17\cdot46)$$

Eqn. (17·46) is the same as Eqn. (17·34) obtained by application of Barkhausen criterion.

Tuned Grid Oscillator

Fig. 17·8 shows the circuit diagram of a tuned grid oscillator,

and provides the necessary feedback from the plate circuit to the grid circuit.

Fig. 17·8. Circuit of tuned grid oscillator.

An analysis similar to that presented for the tuned plat oscillator shows that the frequency of oscillation of the tuned gri oscillator is,

$$\omega=\omega_0\sqrt{\frac{r_p}{r_p+R}}=\omega_0\left(1+\frac{R}{r_p}\right)^{-\frac{1}{2}} \qquad \ldots(17\cdot47)$$

or

$$\omega\approx\omega_0\left(1-\frac{R}{2r_p}\right) \qquad \ldots(17\cdot48)$$

Further the average transconductance g_m is given by,

$$g_m=\frac{\mu RCL}{M(M-L\mu)} \qquad \ldots(17\cdot49)$$

Tuned Grid-tuned Plate Oscillator (*TGTP Oscillator*)

... diagram of a tuned grid-tuned plat ... there are two parallel tuned circuits ... other in the plate circuit. Usuall ... between the coils L_1 and L_2. The ... by the grid-to-plate capacitance C_{gp} ... ficient, even a physical condenser may be connected bet ... The frequency of oscillation is slightly below the resonant frequency of the tuned

circuit on the plate side. The plate tuned circuit then offers an inductive load. This inductive load in the plate circuit along with grid-to-plate capacitance C_{gp} results in a negative value of the effective conductance between the grid and the cathode. This

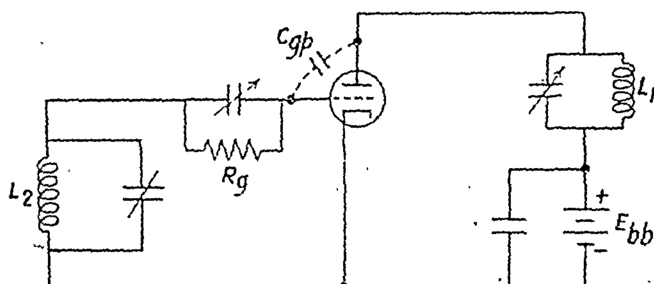


Fig. 17·9. Circuit of tuned grid-tuned plate oscillator.

negative conductance may be interpreted as power supply source to the parallel tuned circuit in series with the grid and thereby it makes possible sustained oscillations.

The input admittance $\mathbf{Y}_i$ between grid and cathode of the oscillator tube is given by,

$$\mathbf{Y}_i = j\omega\, C_{gk} + j\omega\, C_{gp}\,(1 - A_r - jA_i) \qquad ...(17{\cdot}50)$$

where A_r and A_i are the real and imaginary parts of the complex voltage gain of the internal amplifier of the feedback oscillator.

But $$\mathbf{Y}_i = G_i + j\, B_i \qquad ...(17{\cdot}51)$$

where G_i and B_i are respectively the input conductance and the input susceptance.

If the effective load impedance in the plate circuit is given by,

$$\mathbf{Z}_l = R_l + jX_l \qquad ...(17{\cdot}52)$$

the complex voltage gain $\mathbf{A}$ is given by,

$$\mathbf{A} = \frac{-\mu\, \mathbf{Z}_l}{r_p + \mathbf{Z}_l} = A_r + jA_i \qquad ...(17{\cdot}53)$$

or $$\mathbf{A}_r + j\mathbf{A}_i = -\mu \frac{R_l + jX_l}{(r_p + R_l) + jX_l}$$

$$= -\frac{\mu R_l (r_p + R_l) + X_l^2}{(r_p + R_l)^2 + (X_l)^2}$$

$$- j\mu \frac{r_p X_l}{(r_p + R_l)^2 + (X_l)^2} \qquad ...(17{\cdot}54)$$

Hence $$G_i = -\mu\omega\, C_{gp} \frac{r_p X_l}{(r_p + R_l)^2 + (X_l)^2} \qquad ...(17{\cdot}55)$$

This negative conductance G_i causes the necessary oscillations.

Hartley Oscillator

Fig. 17·10 (a) shows the circuit of Hartley oscillator. In this oscillator, the tuned circuit extends from grid to plate. The tuning

coil has two parts. Coil L_1 appears between grid and cathode while the coil L_2 appears between cathode and plate The tuning condenser C is connected between grid and plate terminals G and P. Coils L_1 and L_2 need not have mutual coupling.

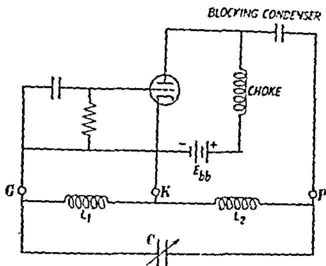


Fig. 17·10 (a). Circuit of Hartley Oscillator.

In practice the tank circuit may be made as shown below. Both G and P tops may be variable.

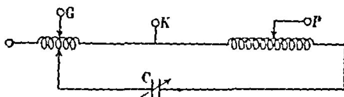


Fig 17·10 (b). Alternative practical form of tuned circuit of Hartley oscillator.

Condenser C is kept variable whenever variable frequency oscillator is required. By the arrangement of Fig. 17·10 (b), the impedance of the tank circuit may be matched to the tube impedance. The grid drive may be varied by top at G.

Frequency of oscillation. The a c. equivalent circuit of Hartley oscillator is shown in Fig. 17·11.

Let us first assume the frequency of oscillation to be the same as the resonant frequency. Then $X_{l1}+X_{l2}=X_c$. At resonance, the impedance $\mathbf{Z}_t$, between the points K and P is a pure resistance. The

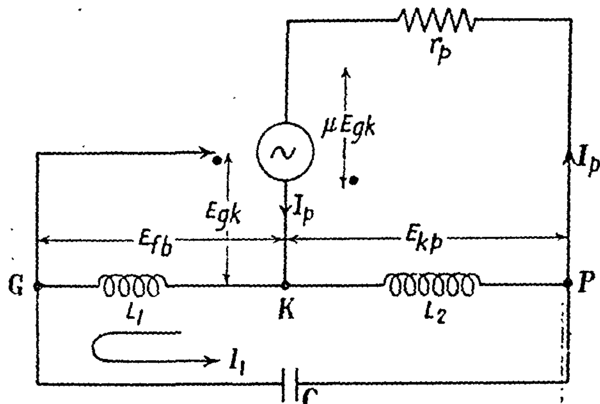


Fig. 17·11. Incremental equivalent circuit of Hartley oscillator.

vector diagram of the voltages and currents is then as shown in Fig. 17·12.

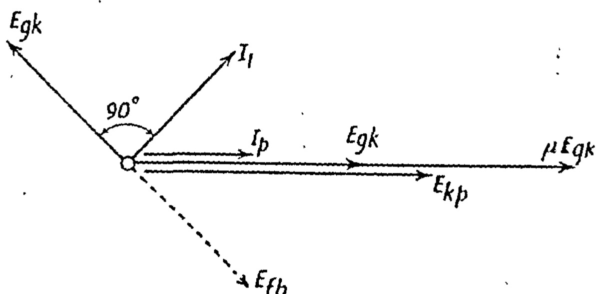


Fig. 17·12. Vector diagram of Hartley Oscillator assuming the frequency of oscillation to be equal to resonant frequency.

Since Z_{kp} is resistive, E_{kp} must be in phase with μE_{gk}. Since both r_p and Z_{kp} are resistive, plate current $\mathbf{I}_p$ must be in phase with $\mathbf{E}_{gk}$. Also as C predominates over L_1, $\mathbf{I}_1$ leads $\mathbf{E}_{kp}$. Further the voltage E_{kg} leads I_1 by 90° as shown. The feedback voltage $\mathbf{E}_{fb}$ which constitutes the grid to cathode voltage $\mathbf{E}_{gk}$ is equal and opposite to $\mathbf{E}_{kg}$. We see that this voltage $\mathbf{E}_{fb}$ and input voltage $\mathbf{E}_{gk}$ are not the same since they are not in phase. Hence we conclude that Hartley oscillator will not oscillate at the resonant frequency of the tuned circuit.

For oscillation to be sustained, feedback voltage $\mathbf{E}_{fb}$ must be the same as the input voltage $\mathbf{E}_{gk}$. $\mathbf{E}_{fb}$ must then come in phase with $\mathbf{E}_{gk}$. Current $\mathbf{I}_1$ must lead $\mathbf{E}_{gk}$ by a much larger angle *i.e.* C must predominate too much over L_1. To achieve this X_{l1} should be decreased and X_c should be increased. This can be done by decreasing the frequency of oscillation below the resonant frequency of the tank circuit. $\mathbf{Z}_{kp}$ will then be inductive and hence $\mathbf{Z}_{kp}$ along with r_p makes $\mathbf{I}_p$ lag behind $\mathbf{E}_{gk}$. The vector diagram is then as shown in Fig. 17·13.

Fig. 17·13. Vector diagram of Hartley oscillator with sustained oscillations.

In this vector diagram E_{bk} is opposite in phase to the input voltage E_{gt} and hence feedback voltage E_{gk} will be in phase with E_{gk}. Hence oscillations will be sustained.

Colpitts Oscillator

Fig. 17·14 (a) shows the circuit of Colpitts oscillator. In this oscillator also, as in the case of Hartley oscillator, the tuned circuit extends from grid to plate. In this oscillator, however, the coil is connected across grid to plate circuit. The tank circuit condenser is split into two parts. Condenser C_1 is connected between grid and cathode and condenser C_2 is connected between cathode and plate. Fig. 17·14 (b) shows the incremental equivalent circuit of the oscillator.

(a) Circuit diagram (b) Incremental equivalent circuit

Fig. 17·14. Circuit diagram and incremental equivalent circuit of Colpitts oscillator.

Let us first assume that the frequency of oscillation be the resonant frequency. $\mathbf{Z}_{kp}$ is then a pure resistance. $\mathbf{I}_p$ is in phase with the voltage $\mu\mathbf{E}_{gk}$. Further since inductance L predominates C_1, current I_1 lags behind E_{kp}. Fig. 17·15 shows the vector diagram. Voltage E_{kg} lags behind current $\mathbf{I}_1$ by 90°. Feedback voltage $\mathbf{E}_{fb}$ is opposite to $\mathbf{E}_{kg}$ and is, therefore, not in phase with the input voltage $\mathbf{E}_{gk}$. Hence the oscillations will not be sustained. The frequency of operation of a Colpitts oscillator is, therefore, not the same as the resonant frequency of the tuned circuit.

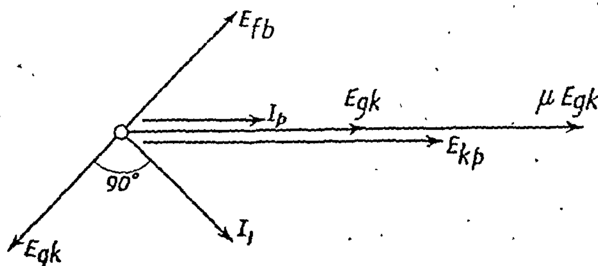


Fig. 17·15. Vector diagram of Colpitts oscillator under the assumption that the frequency of operation is the same as the resonant frequency.

For oscillations to take place, current $\mathbf{I}_1$ must lag behind $\mathbf{E}_{gk}$ by a much larger value *i.e.* L must predominate over C_1 by a larger amount. Hence X_{l1} must be increased and X_{c1} must be reduced.

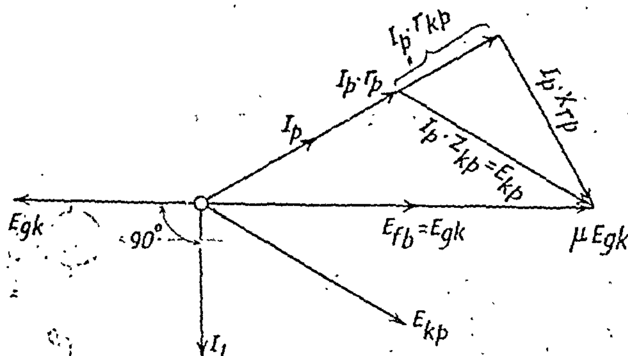


Fig. 17·16. Vector diagram of Colpitts oscillator with sustained oscillations.

This can be achieved by increasing the frequency of operation above the resonant frequency of the tuned circuit. The vector diagram of the oscillator then is as shown in Fig. 17·16.

$\mathbf{Z}_{kp}$ is now capacitive. Hence current I_p leads the voltage $\mathbf{E}_{gk}$. Let $\mathbf{Z}_{kp}=r_{kp}+jX_{kp}$. $\mathbf{E}_{kp}$ lags behind E_{gk}. Current $\mathbf{I}_1$ lags behind $\mathbf{E}_{kp}$ by 90° and for a suitable frequency of operation $\mathbf{E}_{kp}$ is 180° out of phase with $\mathbf{E}_{gk}$. The feedback voltage $\mathbf{E}_{fb}$ is opposite to $\mathbf{E}_{kg}$ and hence is in phase with the input voltage $\mathbf{E}_{gk}$.

Thus $$I_1 R = E_{sk} \qquad \dots (17\cdot56)$$

$$I_2 R = I_1 \left(R + \frac{1}{j\omega C} \right) \qquad \dots (17\cdot57)$$

$$I_3 R = (I_1 + I_2) \frac{1}{j\omega C} + I_2 R \qquad \dots (17\cdot58)$$

and $$E_{pk} = \frac{(I_1 + I_2 + I_3)}{j\omega C} + I_3 R \qquad \dots (17\cdot59)$$

Substituting the values of I_1, I_2 and I_3 from Equations (17·56), (17·57) (17·58) into Eqn. (17·59) we get,

$$E_{pk} = \frac{(I_1 + I_2 + I_3)}{j\omega C} + \frac{(I_1 + I_2)}{j\omega C} + \frac{I_1}{j\omega C} + E_{sk}$$

$$= \frac{3I_1 + 2I_2 + I_3}{j\omega C} + E_{sk} \qquad \dots (17\cdot60)$$

From Eqn. (17·56), $$I_1 = \frac{E_{sk}}{R} \qquad \dots (17\cdot61)$$

From Eqn. (17·57), $$I_2 = \frac{I_1}{R}\left(R + \frac{1}{j\omega C} \right) = \frac{E_{sk}}{R} + \frac{E_{sk}}{R^2 j\omega C}$$

$$= E_{sk} \left[\frac{1}{R} + \frac{1}{j\omega C R^2} \right] \qquad \dots (17\cdot62)$$

and $$I_3 = I_2 + \frac{I_1 + I_2}{j\omega C R}$$

$$= E_{sk} \left[\frac{1}{R} + \frac{1}{j\omega C R^2} + \frac{1}{j\omega C R} \left\{ \frac{1}{R} + \frac{1}{R} + \frac{1}{j\omega C R^2} \right\} \right]$$

$$= \frac{E_{sk}}{R^2} \left[R + \frac{3 + \dfrac{1}{j\omega C R}}{j\omega C} \right] \qquad \dots (17\cdot63)$$

Substituting the values of I_1, I_2 and I_3, from Eqns. (17·61), (17·62) and (17·63) into Eqn. (17·60) we get,

$$\frac{1}{\beta} = \frac{E_{pk}}{E_{sk}}$$

$$= 1 + \frac{\dfrac{3}{R} + 2\left[\dfrac{1}{R} + \dfrac{1}{j\omega C R^2} \right] + \dfrac{1}{R^2}\left[R + \dfrac{3 + \dfrac{1}{j\omega C R}}{j\omega C} \right]}{j\omega C}$$

$$= 1 - j\,\frac{6}{\omega C R} - \frac{5}{\omega^2 C^2 R^2} + \frac{j}{\omega^3 C^3 R^3}$$

$$= \left[1 - \frac{5}{\omega^2 C^2 R^2} \right] + j \left[\frac{1}{\omega^3 C^3 R^3} - \frac{6}{\omega C R} \right] \qquad \dots (17\cdot64)$$

where β is the feedback ratio.

Let **A** be the voltage gain of the internal amplifier and may be expressed as

$$\mathbf{A}=A_r+j\,A_i \qquad \dots (17{\cdot}65)$$

where A_r is the real component of voltage gain and A_i is the imaginary component of voltage gain.

But for the given circuit, assuming the effect of phase shift network on R_l to be negligible,

$$\mathbf{A}=\frac{-\mu\,R_l}{r_p+R_l} \qquad \dots (17{\cdot}66)$$

Obviously $\quad A_i=0$ and $A_r=\dfrac{-\mu\,R_l}{r_p+R_l} \qquad \dots (17{\cdot}67)$

For oscillations to be sustained, $\mathbf{A}\beta=1$ or $\dfrac{1}{\beta}=\mathbf{A}$.

Hence $\left[1-\dfrac{5}{R^2\omega^2C^2}\right]+j\left[\dfrac{1}{\omega^3C^3R^3}-\dfrac{6}{\omega CR}\right]=A_r \quad \dots (17{\cdot}68)$

Equating imaginary parts in Eqn. (17·68) we get,

$$\frac{1}{\omega^3C^3R^3}-\frac{6}{\omega CR}=0 \qquad \dots (17{\cdot}69)$$

or $\qquad \omega^2=\dfrac{1}{6R^2\,C^2}$

or $\quad$ angular frequency

$$\omega=\frac{1}{\sqrt{6}\,RC} \qquad \dots (17{\cdot}70)$$

Eqn. (17·70) may be put as, $X_c=\sqrt{6}R \qquad \dots (17{\cdot}70a)$

where X_c is the reactance of C and is equal to $\dfrac{1}{\omega C}$

Frequency $\quad f(\text{c/s})=\dfrac{1}{2\pi\sqrt{6}\,RC} \qquad (17{\cdot}71)$

Substituting $\quad \omega=\dfrac{1}{\sqrt{6}\,RC}$ we get,

$$A_r=\left[1-\frac{5}{\omega^2C^2R^2}\right]$$

$$=\left[1-\frac{5}{\frac{1}{6}}\right]=-29 \qquad \dots (17{\cdot}72)$$

Hence μ, r_p and R_l should be so chosen that A_r as given by Eqn. (17·67) should be -29.

These phase shift oscillators are used as constant frequency oscillators because the variation of resistances R and capacitances C in the phase-shift network is troublesome.

Wien Bridge Oscillator

Wien-bridge oscillator is really a two-stage amplifier, in which a fraction of the output voltage is feedback to the input terminals. Oscillations are produced if the feedback voltage E_β is equal in magnitude and phase with the voltage initially assumed at the input of the first stage. Fig. 17·20 shows the circuit diagram. The oscillator is termed a Wien Bridge Oscillator because of the similarity of the frequency determining circuit with the basic Wien Bridge.

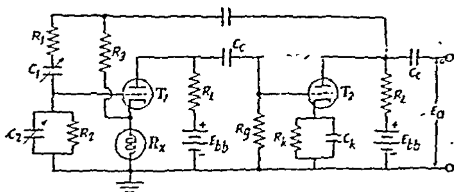


Fig. 17·20. Circuit diagram of Wien Bridge Oscillator.

It has been found that the following relation must hold good.

$$E_\beta = AE_i - \frac{R_2/(1+j\omega C_2R_2)}{\frac{R_2}{1+j\omega C_2R_2}+\left(R_1+\frac{1}{j\omega C_1}\right)} \quad ...(17·73)$$

where **A** is the complex voltage gain of the amplifier

E_i is the input voltage to the first stage

and E_β is the feedback voltage.

Obviously E_β must be equal to E_i for oscillations to be sustained. Hence separating the real and imaginary parts in Eqn. (17·73) we get,

$$f=\frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} \quad ...(17·74)$$

and

$$A=1+\frac{R_1}{R_2}+\frac{C_1}{C_2} \quad ..(17·75)$$

If $C_1=C_2=C$ and $R_1=R_2=R$, then

$$f=\frac{1}{2\pi RC} \text{ and } A=3 \quad ...(17·76)$$

C_1 and C_2 are variable air condensers which have maximum to minimum capacitance ratio of about 10 : 1. Hence in accordance with Eqn. (17·76), the frequency variation in the range 10 : 1 is possible in this oscillator. This ratio is much larger compared with the maximum frequency ratio of approximately 3 : 1 in LC ...

a single dial. Thus if frequency f_1 is fixed at 400 kc/s and f_2 is variable from 380 kc/s to 400 kc/s, the audio frequency voltage available at the output will have a frequency that may be varied from 20 kc/s down to 1 c/s. Beat frequency oscillator has addition advantages of good waveform, constant output level, lightness and compactness. For excellent performance, however, the beat frequency oscillator must be designed with special reference to the following considerations :

(i) **Prevention of interaction between the r.f. oscillators.** The interaction between the two r.f. oscillators causes them to pull each other into synchronism when they have small frequency difference. Interaction may also result in distortion of the output wave into an almost saw-tooth wave at frequency difference slightly greater than that at which pulling takes place. Interaction may be reduced to negligible proportions by proper shielding, correct layout of component parts and by use of *RC* or *LC* filters in the power supply leads. It may also be reduced by using such coupling means from R.F. oscillators to detectors which prevent interaction *e.g.* the use of balanced modulator, use of buffer amplifier between r.f. oscillators and detector, use of mixing bridge etc. Use of a balanced modulator is shown is Fig. 17·22.

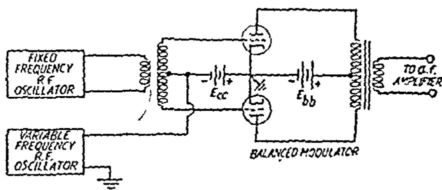


Fig. 17.22. Use of Balanced Modulator in Beat Frequency Oscillator.

(ii) **Elimination of harmonics and other undesired frequencies.** In order to affect this it is simply required to remove the harmonics from the output of one of the two R.F. oscillators and the detector. In that case all undesired frequencies lie far beyond the audio range.

(iii) **Improvement of frequency stability.** In Beat Frequency oscillator the frequency stability is likely to be poor, [illegible] small percentage change in any R.F. oscillator [illegible]

must be very stable.

(iv) **Variation of output level.** The amplitude of a.f. output voltage may vary due to one of the following reasons : (a) due to

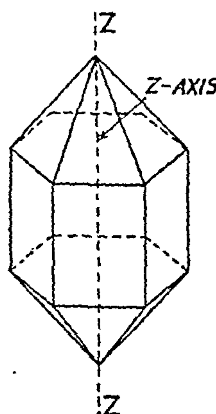
change of the amplitude of the variable frequency oscillator voltage over the tuning range (*b*) due to improper r.f. filter action in the output of the detector and (*c*) due to frequency distortion in the a.f. amplifier. Due attention must be paid to these considerations.

Crystal Oscillator

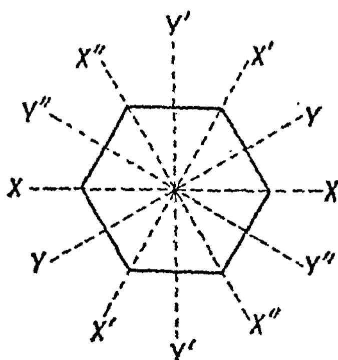
In conventional Radio Frequency Oscillators using tuned circuit, the frequency stability is usually poor because of variations in temperature, humidity, tube and circuit constants etc. For certain applications notably in Radio Broadcast and Telephone Transmitters, it is required that the frequency of oscillation of Master Oscillator be extremely stable. Frequency of an oscillator may be made very stable by replacing the usual resonant circuit by a mechanically vibrating piezo-electric quartz crystal and utilizing its piezo-electric effect. Such crystal oscillators constitute the Master Oscillator of Radio Transmitters and are also used in reception of signal from transmitter stations operating at standard known frequencies.

In its natural form a quartz crystal has a hexagonal cross-section and pointed ends as shown in Fig. 17·23.

Three sets of axes completely define the properties of such a crystal. The axis joining the two pointed ends of the crystal is called the *Z*-axis or "optical axis". Fig. 17·23 (*b*) shows a section of the crystal at right angles to the *Z*-axis. With reference to the section shown in Fig. 17·23, the three axes XX, $X'X'$ and $X''X''$ passing through the corners of the hexagon are known as the *X*-axis or "Electrical axes". The three axes YY, $Y'Y'$ and $Y''Y''$ which are perpendicular to the faces of the crystal are called the *Y*-axes or "Mechanical-axes".



(*a*) Quartz crystal in natural form.



(*b*) Section of quartz crystal perpendicular to *Z*-axis.

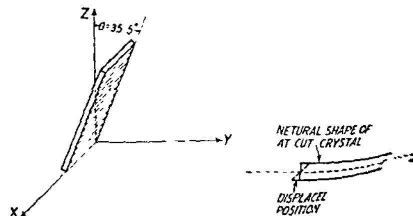
Fig. 17·23. Three sets of axes in piezo-electric quartz crystal.

Piezo-electric effect. In a quartz crystal, if an electric stress is applied in the direction of an X-axis or electrical axis, a mechanical stress is produced in the direction of the Y-axis which is perpendicular to the X-axis. Conversely if a mechanical stress is applied along a Y-axis, electric charges will appear on the faces of the crystal perpendicular to the X-axis which is at right angles to the Y-axis concerned. This phenomenon is called the "piezo-electric effect" and has been utilized in crystal oscillator.

On application of an alternating voltage along an X-axis, the mechanical stresses produced also alternate *i.e.* the crystal vibrates *at the frequency of applied alternating voltage. If, however, the* frequenc [illegible] of mecl [illegible] vibratio [illegible] number [illegible] depend upon the size and shape of the crystal, orientation of the crystal plate cut from the natural crystal, the type of oscillations concerned and upon six elastic constants of the crystal. Further the frequency of resonance is a function of the ambient temperature. The temperature coefficient of frequency *i.e.* frequency variation in cycles per second per degree change in ambient temperature can be calculated from the crystal orientation and temperature coefficients of the elastic constants of the crystal.

Common Crystal Cuts. Usually crystals are cut from the natural quartz crystal, in the form of thin plates of rectangular or circular shape and almost uniform thickness. There are, of course, a large number of shapes of crystal plates possible. Similarly there *are a large number* of orientations of crystals *i.e.* the position of crystal plate cut from the natural quartz crystal. The common crystal cuts are : (*i*) AT cut (*ii*) X-cut or Curie cut and (*iii*) Y-cut or 30° cut.

The AT cut crystal is commonly used for generation of frequencies in the frequency range of 50 kc/s to about 10 Mc/s. AT



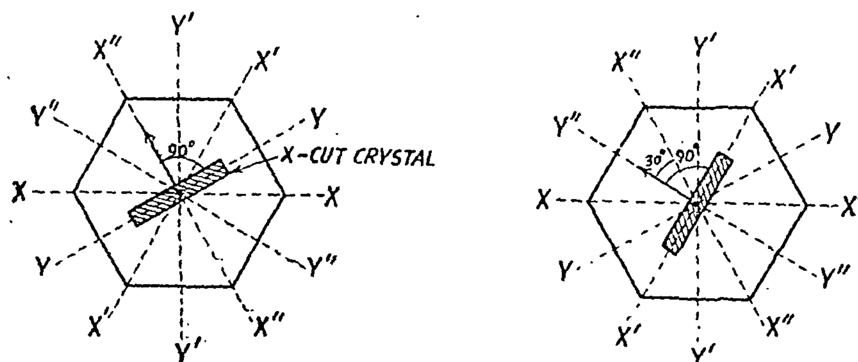
(*a*) Orientation of AT cut crystal (*b*)
Fig 17 24 Orientation and shear vibrations

cut crystal plate is cut from a plane rotated about an X-axis so that the angle θ made with Z-axis is approximately 35·5 degrees as shown in Fig. 17·24 (a). On application of voltage across the flat sides of such a crystal-plate, shear vibrations take place causing displacement as shown in Fig. 17·24 (b).

The AT cut crystal has the following important advantages: (i) zero temperature coefficient at a temperature determined by the value of angle θ, (ii) high piezo-electric effect and (iii) simple frequency spectrum.

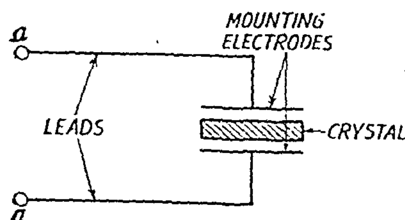
The X-cut or Curie cut crystal is taken out of a face perpendicular to the X-axis as shown in Fig. 17·25 (a).

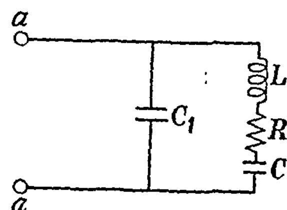
The Y-cut or 30° cut crystal is taken out of a face perpendicular to a Y-axis as shown in Fig. 17·25 (b). The Y-cut crystal has a number of resonant frequencies.



(a) Orientation of X-cut crystal. (b) Orientation of Y-cut crystal.
Fig. 17·25. Orientation of X-cut and Y-cut crystals.

Equivalent electrical circuit of a quartz crystal.



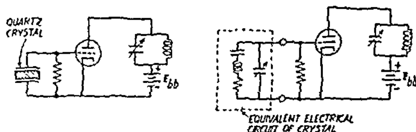


(a) Quartz crystal with mounting electrodes. (b) Equivalent electrical circuit.
Fig. 17·26. Equivalent electrical circuit of a quartz crystal with mounting plates.

Physically the crystal plate cut from the natural quartz crystal is mounted between two mounting plates and two leads are connected to the two mounting plates. Fig. 17·26 (a) shows the crystal and the mounting plates. Fig. 17·26 (b) shows the equivalent electrical circuit of the crystal mounted between electrodes. In Fig. 17·26 (b), C_1 represents the electrostatic capacitance between

the crystal electrodes when the crystal is *not* vibrating. Series elements L, C and R represent the electrical equivalent of the vibrational characteristics of the quartz. Here inductance L represents the electrical equivalent of the crystal mass in vibration. C is the electrical equivalent of the effective mechanical compliance (elasticity) of the crystal. R represents the electrical equivalent of the mechanical friction. In the analysis of an electrical circuit containing a quartz crystal, the crystal itself may be replaced by its equivalent electrical circuit.

Crystal oscillator circuit. In a crystal oscillator, a quartz crystal is put instead of resonant circuit which determines the frequency of oscillation. Fig. 17·27 (*a*) shows the circuit of the crystal oscillator using Miller circuit. Fig. 17·27 (*b*) gives the equivalent electrical circuit of oscillator. A study of equivalent circuit of Fig. 17·27 (*b*) shows that the circuit of the crystal oscillator is equivalent to a tuned grid-tuned plate oscillator. To generate oscillations, the tuned circuit in the plate side is tuned to a frequency higher



(*a*) Crystal oscillator using Miller circuit. (*b*) Equivalent electrical circuit of crystal.

Fig. 17·27. A common crystal oscillator circuit and its equivalent electrical circuit.

than the resonant frequency of the crystal. The reactance of the plate circuit at crystal resonant frequency is then inductive. The amplitude of oscillation is determined by the inductive reactance of the plate circuit and by the grid-to-plate tube capacitance.

The frequency of oscillation of crystal oscillator lies in the range of about 50 kc/s to 15 Mc/s. The frequency of resonance for any crystal at a given temperature depends upon the elastic constants of the crystal, *orientation* of the crystal and the size of the crystal. As the crystal is made more and more thinner the frequency of resonance increases. The lowest frequency limit arises from the difficulty of obtaining large size quartz crystals while the high frequency limit results because of fragility of the thin crystal plate.

Negative Resistance Oscillator

A negative resistance is said to be present when an [illegible] [illegible] result of a decreased applied voltage. [illegible] [illegible] tions are to occur, [illegible] [illegible] tralize all the [illegible] [illegible] the circuit. Two [illegible] tive resistance oscillators are commonly used : (*i*) [illegible] and (*ii*) transitron oscillator.

Dynatron Oscillator. This makes use of the portion of the plate characteristics of a tetrode which displays negative resistance. Fig. 17·28 shows the plate characteristics of tetrode for a fixed value of screen grid voltage, say 100 volts. As the plate voltage is increased from zero, the plate current increases as normal. At a plate voltage e_{b1} reaches about 15 volts secondary emission from the plate begins. Screen grid voltage is then much higher than the plate voltage and hence secondary electrons get collected by the screen grid. The number of secondary electrons so lost by the plate may exceed the number of primary electrons so that the net plate current actually decreases with the increase of plate voltage accounting for the negative resistance property in the region *a* to *b* of the characteristic. Beyond plate voltage e_{b2} secondary electrons are

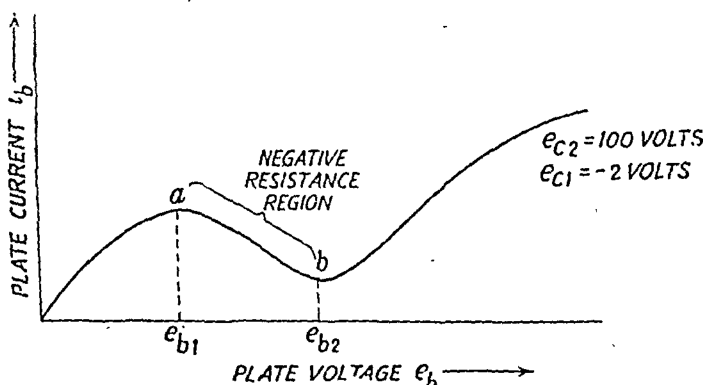


Fig. 17·28. Negative resistance characteristic of a tetrode.

drawn back to the plate because at this potential the electrostatic field of the anode is sufficiently large to hold them. This negative resistance portion *ab* of the tetrode is used in Dynatron oscillator, circuit of which is given in Fig. 17·29. The plate voltage is kept at a suitably low voltage with respect to the screen grid voltage. The tuned circuit is adjusted to resonate at the desired frequency of oscillation.

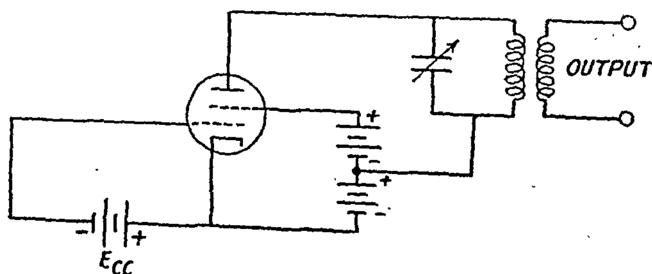


Fig. 17·29. Circuit of a dynatron oscillator.

Transitron Oscillator. This oscillator uses a pentode for obtaining the negative resistance. A pentode exhibits negative

resistance characteristic without using secondary emission. Thus if the plate potential
the suppressor gr,
cathode, electrostat
rons which would
screen grid. The
suppressor grid is made slightly less negative, some of the electrons formerly repelled by the suppressor grid may now get through to the anode thereby reducing the screen grid current. Thus an increase of suppressor grid v[illegible] increases the screen grid current. But to
constitute a nega[illegible]
current with incre
we may raise the
taneously by the
grid voltage rise is much greater than the influence of screen grid voltage so that there results a net reduction in screen grid current with increase in screen grid potential. This property of a pentode is used in the transitron oscillator of Fig. 17 30.

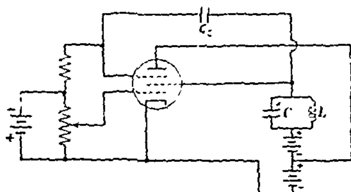


Fig. 17·30. Circuit of transitron oscillator.

Condenser C_c forms the coupling condenser It couples the variation of *screen grid voltage to suppressor grid*. *Frequency* of oscillation is governed by the tuned circuit in the screen grid circuit.

Other devices such as a transistor and thermistors are sometimes used to exhibit negative resistance and produce oscillations thereby.

Stabilization of frequency of feedback oscillators

The frequency of oscillation in a vacuum tube oscillator may vary due to *one of the following reasons* -

(i) *Change in valve characteristics*. Variations in interelectro[illegible] capacitances and tube characteristics are brought about by variatio[illegible] in the temperature and by variations in the electrode voltages.

(ii) *Change in circuit parameter* Inductance and capacitance the tuned circuit may change due to variation in tempera[illegible]

(iii) Change in loading of the tuned circuit. A change in the coupled load causes a change in the shunt resistance of the tuned circuit, with the consequent change in frequency.

For oscillations to take place, $A\beta=1$. If the frequency varies over a small range, then at all the frequencies, the amplitude $|A\beta|$ and the total phase shift in the feedback loop must have such values as to satisfy the above mentioned criterion for oscillation. Then the phase shift of the β network must be such as to keep the total phase shift of the circuit substantially constant. The frequency stability S_f of an oscillator is defined by the relation,

$$S_f=\omega_0 \frac{d\theta}{d\omega}=\frac{d\theta}{\frac{d\omega}{\omega_0}} \qquad \text{...(17·77)}$$

where ω_0 is the mean frequency of the oscillator, and $\frac{d\theta}{d\omega}$ is the rate of change of phase with frequency. More the value of S_f, more stable is the system. Ideally S_f should be infinite which gives completely frequency stable oscillator.

A number of means are adopted to improve the frequency stability of an oscillator. One method consists in use of such inductors and capacitors as have negligibly small temperature coefficients or use of such inductors and capacitors that any change in one parameter is exactly counteracted by an opposite change in the other parameter. Another measure includes the use of highly stabilized power supply.

In order to eliminate the changes in frequency due to variation of load impedance, an isolating or buffer amplifier is used between the oscillator and the load. The buffer amplifier draw negligible grid current and hence produces no loading of the oscillator. Usually the buffer amplifier also acts as a frequency multiplier. This system consisting of Master Oscillator and buffer amplifier is called a "Master Oscillator Power Amplifier" and is usually abbreviated as MOPA.

In addition to the above basic means adopted to reduce frequency variations, a number of additional measures are adopted to improve further the frequency stability of oscillators. One is the use of such compensating reactances as result in virtual isolation o the tuned circuit from the rest of the oscillator. Sometimes a quartz crystal is used instead of a tuned circuit to improve frequency stability. The crystal used is such as has a very small temperature coefficient of frequency. Further the crystal may be put in a thermostat chamber to reduce temperature variation.

Resistance stabilization of frequency. This method is commonly used to improve stability of frequency of oscillation of an oscillator. The method is due to Horton. It consists in insertion of a high resistance R_{st} in the plate circuit as shown in Fig. 17·31. The primary function of this resistance R_{st} is to make the total resistance of the plate circuit so high that changes in the plate

resistance of the tube have negligible effect on the frequency of oscillation. An additional function performed by this resistance R_{st} is to provide a convenient means of controlling the feedback and hence the amplitude of oscillation. Terman analysed this method of stabilization and found that for best stabilization, R_{st} must have

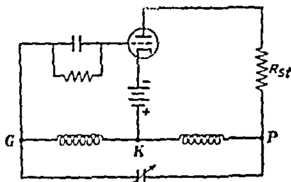


Fig. 17·31. Resistance stabilization of frequency of oscillation of an oscillator.

a value given by the relation,

$$R_{st}=Z_{pk}\,(\mu-1)-r_p \qquad \text{...(17·78)}$$

where Z_{pk} is the impedance of the tuned circuit between P and K looking from the anode side. This value of R_{st}, however, just blocks the oscillations. Hence in practice, R_{st} is kept about 10% below the value given by Eqn. (17·78) in the beginning but full value of R_{st} is put in as soon as oscillations have built up. Since this stabilization resistance R_{st} is put in the plate circuit, this method of resistance stabilization is called the *plate stabilization.*

In 1940, Tuttle suggested the use of stabilization resistance in the cathode lead instead of plate lead as shown in Fig. 17·32. Frequency stability is increased exceedingly by this means. This

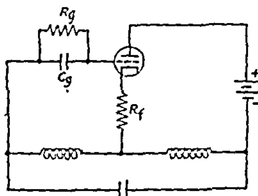


Fig. 17·32. Bridged-T Oscillator

oscillator is called the "Bridged-T Oscillator" because of the shape of the feedback circuit shown in Fig. 17·33.

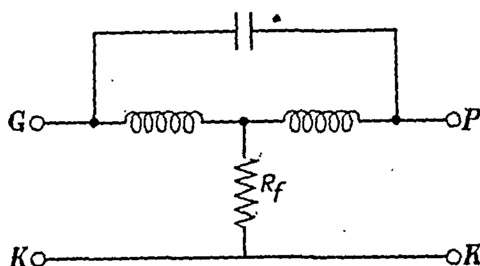


Fig. 17·33. Feedback network in Bridged-T oscillator of Fig. 17·32.

In 1943, Shepherd and Wise suggested the use of modified feedback network shown in Fig. 17·34. The oscillator using this feedback network is called "parallel-T oscillator" because of the specific configuration of the feedback network. This oscillator has excellent frequency stability. It has the additional advantage of

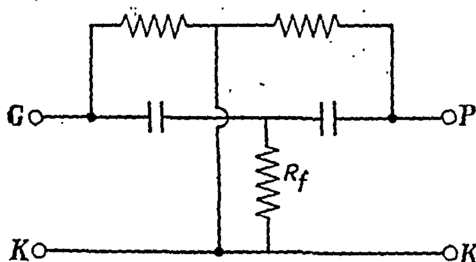


Fig. 17·34. Feedback network in parallel-T oscillator.

requiring no inductances and hence this oscillator is cheaper but reliable at the same time.

Relaxation Oscillator

A relaxation oscillator generates voltages or currents which very abruptly at one or more times in a cycle of oscillation.

In certain applications, relaxation oscillators have a number of advantages over sinusoidal oscillators. These are enumerated below :

(*i*) Number and amplitude of harmonics in the output.

(*ii*) The ease of synchronization of frequency of oscillation by introduction of small voltages of frequency close to the oscillator frequency.

(*iii*) Possibility of frequency multiplication and division, in a few types of relaxation oscillators.

(*iv*) Wide frequency range coverage with a single oscillator.

(*v*) Compactness, simplicity of construction and comparatively low cost.

These relaxation oscillators, like the feedback oscillators, have in general a feedback loop. In this case, however, the feedback potential is so chosen that the tube is driven beyond cutoff. Hence the output voltage is heavily distorted. *The important relaxation* oscillators are :

(a) Linear sweep oscillators or saw-tooth oscillators : These include neontimebase circuit, Puckle's Hard Valve Timebase circuit, thyratron Timebase and number of other similar sweep circuits.

(b) Multivibrator.

(c) Blocking oscillator.

In relaxation oscillators, the output waveforms are not

MULTIVIBRATOR

Multivibrator is a device which produces extended voltage waveforms of almost square shape and also voltage pulses occurring periodically.

Multivibrator may be used *for one of the following purposes :—*

(i) Generation of pulses occurring periodically

(ii) Generation of extended waveform.

(iii) Synchronized generation of pulses or extended waveforms.

(iv) For frequency multiplication.

(v) For introduction of time delay.

Classification of multiplicators. According to one method of classification, multivibrators may be classified as,

(*A*) *"Astable" or "Free-running" Multivibrator*—This may generate pulses and extended waveforms independently without the necessity of any driving or external synchronizing voltage pulse.

(*B*) *"Monstable" or "Single-shot" Multivibrator*—This requires one driving pulse for generation of each cycle of waveform

(*C*) *"Bistable" Multivibrator*—This requires two driving pulses one for each half cycle of the waveform.

Free Running Plate Coupled Multivibrator. It is essentially a two-stage R.C. coupled amplifier in which the output of the second stage is feedback to the input of the first stage. Fig. 17·35 shows the circuit diagram. Such a device will oscillate to produce periodic voltage waveforms because each tube introduces a phase shift of 180°. The circuit is redrawn in Fig. 17·36 in the form that is particularly convenient for tracing instantaneous voltage changes. This circuit represents a "symmetrical multivibrator" since both the tubes are similar and the components associated with each tube are equal in magnitude and are symmetrically placed.

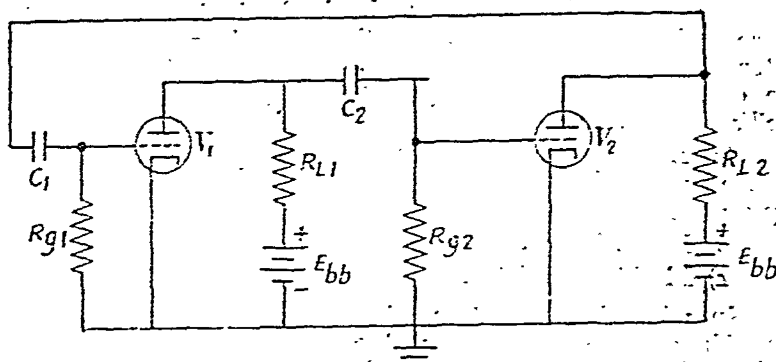


Fig. 17·35. Free running plate-coupled multivibrator.

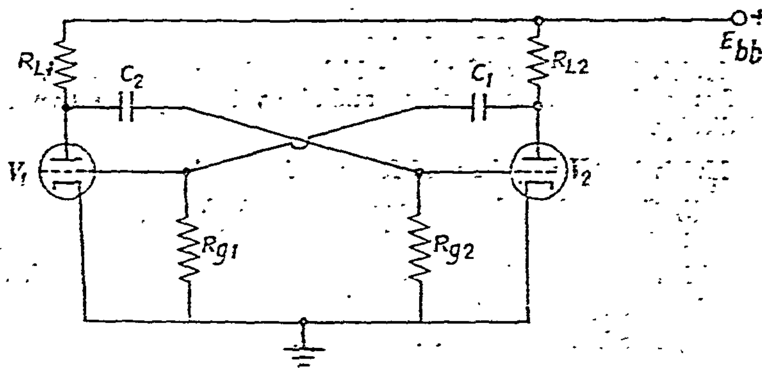


Fig. 17·36. Rearranged form of free-running plate coupled multivibrator.

The operation of this multivibrator is as follows : At any instant of time only one tube conducts while the other is biased beyond cutoff. Operation of the multivibrator can be understood with reference to voltage waveforms shown in Fig. 17·37. Let a small positive voltage appear at the grid of tube V_2. This voltage gets amplified by the two tubes and it reappears at the grid of tube V_2. This action is instantaneous and cumulative and is repeated a number of times with the end result that the grid potential e_{c2} of V_2 rises suddenly to zero, while the grid potential e_{c1} of tube V_1 suddenly drops to a value far below cutoff. Hence valve V_2 draws a heavy current. Its plate voltage drops by an amount E_o and hence grid voltage e_{c1} of V_1 drops by the same amount E_o. Valve V_1, therefore, gets cut off. This condition corresponds to time t_1 in waveforms of Fig. 17·37.

At the instant t_1, condenser C_1 gets charged to voltage E_o. But soon thereafter, it discharges through resistor R_{g1} and hence the potential e_{c1} of tube V_1 rises exponentially toward zero. After a time interval T_1 *i.e.* at time t_2, the grid voltage e_{c1} reaches the cutoff

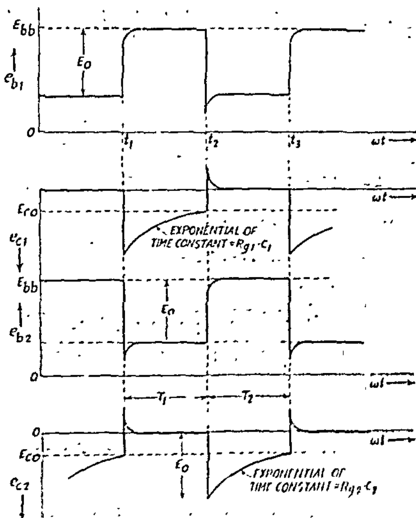


Fig. 17·27. Waveforms of grid and plate voltages in a free running plate-coupled symmetrical multivibrator

value E_{co} and tube V_1 begins to conduct. This causes the plate voltage e_{b1} of V_1 to fall and hence grid voltage e_{c2} of valve V_2 falls; plate voltage e_{b2} of V_2 rises and grid voltage e_{c1} rises. This is a cumulative action which results in e_{c1} suddenly becoming zero or even positive and e_{c2} highly negative. V_2 is cut off. Plate voltage e_{b1} falls by E_o and grid voltage e_{c2} of V_2 drops by the same amount E_o. Plate voltage e_{b2} of V_2 becomes E_{bb}. This situation corresponds to time t_1. Condenser C_2 gets charged to a voltage E_o. From there [illegible] C_2 discharges through R_{g2} and after a time interval T_1 [illegible] grid voltage e_{c2} reaches cutoff voltage E_{co} [illegible] cumulative action takes place resulting in full conduction [illegible] valve V_2. Plate voltage e_{b2} of V_2 drops by [illegible] by E_o to reach the value E_{bb}. Condition at [illegible] to that at time t_1

One cycle of operation corresponds to time interval from t_1 to t_3. Next cycle of operation is similar to this. Time interval T_1 is the period of nonconduction of tube V_1 and period of conduction of V_2 while time interval T_2 is the period of nonconduction of valve V_2 and period of conduction of V_1.

Time intervals T_1 and T_2 depend upon the voltages E_o and E_{co} and upon time constants $R_{g1}C_1$ and $R_{g2}C_2$. In a symmetrical multivibrator $R_{g1}=R_{g2}=R_g$, $C_1=C_2=C$ and $R_{l1}=R_{l2}=R_l$. Hence time intervals T_1 and T_2 are equal. The total periodic time T of the output voltage waveform is given by :—

$$T=T_1+T_2=(R_{g1}C_1+R_{g2}\ C_2)\ \log_e \frac{E_o}{E_{co}} \qquad \ldots(17\cdot79)$$

where cutoff voltage $E_{co}\approx\frac{E_{bb}}{\mu}$.

Hence $$T=(R_{g1}\ C_1+R_{g2}\ C_2)\ \log_e \frac{\mu E_o}{E_{bb}} \qquad \ldots(17\cdot79a)$$

Hence frequency of oscillation

$$f=\frac{1}{T}=\frac{1}{(R_{g1}C_1+R_{g2}C_2)\ \log_e \frac{\mu E_o}{E_{bb}}} \qquad \ldots(17\cdot80)$$

In accordance with Eqn. (17·80) the frequency of oscillation increases with the reduction of resistances R_{g1} and R_2 and capacitances C_1 and C_2. The output may be derived from any of the circuit elements. Except when the impedance of the load is very large, it is desirable to connect to load to the output of an amplifier which is coupled to the multivibrator. This avoids direct loading f the multivibrator and hence avoids the possibility of frequency variation due to loading.

The voltage drop E_o may be found from the tube characteristics and the load line for resistance R_l as shown in Fig. 17·38.

Fig. 17·38. Graphical construction for determination of voltage E_o.

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